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ELECTRIC POWER TRANSMISSION

ELECTRIC POWER TRANSMISSION

PRINCIPLES AND CALCULATIONS

INCLUDING A REVISION OF
"OVERHEAD ELECTRIC POWER TRANSMISSION"

BY

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PREFACE TO SECOND EDITION

When this book was published originally under the title "Overhead Electric Power Transmission," its suitability for use as a College text had not been seriously considered. It has, however, been used by a large number of technical schools and colleges, and while the restricted scope of the book may limit its suitability as a text for college students, the changes and additions which will be found in this new edition should, in the author's opinion, enhance its value as a college text without detracting from its usefulness in the field of practical engineering.

The principal addition—which has necessitated an alteration in the title—is an entire chapter treating of Underground Conductors. This has been written with the kind assistance of Mr. C. J. Beaver who has not only furnished most of the data relating to underground cables, but has also read and criticised the matter presented in Chapter VII.

A portion of the material which was originally contained in the Appendix has been incorporated in the text; but much of the first edition has been entirely omitted, either because its inclusion is no longer necessary owing to the rapid strides that have been made of late years in the general knowledge of electrical power transmission, or because it has been replaced by new material believed to be of more value to the student or practical engineer.

The chapter describing the Thury system of transmission by continuous currents has been retained with only slight changes and additions. It has not been deemed expedient to omit this entirely because, although few American engineers have taken the trouble to familiarize themselves with this system of transmission, there are conditions under which it has certain indisputable advantages which European engineers have not been slow to recognize.

The costs, both of material and labor, which are given in Chapter III, are not representative of market conditions on or about the date of publication of this book. Their principal use is to give an idea of the relative costs of different parts of a transmission system. They are based on trade conditions prevailing during the two or three years immediately preceding the war.

A. Still.

Purdue University,
La Fayette, Indiana,
June, 1919.

PREFACE TO FIRST EDITION

Although this book treats mainly of the fundamental principles and scientific laws which determine the correct design of overhead electric transmission lines, it has been written primarily to satisfy the needs of the practical engineer. An attempt has been made to give the reasons of things—to explain the derivation of practical methods and formulas—in the simplest possible terms: the use of higher mathematics has been avoided; but vector diagrams, supplemented where necessary with trigonometrical formulas, have been freely used for the solution of alternating-current problems. It is therefore hoped that the book may prove useful, not only to the practical designer of transmission lines, but also to those engineering students who may wish to specialize in the direction of Power Generation and Transmission, for these will find herein a practical application of the main theoretical principles underlying all Electrical Engineering.

The subject is treated less from the standpoint of the construction engineer in charge of the erection work, as of the office engineer whose duty it is to make the necessary calculations and draw up the specifications. The considerations and practical details of special interest to the engineer in charge of the work in the field have already been presented in admirable form by Mr. R. A. Lundquist in his book on Transmission Line Construction.

Much of what appears in these pages is reprinted with but little alteration from articles recently contributed by the writer to technical journals; but in the selection and co-ordination of this material, the scheme and purpose of the book have steadily been kept in mind.

Systems of distribution, whether in town or country, are not touched upon: the subjects dealt with cover only straight long-distance overhead transmission. It is true that, when treating of lightning protection, it is the machinery in the station buildings rather than the line itself that the various devices referred to are intended to protect; and, when considering the most economical

system of transmission under given circumstances, a thorough knowledge of the requirements and possibilities in the arrangement of generating and transforming stations is assumed; but these engineering aspects of a complete scheme of power development are not included in the scope of this book.

In the Appendix will be found reprints of some articles dealing with theoretic aspects of long-distance transmission which, although believed to be of interest to anyone engaged on the design of transmission lines, are not essential to the scheme of the book. In the Appendix will also be found complete specifications for a wood pole and steel tower line respectively: these should be helpful, not so much as models for other specifications —every engineer is at liberty to draw these up in his own way— but rather as containing suggestions and reminders that may be of service when specifying and ordering materials for an actual overhead transmission.

The writer desires to thank the editors of the following technical journals for permission to reprint articles or portions of articles which they have published from time to time: *Electrical World*, New York; *Electrical Times*, London; *Canadian Engineer*, Toronto; *Western Engineering*, San Francisco; *Journal of Electricity, Power, and Gas*, San Francisco.

Purdue University,
La Fayette, Indiana,
August, 1913.

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LIST OF SYMBOLS

A = area of cross section.
a = temperature-elongation coefficient.
a = percentage to cover annual interest and depreciation.
$a = \frac{d}{2r}$ in capacity formulas.
B = magnetic flux density (gauss).
B. & S. = Brown and Sharpe wire gauge.
b = barometric pressure: cm. or inches of mercury.
C = electrostatic capacity (farad).
C_m = electrostatic capacity; microfarads (usually per mile of conductor).
C_e = effective equivalent capacity, core to neutral, of underground cables.
D = flux density in electrostatic field (coulombs per sq. cm.).
D = butt diameter of wood pole.
d = distance between centers of parallel wires.
d = diameter of wood pole.
d_g = diameter of wood pole at ground level.
E = electromotive force (e.m.f.): difference of potential (volts).
E = voltage between lines at receiving end.
E_a = voltage, line to ground, on A. C. system (in comparison with D. C.).
E_n = voltage between wire and neutral (usually at receiving end).
E_o = disruptive critical voltage (corona)—(r.m.s. value of sine wave).
E_r = "economic" ohmic drop, in volts per mile of conductor.
E_v = visual critical voltage (corona).
e = e.m.f.—usually volts.
e.m.f. = electromotive force.
F = wind pressure: lb. per sq. ft.
f = frequency (number of periods per second).
G = potential gradient $= \frac{D}{Kk}$ (volts per centimeter).
G_v = potential gradient at which corona begins to appear.
H = height of pole or tower (feet or inches; refer to text).
H = intensity of magnetic field; magnetizing force (gilbert per centimeter; or gauss).
h = depth of footing below ground level (Art. 167)—feet.
h = difference in level between two points of support of overhead wire (feet).
hp. = horsepower.
I = current (amperes).

I = moment of inertia of pole section (Art. 151).
I_a = current in three-phase line (in comparison with D. C.).
I_c = capacity—or charging—current (amperes).
I_e = total line current at a given point when current varies on account of distributed capacity.
i = current—usually in amperes.
K = a numerical constant (in capacity formulas $K = 8.84 \times 10^{-14}$).
k = a numerical constant (steel tower design; Art. 164).
k = "skin effect" factor.
k = specific inductive capacity; dielectric constant; permittivity.
$k = \frac{w}{8A}$ = 1.5 times the wt. in lb. of a cubic inch of conductor material.
k.v.a. = kilovolt-amperes.
k.w. = kilowatts.
L = distance of transmission (miles).
L = inductance, or flux-linkages per unit current (henry).
$\mathbf{L}_i$ = internal inductance of straight conductor.
l = a length = length of dielectric flux line (cm.).
l = length of span—horizontal distance between supports (feet).
l = length of wire or cable (cm.).
l = length of wood strut and of compression members of steel towers (inch).
l' = straight line distance between supports of wire span.
$\left.\begin{matrix} l_A \\ l_B \end{matrix}\right\}$ refer to Fig. 93.
M = modulus of elasticity—ratio $\frac{\text{stress}}{\text{extension}}$.
(M) = circular mils per ampere.
m = ratio $\frac{\text{mutual capacity}}{\text{capacity to ground}}$ in formulas for suspension insulators.
(m) = circular mils.
$\left.\begin{matrix} m_o \\ m_v \end{matrix}\right\}$ correction factors in corona formulas.
m.m.f. = magnetomotive force (gilbert).
n = number of phases or conductors of a polyphase system.
n = number of units in a string of suspension insulators.
n = ratio $\frac{\text{resultant loading per foot of wire}}{\text{loading per foot due to wt. of wire only}}$.
P = power.
P = permeance.
P = pull; tension; force; (lb.).
P_h = horizontal component of total tension in suspended wire.
p = wind pressure per foot length of wire (lb.).
p = price of 100 lb. of transmission line conductor.
p_1 = cost per kilowatt-year of I^2R losses.
Q = quantity of electricity.
R = inside radius of metal cylinder surrounding a conductor.

R = ground-level radius of cone of earth to be lifted by tower leg (ft.).

R = resistance (ohm): Resistance of one conductor of a transmission line. Resistance per mile of one conductor.

R = reluctance (oersted).

R_a = non-inductive resistance in ground connection from lightning arrester.

R_a = resistance per mile of wire (three-phase, in comparison with D. C. system).

R_i = insulation resistance (megohms of one mile of cable).

R_p = joint resistance of all conductors of a transmission line connected in parallel.

r = radius or semi-diameter of cylindrical conductor.

r = least radius of gyration (inches).

S = stress (lb. per square inch).

S = vertical sag (feet).

S' = maximum deflection of wire from straight line joining points of support (line on an incline).

S_c = stress at "critical" temperature.

S_c = sag at "critical" temperature.

S_m = maximum stress.

T = temperature rise (degrees Centigrade).

t = temperature (degrees Centigrade or Fahrenheit; refer text).

t = interval of time (usually seconds).

t = constant defining taper of wood poles (Art. 147).

t_c = "critical" temperature—degrees Fahrenheit (sag-temperature calculations).

V = velocity of wind (miles per hour).

V = volume of frustrum of cone (cubic feet).

V = volts between lines at generating end.

V_i = reactive voltage drop due to "internal reactance" only.

V_n = volts between wire and neutral at generating end.

W = power (watts).

W = resultant pull on corner pole (lb.).

w = total I^2R loss in conductors of a transmission line.

w = weight per foot length of overhead wire (lb.).

w_r = resultant or total load on wire (lb. per foot).

X = reactance (ohms).

Z = impedance (ohms).

Z = section modulus; being ratio

$$\frac{\text{moment of inertia of section}}{\text{distance of center of gravity from edge of section}}.$$

Δ = current density (amperes per square inch).

δ = air density factor (corona formulas).

δ = deflection at top of pole or tower (inch).

θ = an angle: $\cos\theta$ = power factor of load (usually at receiving end of line).

θ = angle between direction of transmission line and horizontal line in the same vertical plane.

θ = angle of natural slope of earth.
λ = length of wire between two points of support (feet).
λ_c = length of wire at "critical temperature" in still air.
λ_e = change in length of overhead wire (feet).
μ = magnetic permeability = B/H.
π = 3.1416 (approximately).
ρ = resistivity, or specific resistance (megohms per centimeter cube).
Φ = magnetic flux (maxwell).
φ = an angle; usually the power factor angle at sending end of line.
$\cos \varphi$ = power factor of insulated cables on open circuit.
Ψ = dielectric flux (coulomb).
$\omega = 2\pi f$.

ELECTRIC POWER TRANSMISSION
PRINCIPLES AND CALCULATIONS

CHAPTER I

INTRODUCTORY AND GENERAL

Energy can be transmitted electrically by conductors placed either above or below ground. The cost of a system of underground insulated cables is always higher than that of an equivalent overhead transmission; but there are conditions, especially in Europe, under which overhead wires are not desirable or permissible, and the whole or a portion of the transmission line must then be placed underground.

The electrical transmission of energy over long distances at high pressures must necessarily be by overhead conductors, and even at the lower pressures—up to about 45,000 volts, beyond which underground cables would not be suitable—by far the greater number of transmission lines are overhead. It follows that this book is mainly concerned with the problems of overhead electric power transmission; but since underground cables may have to be used in certain sections of a proposed transmission scheme, their characteristics, uses, and limitations, will be discussed in Chapter VII.

An overhead electric power transmission line, consisting as it does of wires stretched between insulators on poles or structures the main purpose of which is to maintain the conductors at a proper distance above the ground level, may appear at first sight to be a very simple piece of engineering work. It is indeed true that the erection of an overhead line of moderate length, capable of giving good service on a comparatively low-pressure system, does not present any insurmountable difficulties to a man of ordinary engineering ability; but whether or not such a line will be the best possible line for the particular duty required of it,

depends very much upon the knowledge, skill, and experience of the designer. By the best line should be understood a line which is not only substantially and lastingly constructed, but in connection with which economic considerations have not been overlooked.

It is an easy matter to design a bridge of ample strength for the load it has to carry, or a transmission line with conductors of so large a size, insulators with so large a factor of safety, and supports so closely spaced and strong, that the electrical losses will be small and the risk of mechanical failure almost nil; but neither the bridge nor the transmission line will reflect credit on the designing engineer unless he has had before him constantly the commercial aspect of the work entrusted to him, and has so chosen or designed the various parts, and combined these in the completed whole, that all economic requirements are as nearly as possible fulfilled.

In the construction of electrical plant and machinery, such as generators, transformers, and switching apparatus, the economic conditions are, as it were, automatically fulfilled, owing to the competition between manufacturers, each one of which is a specialist in his own particular line of business. This competition, it should be observed, is not merely in the matter of works cost or selling price, but in works cost *plus* efficiency and durability. It is not necessarily the cheapest nor the most costly manufactured articles that wins in the long run, but the one which is commercially best suited to the needs of the user.

In the lay-out of power plants; in the development of natural power resources and the transmission of electric energy from water falls or coal fields to the industrial centers, the engineer, who may or may not be influenced by possibly conflicting financial interests, has much scope for the reckless and unwise expenditure of other people's money. He must resist this temptation—if temptation it be—and devote himself to the careful study of all engineering problems from the economic standpoint.

The cost per mile of a finished transmission, whether by overhead or underground conductors, is not all-important. It may frequently be said to be of importance only in so far as it influences the *annual cost* of the line, which annual cost is understood to include interest on the capital sum expended on the line. If a heavy section of copper is used for the conductors, the loss of energy in overcoming resistance will be less than with a lighter

section, but the initial cost will be greater: there is only one particular size of conductor which is economically the right size for any given line operating under definite conditions, and this is by no means easy to determine notwithstanding the apparent simplicity of what is usually referred to as Kelvin's Law.

Efficiency of service, which includes reasonably good voltage regulation and freedom from interruptions, must necessarily be merged into the all-important question of cost. By duplicating an overhead transmission line and providing two separate pole lines, preferably on different and widely separated rights of way, insurance is provided against interruption of service over an extended period of time; but whether or not such duplicate lines shall be erected must be decided on purely economic grounds.

Again, lightning arresters may be provided in abundance at frequent intervals along an overhead transmission line, and assuming—what is not necessarily the case—that such profusion of protective devices will prevent interruptions which are otherwise liable to occur through lightning disturbances, it does not follow that they should be installed. Examples of this kind can be cited to an almost unlimited extent, and, in Chapter III which deals especially with economics, an attempt will be made to indicate a mode of procedure in designing a transmission line from this, the only standpoint of importance to the engineer; but the question is a large one which cannot adequately be dealt with by set rules or formulas. In such cases as the design of supporting structures, when the calculations for strength have been made, it is the designer of the transmission line and not the manufacturer of the steel tower who shall decide upon the factor of safety to be used, for this is the prerogative of the man who is going to be held responsible for the commercial success of the undertaking. If he is incompetent or timid, he will allow too high a factor of safety, or follow blindly in the footsteps of others who may have been equally incompetent or timid. If he is sure of himself, and has carefully checked his calculations and deductions, he may depart from precedent and construct a line which is cheaper—not only in appearance, but in fact—that any line previously constructed under similar conditions and within the same limitations.

It is commonly supposed that wood poles with pin type insulators, spaced something less than 200 feet apart, should be used on small-power lines working at low or medium pressures;

yet there are conditions under which light steel "A" frames, or latticed poles, may with advantage replace wooden poles. In some cases taller steel structures with long spans might even prove to be the better construction from the economical standpoint. On the other hand, when a large amount of power has to be transmitted over long distances at high pressure—in which case the suspension type of insulator is almost a necessity—it does not follow that tall, rigid, wide-base, steel towers, spaced about eight to the mile, should necessarily be adopted. The so-called "flexible" steel structures, with somewhat closer spacing, may prove more desirable for several reasons apart from the costs of material and erection. Even wood poles may have advantages over steel structures, especially when the right kind of wood pole is readily obtainable, and when the cost of transporting the steel structures from factory to site is an important item of the total cost. The engineer should be slow to decide upon the kind of structure and average length of span best suited to the character of the country through which the line will be carried. As an illustration of the fact that no particular type of construction is suitable under all conditions, it is interesting to note that, of the important electric power lines in all parts of the world, transmitting energy at pressures exceeding 50,000 volts, the total mileage is about equally divided between the wood pole and steel tower types of construction. A good example of wood poles used for high-pressure power transmission is the 100,000-volt three-phase transmission line of the Montana Power Company, where each supporting structure consists of two 45-foot cedar poles connected at the top by a single cross-arm, but provided with no additional bracing or stiffening members between the poles. This line runs from Great Falls to Deer Lodge, Montana, a distance of 140 miles; it consists of three No. 4/0 copper conductors, and supplies power to the Chicago, Milwaukee & St. Paul Railway.

The climate and probable weather conditions will obviously have an important bearing on the safe span limit and mechanical design of the line generally. The effects of wind and ice will be referred to in Chapter IX.

A knowledge of the country through which an overhead transmission line is to be carried is essential to the proper design of the line and supporting structures. Without a knowledge of the natural obstacles to be reckoned with, including the direction

and probable force of wind storms, and whether or not these may occur at times when the wires are coated with ice, the nature of the supports and the economical length of span cannot properly be determined. On the Pacific coast, where there is rarely, if ever, an appreciable deposit of sleet on overhead conductors, it is possible that the spacing of supports may generally be greater than in countries where the climatic conditions are less favorable. At the same time, it has been observed, in districts where the winters are severe and sleet formation on conductors of frequent occurrence, that the effects of storms in winter on wires heavily weighted with ice, and offering a largely increased surface to the wind, are less severe than in summer when much higher wind velocities are sometimes attained. These examples are here mentioned to emphasize the necessity for a thorough investigation of local conditions before starting upon the detailed design of a proposed transmission line.

There are obviously many preliminary matters to be considered and dealt with before the actual details of design can be proceeded with; but, although many of these are partly, if not wholly, engineering problems, they cannot adequately be dealt with in the limits of this book, or indeed within the limits of any book, since the differences in local conditions, in the scope and commercial aim or end of a transmission system, makes it next to impossible to formulate rules or devise methods of procedure which can be of general utility.

Assuming that it is proposed to transmit energy electrically from a point where the power can be cheaply generated to an industrial or populous center where there is a demand for it, a straight line drawn on the map between these two points will indicate the route which, with possibly slight deviations to avoid great differences in ground level, would require the smallest amount of conductor material and the fewest poles or supporting structures. There may be natural obstacles to the construction of so straight a line, as for instance, lakes that cannot be spanned, or mountains that cannot be climbed; but even the shortest route which natural conditions would render possible is by no means necessarily the best one to adopt. The right of way for the whole or part of the proposed line may have to be purchased, and the cost will often depend upon the route selected. By making a detour which will add to the length of the line, it may be possible to avoid crossing privately owned lands where a high

annual payment may be demanded for the right to erect and maintain poles or towers. Again, by paralleling railroads or highways, the advantage of ease of access for construction and maintenance may outweigh the disadvantage of increased length. A slightly circuitous route may take the transmission line near to towns or districts where a demand for power may be expected in the near future; and such possibilities should be taken into account. The engineer in charge of the preliminary survey work (a section of transmission-line engineering which is not dealt with in this book) should bear all such points in mind and compare the possibilities of alternative routes. On a long and necessarily costly transmission line, it is rarely possible to spend too much time and though on the preliminary work. Money so spent is usually well spent, and will result in ultimate economies.

Coming now to the problems of a more strictly engineering nature, one of the first things to be decided upon is the system of electric transmission, whether it shall be by continuous currents with its simple two-wire circuit and ideal power factor, or by single- two- or three-phase alternating currents with manifold advantages in respect to pressure transformations and adaptability for use with commutatorless motors, but handicapped by low power factors and other complications due to the inductance and electrostatic capacity of the circuit.

Although nearly all long-distance transmissions—especially on the continent of America—are by three-phase currents, the other systems will be referred to briefly in the following chapter; and since, with the latest improvements in continuous current machinery, the series system of power transmission by continuous currents may, under favorable conditions, hold its own in this country as it does in Europe, an entire chapter will be devoted to a discussion of the points for and against the use of continuous currents on long-distance power transmission lines.

The question of underground cables *versus* overhead conductors does not give rise to so much discussion in America as it does in Europe. Apart from the limitation of voltage, which renders underground cables unsuitable for long-distance transmission, the cost of an underground system of conductors will generally be at least twice that of the equivalent overhead construction. The difference in cost is more noticeable in America where reasonable factors of safety in overhead construction have been adopted, than in Europe, and especially in England where

legislation has in the past been generally unfavorable to the development of electrical transmission. Factors of safety of 10 for wooden poles, 6 for steel structures, and 5 for overhead conductors, with a maximum wind pressure of 30 lb. per square foot, are not likely to encourage the construction of overhead lines; yet these figures are insisted upon by the British Board of Trade. A factor of safety of 5 for an *elastic* and moderately *flexible* system of overhead wires in a country rarely subject to severe sleet and wind storms, seems to have been selected on purely arbitrary grounds. So high a figure is especially objectionable in that it involves extra tall structures or uneconomically short spans, if the wires are to clear the ground at a reasonable height. In any case, overhead conductors hanging in festoons would not be considered good engineering practice in this country. From the artistic point of view, the unsightly appearance of innumerable poles and wires does not appreciably perturb the average American, whereas it may surely be said that a congestion of high- and low-voltage circuits as seen in Fig. 1,[1] would not be tolerated in the vicinity of the smallest hamlet in England. These reasons account for the more frequent use of high-tension underground cables in Europe than in America, notwithstanding the fact that interruptions due to lightning are far more liable to occur here than there.

The choice of system and determination of the most economical transmission voltage involve a knowledge of the cost and efficiency of generating and transforming machinery and controlling gear. It is obvious that a system of transmission that appears good owing to the low cost and high efficiency of the line itself, may yet be unsuitable and uneconomical because of the high cost or unsatisfactory nature of the machinery in the generating and receiving stations.

Apart from capital investment and power efficiency, a factor of the greatest importance, almost without exception, is efficiency and continuity of service. At the present time, the weakest link in a power system with long-distance transmission is probably the line itself. Electrical troubles may be due to faulty insulation, or they may have their origin in lightning or switching operations causing high frequency oscillations and abnormally high voltages, leading to fracture of insulators or breakdown of machinery. Troubles are more likely to be due to mechanical

[1] Photograph kindly supplied by Messrs. Archbold Brady and Co.

defects, or mechanical injuries sometimes difficult to foresee and guard against. Trees may fall across the line, landslides may occur and overturn supports, or severe floods may wash away pole foundations; and against such possibilities the engineer must, by the exercise of judgment and foresight, endeavor to protect his work. The width of the right of way should depend upon the height of trees, and be so wide that the tallest tree cannot fall across the wires; poles and towers should, if possible, be kept away from the sides of steep hills where the nature of the ground suggests the possibility of falling stones or of landslides; and, in regard to floods, the inhabitants of the districts through which the line passes are usually able to furnish information of use in indicating where trouble from this cause may be expected. Other causes of mechanical failure are storms of exceptional violence, either with or without a heavy coating of ice on the conductors. When strong winds blow across ice-coated wires, the danger is not only that the wires themselves may break, but also that the resulting horizontal loading of the poles or towers may be great enough to break or overturn them.

Faulty mechanical design of the line as a whole, and improper supervision or inspection during construction, will account for many preventable interruptions to service. The transmission line considered as a mechanical structure will be dealt with at some length in Chapters IX and X, and the sample specifications for a wood pole and steel tower line respectively (see Appendices II and III), together with detailed material schedules with approximate costs, given in Chapter III, will cover some practical details which should be helpful when designing a transmission line to operate under generally similar conditions.

Although the electrical and mechanical qualities of insulators and conductor materials are necessarily somewhat dependent upon each other, an attempt will be made to deal almost exclusively with electrical calculations and the electrical characteristics of transmission lines in the following chapter, and again in Chapters IV, V, and VI. The chapter treating particularly of the economic aspect of transmission-line design will follow immediately after Chapter II, because it is well to determine provisionally the system and voltage likely to be most suitable for a given scheme, before entering into the more detailed calculations of line losses and regulation, and considering the practical requirements in matters of insulation and lightning protection.

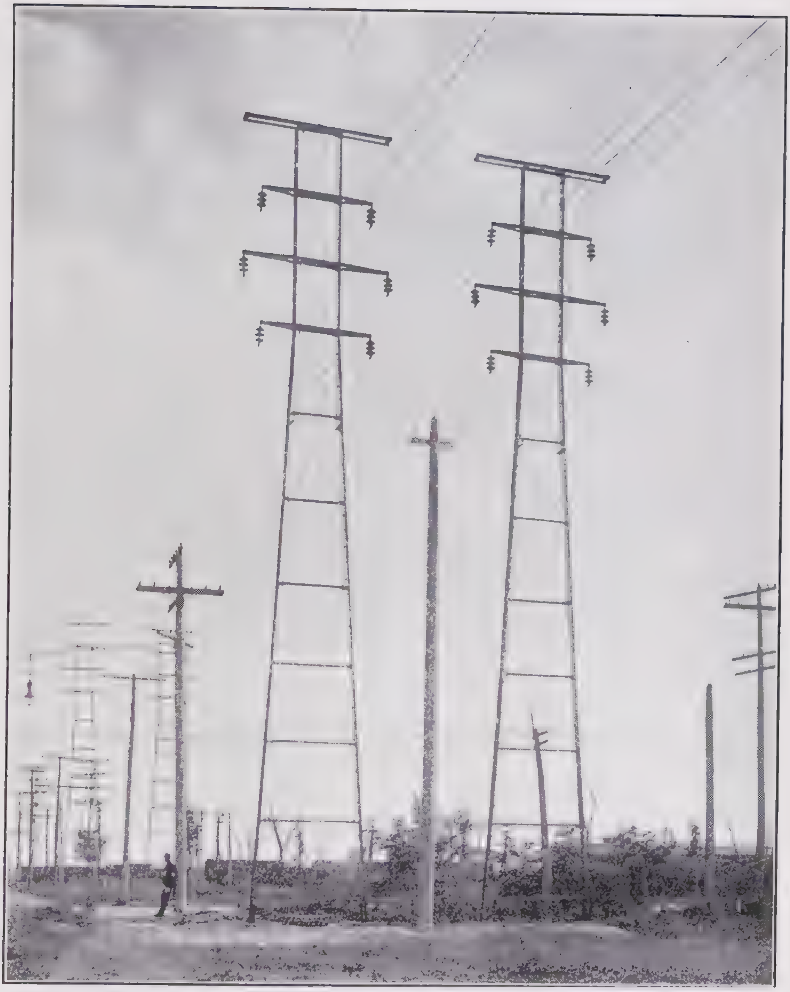


FIG. 1.—Overhead wires, steel towers, and poles.

(*Facing Page* 8)

Before closing this chapter it may be well to refer to a few matters of general interest that are not dealt with in succeeding chapters.

Except for the sample specifications in the Appendix, previously referred to, no attempt has been made to describe complete transmission lines, with details of construction and operation; but the more purely practical details of construction have already been admirably presented by Mr. R. A. Lundquist in his book on Transmission Line Construction,[1] to which the reader is referred. Descriptions of underground cable systems, together with a complete presentation of the more practical aspects of energy transmission by underground cables, will be found in Mr. E. B. Meyer's book on Underground Transmission and Distribution.[1]

In the matter of crossing highways or railroads with high-tension conductors, the engineer will usually have to abide by the rules and regulations of the local authority or railroad company, however unreasonable or unnecessary the particular requirements may be. The modern tendency is to avoid a multiplicity of devices intended to catch falling wires or ground them in the event of a breakage, and to rely mainly on short spans and exceptionally sound mechanical construction at places where the falling of charged wires would be a menace to life, and cause an interruption to traffic.

Reverting again to the all-important matter of uninterrupted service, it is obvious that reasonable provision should be made for the early resumption of service in the event of stoppage. When two parrallel lines are run all the way between generating and receiving stations, it is usual to provide section switches and by-pass connections about every 20 miles, at the points where patrolmen are stationed. Even if the line is not duplicated, it is usually wise, on important systems, to have patrolmen's houses every 15 or 20 miles, depending on the character of the country. Emergency houses, containing tools and sundry line materials such as wire and spare insulators, should be provided midway between the patrol houses.

For communication between generating station and patrolmen and substations, a telephone circuit is almost essential. This can be run on the same poles or towers as the high-tension conductors; but, if possible, it should be run on separate poles. The extra

[1] McGraw-Hill Book Co.

cost of a separate pole line for telephone wires is, however, the reason for these wires being frequently supported on the same poles as the transmission wires. This leads to trouble in the case of a ground on the high-tension wires and, indeed, almost invariably when there is a fault on the power system, except, of course, when the power service is entirely interrupted. Even when carried on a separate pole line, the telephone circuit is liable to be useless at times when it is most needed, and for this reason it is not unusual, on important lines, to provide telegraphic instruments in addition to the telephones, and men that are telegraph operators at the ends of the line and also at any intermediate points where switching stations may be provided.

Closely related to the matter of continuous and efficient service is the question of duplication of lines, already referred to. This has to be decided mainly on economic grounds; but, at the same time, the purely engineering difficulties may become very serious if an attempt is made to transmit very large amounts of power on a single set of conductors. It is not possible to lay down definite rules as to the practical limit for a single line; this will depend on the distance and voltage, and therefore on the current to be carried per conductor; but if the power to be transmitted exceeds 30,000 kw., it would generally be wise to duplicate the conductors, even if carried on the same set of supporting towers; and if the total power exceeds 60,000 kw., two separate tower lines, either on the same right-of-way or (preferably) following different routes, will, in most cases, lead to ultimate economy.

Among recently established long-distance high-voltage transmission systems, the Big Creek to Los Angeles line of the Pacific Light and Power Co. is one of the most important. It is designed for an operating pressure of 150,000 volts and an ultimate capacity of 300,000 kilowatts. At present there are two steel-tower three-phase lines, running parallel over a total distance of 240 miles, with an average distance of about 660 feet between towers. The line was put into operation in 1913. Other important lines are those of the Au Sable Electric Co. from Battle Creek to Au Sable, Mich. (245 miles), and of the Southern Sierras Power Co. (238 miles), both operating at 140,000 volts. The latter which was put into operation in 1915, departs somewhat from previous practice in that it includes many outdoor substations.

The 60,000-volt system of the Texas Power and Light Co. is also noticeable for its outdoor substations, which are used exclusively throughout the system. The present-day tendency appears to be toward the more frequent use of outdoor substations where not only the section disconnecting switches, but also transformers, lightning arresters, and indeed all high-tension equipment, are exposed to the weather. The control of local feeder circuits at the lower pressures would be by means of switch-panels in cheaply constructed houses erected near the transformers.

It is well to bear in mind that the introduction of automatic switches and similar devices designed to save labor and ensure the rapid changing over of the load from a faulty section to a sound section on a duplicated transmission line, is liable to lead to unlooked-for troubles; and even the generous provision of lightning arresters, especially on the extra-high-tension lines, is not necessarily good policy. Simplicity, and the avoidance of unnecessary joints, rubbing contacts (as in switches or cut-outs), fuses in the stations, and spark gaps or arresters along the line, should generally be aimed at; but there will always be exceptions to such rules.

Careful design in the matters of material, size and spacing of conductors, and in methods of support and insulation, together with scientific selection and lay-out of poles or towers, will lead to the construction of transmission lines which may ultimately prove to be the strongest instead of the weakest links in power transmission schemes; and this without the addition of more or less complicated and unreliable automatic and so-called protective devices, and at a cost which will make long-distance power transmission propositions more attractive from the stockholder's point of view than they have been in the past.

There has been too much speculation and too little business connected with many of the existing power transmission undertakings, especially where water power is available. The almost ideal conditions at Niagara, due to an enormous amount of energy being available in the proximity of industrial cities and centers of population, are probably unique. There are few, if any, other water power sites in the world where the conditions are so favorable. If competent and honest engineers were always employed by promoters of water power developments, for electrical transmission purposes, and if the reports or advice of such experts were

acted upon, there would be fewer undertakings carried out in the wrong manner. In determining the magnitude of any proposed development, and the distance beyond which it would be uneconomical to transmit the energy, a valuation should be made on the basis of the kilowatt-hours per year available *for which there is, or is likely to be, a market.* This involves experience, together with business and engineering judgment of a high order, on the part of those responsible for the capital invested; and although there are many water power sites—especially with comparatively low heads—which still await development, the engineering problems in connection therewith are never so simple as to render unnecessary a great deal of investigation and careful thought before the nature of the development and the best manner of carrying out the work can be properly determined.

CHAPTER II

ELECTRICAL PRINCIPLES AND THEORY—ELEMENTARY

The purpose of an electric transmission line is to transmit energy from one place to another; and it is the engineer's business to design and construct such a line to fulfil its purpose in the best and most economical manner.

The system to be adopted will affect the design of the generating plant and of the motors or other devices through which the electric energy at the receiving end of the line is converted for industrial purposes or public utility; but, in this chapter, references to alternative systems will be made only for the purpose of comparing them in the matter of line efficiency.

1. Losses in Transmission.—The principal cause of loss of power in a transmission line is the resistance of the conductors. For a given section of conductor, the power dissipated in the form of heat, in overcoming the ohmic resistance, is proportional to the square of the current. A definite amount of power can therefore be transmitted with less loss when the voltage is high than when it is low; but, on each particular transmission, there is a limit to the pressure beyond which there is nothing to be gained in the ma ter of economy. This limit is determined by the cost of generating and transforming apparatus (which will be greater for the higher voltages), by the greater cost of insulators and of the line generally—owing to the larger spacing required between wires—and also, when extra high pressures are reached, by the fact that the power dissipated is no longer confined to the I^2R losses in the conductors, but occurs also in the form of leakage current over insulators, and in the air surrounding the conductors. The means of calculating the dielectric losses will be explained in Chapter V, when treating of corona formation; and a method for determining the best voltage of transmission under any given conditions, will be outlined in Chapter III, because this is essentially an economic problem. For the present it is assumed that a total amount of power amounting to W watts has to be transmitted over conductors of known resistance; and losses through leakage or corona will be considered negligible.

2. Transmission by Continuous Currents.—If E is the voltage between the outward and return wires at generating end, the current is

$$I = \frac{W}{E}$$

and the losses in transmission are,

$$2R \times I^2$$

where R is the resistance of one conductor only. The loss of pressure per wire is

$$R \times I$$

The fact that continuous currents are not extensively used for the transmission of power to a distance is due mainly to the difficulty of providing sufficiently high pressures to render such transmission economical, and also to the necessity for using rotary machines with commutators to convert the transmitted energy into convenient form at the distant end of the line. The modern aspect of long-distance transmission by means of continuous currents will, however, be dealt with at some length in Chapter VIII.

3. Transmission by Single-phase Alternating Currents.—The advantage that alternating currents have over direct currents is in the ease with which pressure transformations can be effected be means of static converters. On a constant-potential system, the distribution of power in scattered districts, at any voltage desired by the consumer, is a very simple matter.

In a single-phase two-wire transmission, the conditions would be similar to those of a direct-current transmission if not only the load, but the line also, could be considered as being without inductance or electrostatic capacity. The current and the line losses would then be the same as if the transmission were by continuous instead of alternating currents.

In practice the inductance must always be reckoned with where alternating currents are used; this inductance is not only that introduced by the load (usually consisting in large part of induction motors), but is partly in the line itself, owing to the loop formed by the outward and return conductors. The charging current due to the capacity of the line is of less account on low-voltage transmissions, but becomes of considerable importance on long lines working at high pressures. The effects of inductance and capacity will be explained later.

Another difference between alternating and continuous currents is the fact that an alternating current has the effect of apparently increasing the resistance of the conductor; this is due to the uneven distribution of the current over the cross-section of the conductor. A small percentage of the alternating flux of induction is in the material of the conductor itself, and this generates counter e.m.fs. which are somewhat greater near the center of the wire than at the circumference, the result being that the current density becomes greater near the surface of the wire than in the center portions. This phenomenon is known as the *skin effect.* The additional resistance offered to the passage of alternating currents, and the correspondingly increased I^2R losses are, however, small and generally of negligible amount on low frequencies, unless the cross-section of the conductor is very large; it is with the higher frequencies that this effect becomes of importance. Means of calculating increase of resistance due to skin effect will be given in Chapter IV.

4. Transmission by Two-phase Currents.—If four separate wires are run from generating to receiving station, as indicated

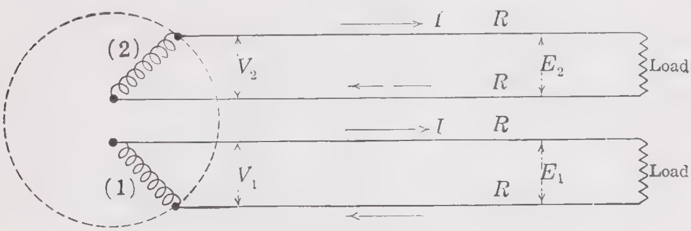


FIG. 2.—Two-phase transmission with four wires.

in Fig. 2, and if the load is the same on both circuits, the total power transmitted, on the assumption of negligible inductance, is

$$W = 2EI$$

where E stands for the terminal voltage E_1 or E_2 of either section at the receiving end; these pressures being assumed equal. The pressure lost in transmission is $2R \times I$, and the watts lost are $2(2R \times I^2)$; the system being simply a transmission of energy by means of two independent single-phase circuits. It will be seen, however, that, by combining two of the conductors to form a common return path for the current, the transmission of two-phase currents can be effected with only three wires, as indicated

in Fig. 3. The vector diagram for such a system of transmission is easy to construct *if it is permissible to assume the resistance R′ of the common conductor to be negligible;* the current relations, with a quarter period time displacement between the currents in the two phases, being as indicated in Fig. 4. The current I_3 in the

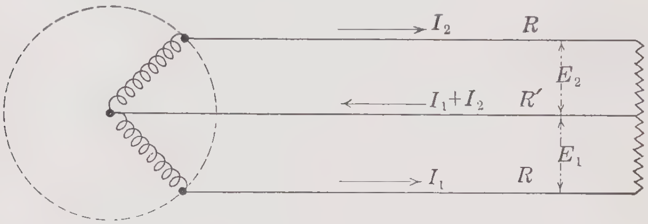


FIG. 3.—Two-phase transmission with three wires.

common conductor is the vectorial sum of the currents I_1 and I_2. It has been drawn equal but opposite to the dotted resultant OA because it is generally convenient to assume the direction of all currents to be positive when flowing away from the source of supply, in which case the condition

$$I_1 + I_2 + I_3 = O$$

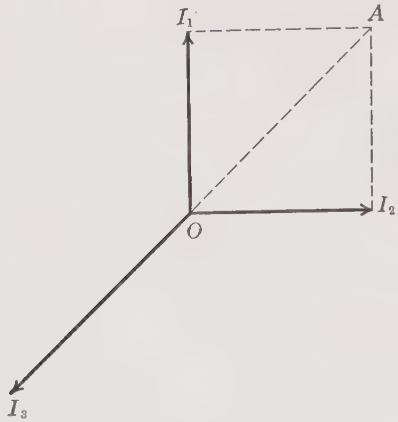


FIG. 4.—Vector diagram of currents in two-phase three-wire transmission.

must be satisfied. The arrow on the central conductor in Fig. 3 indicates a flow of current *opposite* to the currents in the other two wires; but this is done merely to suggest the idea of currents going away from the source of supply by the two outer wires, and *returning* by the common wire. When dealing with poly-

phase currents, such mental pictures of the actual physical occurrences are liable to be misleading, and they should be used sparingly.

On a long transmission, or in cases where the resistance of the common conductor cannot be neglected, the problem of two-phase transmission by means of only three wires becomes complicated. The resistance of the common wire has the effect of disturbing the phase relations, which are no longer the same at the receiving end as at the generating end of the line. It is difficult to explain in simple terms exactly how this occurs; and since very few systems of transmission are by means of two-phase currents it has not been thought necessary to take up space in the study of this peculiarity. Those interested in the question will find a complete discussion of the problem of two-phase transmission by means of three conductors in Appendix C of the first edition of this book. On the few long-distance transmissions by two-phase currents, that are operating successfully, four conductors are used. The advantage of three wires over four is, of course, in the saving of cost on the conductors. Instead of requiring four wires, each carrying I amperes, it is only necessary to provide two wires to carry I amperes and one wire of a cross-section sufficient to carry, not $2I$ amperes, but $\sqrt{I^2 + I^2}$ or $I \times \sqrt{2}$ amperes; which leads to an appreciable saving in the total weight of conductors, if the current density is the same in all.

5. Transmission by Three-phase Currents.—If six separate conductors are run from generating to receiving station, as indicated in Fig. 5, the transmission is equivalent to three independent single-phase two-wire circuits; and if E_n is the potential difference at the terminals of each circuit, and I the current in each wire, the total power transmitted will be

$$W = 3(E_n \times I)$$

the assumption being, as in previous cases, that both inductance and capacity are of negligible amount.

The pressure lost in transmission will be $2R \times I$ and the total power lost in the three lines will be $3 \times I^2 \times 2R$.

Consider now the arrangement as in Fig. 6, where the three circuits have a common terminal at each end of the transmission and three of the wires of the six-wire transmission are replaced by a common return conductor. The pressure at the receiving

end, between each of the three terminals and the common return, or neutral point, is still E_n volts; and the total power transmitted is still $W = 3(E_n \times I)$; but, owing to the fact that the sum of the three outgoing currents is zero (since they differ in phase by 120 time degrees, as shown in Fig. 7, and any one current, such as OB, is exactly equal and opposite to the resultant of the other

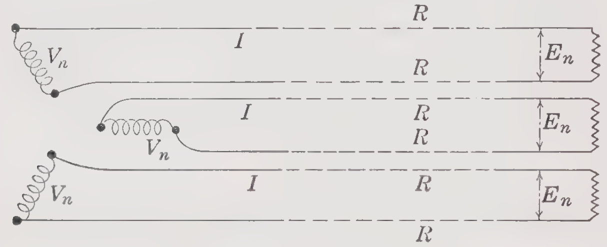


FIG. 5.—Three-phase transmission with six wires.

two currents), there will be no current flowing in the common return conductor, which can therefore be omitted; and it follows that both pressure drop and I^2R losses in the lines are reduced to *one-half* of what they were with the arrangement of three separate circuits; the power loss in the lines being now $3I^2R$. This clearly shows how the transmission by three-phase currents is more economical as regards line losses than single-phase trans-

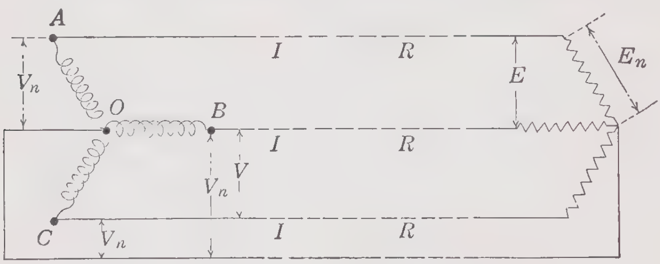


FIG. 6.—Three-phase transmission with three wires.

mission. But it must not be overlooked that, in order to obtain a reduction by half of the weight of copper in the lines, the pressure between the wires is greater on a three-phase system than on a single-phase system transmitting the same amount of power. Thus, the pressure, V (Fig. 6), between any two of the three transmission wires is the *difference* between two of the star voltages, as indicated in Fig. 8. Here the e.m.fs. in the three sections

of the alternator windings are represented by the vectors OA, OB, and OC; and since the e.m.f., V, between any two terminals, such as B and C (Fig. 6), is the resultant of the e.m.fs. acting in the two windings OB and OC connected in series, one of these (as OC) must be subtracted from the other (OB). Thus the resultant is the vector OV (Fig. 8), obtained by *adding* to the vector OB an imaginary vector, OC', exactly equal, but opposite, to OC. This resultant is evidently equal and parallel to the line CB, joining the ends of the two vectors OB and OC, and since the

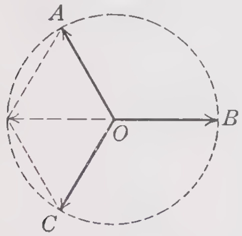


FIG. 7.—Vector diagram of currents in three-phase transmission.

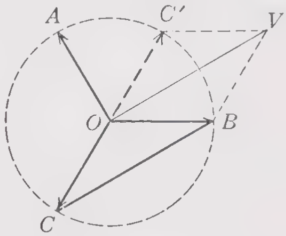


FIG. 8.—Vector diagram of e.m.fs. in three-phase star-connected system.

angle $C'OV$ is 30 degrees, the length OV is equal to $2OC \cos 30°$, or $\sqrt{3}$ times the length of any one of the vectors representing e.m.fs. between conductor and neutral point. Thus,

$$V = 1.732V_n$$

and since the resistance drop is the same in all three conductors, a similar condition exists at the receiving end of the line, and we may also write,

$$E = 1.732E_n \qquad (1)$$

The *power* of a three-phase circuit, which is three times $E_n \times I$, can evidently alsc be written

$$W = 3\left(\frac{E}{\sqrt{3}} \times I\right)$$

or,

$$W = \sqrt{3}EI \qquad (2)$$

where E is the pressure between any two of the three wires.[1]

6. Relative Cost of Conductors Required on the Various Systems.—Apart from all questions of voltage, or necessary insulation and spacing required between adjacent conductors

[1] The *power factor* ($\cos \theta$) does not appear in this formula because, owing to the assumed absence of inductance or capacity from both line and load, it is equal to unity.

and between the conductors and the supporting structures, the total I^2R losses will be the sum of the losses occurring in each conductor of the transmission system. Each wire may be considered as the outgoing conductor of a two-wire single-phase system in which the return wire has no resistance. Thus, in a balanced three-phase system as illustrated in Fig. 6, wherein the common return wire is not required (since it carries no current), the total losses in the transmission wires are

$$3(I^2 \times R)$$

But this may be written,

$$(3I)^2 \times \frac{R}{3}$$

which shows that the total losses can be calculated by adding the currents in the respective conductors regardless of phase relations, and considering this total current as being transmitted over a single wire of the same weight or cross-section as would be obtained by connecting the individual conductors in parallel. This applies to any polyphase system with wires of equal resistance carrying equal amounts of current.

When comparing different systems of transmission, it is necessary to make some assumptions in regard to the voltage, so as to have a common basis of comparison. For instance, if it is desired to compare three-phase and single-phase transmission on the basis of *the same potential difference between wires*, apart from any question of voltage between wires and ground, the total power on the three-phase system will be

$$W = \sqrt{3}E \times I$$

and the (equal) power on the single-phase system would be written,

$$W = E \times (\sqrt{3}I)$$

Let R_3 = the resistance of each conductor on the three-phase system, and

R_1 = the resistance of each conductor on the single-phase system,

then, for equal total line losses,

$$3I^2R_3 = 2(\sqrt{3}I)^2R_1$$

whence

$$R_3 = 2R_1$$

Since the weight of copper in each wire of either system is inversely proportional to the resistance, it follows that,

$$\frac{\text{Weight of copper, single-phase}}{\text{Weight of copper, three-phase}} = \frac{2R_3}{3R_1}$$

$$= \frac{2(2R_1)}{3R_1}$$

$$= \frac{4}{3}$$

which indicates a saving of 25 per cent. of conductor material in favor of the three-phase system.

Consider now the condition of the various systems on the basis of the same efficiency (as in the above example), but on the

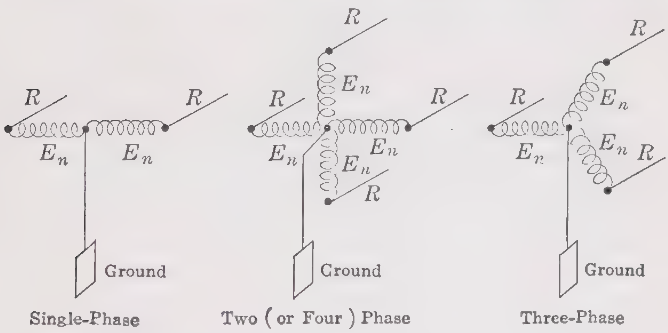


FIG. 9.—Different systems with same potential above ground.

further assumption that the potential difference between the earth or supporting structures and any one of the conductors is constant. This is equivalent to stating that the pressure stress at every point of support, where an insulator carries the conductor, is the same on all systems. This is shown diagrammatically in Fig. 9, where the voltage E_n *per phase* is the same in all systems.

If I is the current per wire, and n the number of wires (or phases), the total power transmitted is

$$W = E_n I \times n$$

provided the power factor is unity. On the assumption of a balanced load, with the current lagging behind the voltage by the same number of time-degrees on each phase of all the systems, no complication will arise if the power factor of the load is taken

into account. The total power transmitted, in every case, may, therefore, be written

$$W = E_n I \cos\theta \times n \qquad (3)$$

If R is the resistance of each line conductor, the total line loss for any system will be

$$w = I^2 R \times n$$

and, for the same line efficiency, the weight of copper per kilowatt transmitted will evidently be the same in all cases.

This leads to the conclusion that, *for any balanced polyphase system,* the power lost in the line depends only upon the joint resistance of the conductors, the power transmitted, and the power factor, *provided the pressure between the conductors and the neutral point is constant.*

Still neglecting the inductance and capacity of the line itself, the *percentage power lost in transmission is*

$$\frac{w}{W} \times 100$$

If the loss w be expressed in terms of the total power W, it will be found that this ratio can be put in an interesting form. The symbol R_p will be used to denote the joint resistance of all the conductors in parallel; that is to say,

$$R_p = \frac{R}{n}$$

The power lost is

$$\begin{aligned} w &= nI^2R \\ &= n^2 I^2 R_p \qquad (4) \end{aligned}$$

but for n^2 may be substituted its equivalent value

$$n^2 = \frac{W^2}{E_n{}^2 I^2 (\cos\theta)^2}$$

obtained from equation (3), whence (4) becomes,

$$w = \frac{W^2 R_p}{E_n{}^2 (\cos\theta)^2}$$

which shows how, for any given amount of power transmitted at a given pressure, the I^2R loss is directly proportional to the joint resistance of all the conductors, and inversely proportional to the square of the power factor of the load.

By substituting this value for w in the ratio for percentage efficiency, the latter quantity becomes,

$$\left.\begin{array}{c}\text{Percentage power lost in } \textit{any} \\ \text{balanced polyphase system}\end{array}\right\} = \frac{WR_p}{E_n{}^2 \cos^2\theta} \times 100 \qquad (5)$$

These formulas show very clearly the advantages of high power factors where economy of transmission is important.

7. Grounding the Neutral on High-tension Overhead Transmissions.—The above comparisons of losses have been made on the assumption of a grounded neutral, but whether this common terminal of the polyphase circuit is grounded or not does not alter the fact that, *under normal conditions of working*, the neutral point is usually at about the same potential as the ground. Further, whether the generator or transformer windings on the high-tension side of a three-phase transmission system are delta connected or star connected is of very little importance if it is decided to operate without a grounded point. The fact that the neutral point on a star-connected system is available for grounding purposes does not mean that it must necessarily be grounded.

The chief arguments in favor of grounding the neutral are (1) that the difference of potential between any conductor and the supporting structure or earth remains unaltered, and cannot become excessive in the event of the grounding of a high-tension conductor, and (2) that it is possible to detect instantly, and disconnect by automatic devices or otherwise, any portion of the system that may become accidentally grounded. The chief objection is that under such conditions, the grounding of any one conductor causes a short-circuit, and even if disconnected by the opening of a switch, leads to an interruption of supply. By inserting a resistance between the neutral and the ground connection, the current through the fault can be limited to just so large an amount as may be necessary to operate an automatic device, or give an indication that there is a fault on the line. Instead of opening the switches and disconnecting the line, the ground connection to neutral may be opened, thus leaving the conductor grounded until such time as it is convenient to carry out repairs; but this would be equivalent to running normally without grounded neutral. The chief advantage of transmitting with ungrounded neutral is that the grounding of one conductor only does not lead to immediate interruption of service. The chief disadvantage is probably the fact that the potential between earth and the other conductors is immediately raised; being $\sqrt{3}$ times greater, in the case of a three-phase transmission, than under normal conditions, when the voltages are balanced.

It is doubtful whether the question of grounding the neutral

on a high-tension transmission can be so settled as to be applicable to all systems and voltages. A few years ago there was an undoubted tendency on the part of engineers to ground all metal that would under normal conditions be at ground potential At the present time the tendency appears to be in the direction of providing substantial insulation throughout the system, and omitting the grounding of the neutral. In an age when the individual appears to distrust the conclusions of his own intellect, there would appear to be much wisdom in the advice once given by Dr. Steinmetz, who suggested that if the engineer is in doubt as to the better course to pursue when two alternatives present themselves, he should *not* follow the one most favored by his fellow engineers, because in so doing he would in all probability merely adopt what happens to be the fad of the day. It is perhaps best practice to avoid grounding any point on a high-tension transmission unless the conditions are such that the grounding of the neutral point would appear to be the obvious remedy for troubles that may have been experienced or that are liable to occur.

Regulation—Effect of Line Inductance on the Transmission of Alternating Currents.—On account of the necessary space between the wires, the loops formed between outgoing and return conductors are of considerable area on a long-distance transmission; and the changing flux of induction in these loops will generate counter e.m.f.'s in the conductors, which may be of considerable importance, especially in regard to their effect on the voltage regulation. Whether dealing with single-phase or polyphase transmissions, it will be found convenient to make calculations on single conductors only. Thus, instead of considering the resistance of the *complete circuit* (which is not convenient in the case of polyphase transmissions), the resistance of one conductor only, or the resistance *per mile* of single conductor is considered, and the ohmic voltage drop calculated for that portion of the complete circuit only. Similarly, in the matter of the counter e.m.f. due to the self-induction of the line, calculations are based, not on the total flux of induction in the loop or loops formed by outward-going and return wires, but on that portion of the total flux which is included between the center line of any one conductor and the *neutral* plane or line. Thus the induced volts per single conductor, or per mile of single conductor, can be calculated, and the resulting total voltage drop can be

computed for each conductor independently of the others. In the case of a single-phase two-wire transmission, the total loss of pressure is evidently just twice the amount so arrived at for a single conductor. In a polyphase transmission, due attention has to be paid to the phase relations between the currents in the various conductors; but the same principle holds good, and calculations of any polyphase transmission can be made by considering each conductor separately, as will be explained later.

The induced volts will be directly proportional to the current, and will depend on the diameter of the wire and its distance from the return conductors. This will be again referred to in Chapter IV, but for the present the induced pressure may be calculated by means of the following formula:

$$\left.\begin{array}{l}\text{Volts induced per mile}\\ \text{of single conductor}\end{array}\right\} = 0.00466 \times f \times I \times \log_{10}\left(1.285\frac{d}{r}\right) \quad (6)$$

where d and r stand respectively for the distance between outward and return (parallel) conductors and the radius or half diameter of the wire; these being expressed in the same units. The frequency f is expressed in cycles per second, and the current I, in amperes. In nearly all pocket books or hand books for the use of electrical engineers, tables are published giving inductive pressure drop for different diameters and spacings of wires; the assumption being always, as in the case of formula (6), that the current variation is in accordance with the simple harmonic law (sine wave). The special case of magnetic conductors such as iron or steel will be referred to in Chapter IV.

9. Fundamental Vector Diagram for Line Calculations: Capacity Neglected.—In the diagram Fig. 10, the various quantities are represented as follows:

OA, or (I), is the current vector.

OB, or (E_n), is the vector corresponding to the pressure (wire to neutral) at the receiving end.

θ is the time angle by which the current lags behind the pressure at receiving end: $\cos\theta$ being the power factor of the load.

BC, or (IR), which is drawn parallel to OA, is the quantity $I \times R$; being the voltage component required at the generating end to compensate for ohmic drop of pressure in the conductor.

CD, or (IX), which is drawn at right angles to *OA*, is the quantity calculated by formula (6), being the voltage component required at generating end to compensate for loss of pressure due to the inductive reactance of the conductor.

BD is the sum of the vectors *BC* and *CD*; being the total additional voltage required at the generating end to compensate for the impedance of the conductor.

OD, or (V_n), is the vector corresponding to the pressure (wire to neutral) at generator end of the line, required to maintain the pressure (E_n) at the receiving end when the current in the conductor is I amperes.

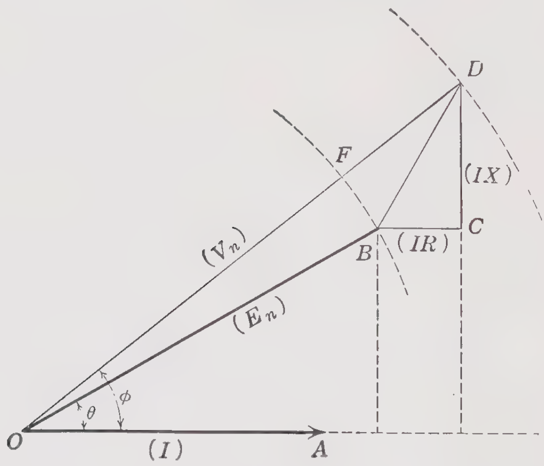


FIG. 10.—Vector diagram for line calculations—capacity neglected.

φ is the time angle by which the current lags behind the pressure at the generating end; $\cos \varphi$ being the power factor of the total load as measured at generating end.

FD is the (arithmetical) difference between pressures at receiving and generating ends of the conductor. The *percentage* loss of pressure being

$$100 \times \frac{\text{length } FD}{\text{length } OF}$$

$$= 100 \times \frac{\text{length } FD}{\text{length } OB}$$

(The dotted circles being described from the center *O*.)

All graphical solutions of transmission line problems are based on this fundamental diagram. Some of them give results that are theoretically correct, while in others certain assumptions are made to simplify the construction without introducing any appreciable error in the solution of practical problems.

Graphical and semi-graphical methods of predetermining the voltage regulation of transmission lines are often convenient. Such methods have been proposed by Messrs. F. A. C. Perrine and F. G. Baum, by Prof. L. A. Herdt, by Mr. R. D. Mershon and others. A convenient diagram for determining regulation, as used by the writer, will be explained in Chapter IV.

For those who prefer to use tables of trigonometrical functions, the required relations can easily be obtained from Fig. 10.

In the first place, the functions of the angle φ are:

$$\sin \varphi = \frac{IX + E_n \sin \theta}{V_n} \tag{7}$$

$$\cos \varphi = \frac{IR + E_n \cos \theta}{V_n} \tag{8}$$

$$\tan \varphi = \frac{IX + E_n \sin \theta}{IR + E_n \cos \theta} \tag{9}$$

From formula (8) it is seen that the required voltage at generating end is,

$$V_n = \frac{IR + E_n \cos \theta}{\cos \varphi} \tag{10}$$

and the volts required to overcome ohmic resistance are:

$$IR = V_n \cos \varphi - E_n \cos \theta \tag{11}$$

As an example of the use of these formulas assume, in the first place, that the *material, size* and *spacing* of conductors is known, and also—in all cases—the power factor of the load ($\cos \theta$), and therefore the other trigonometrical functions of the angle θ, such as $\sin \theta$. Under these conditions, the quantities IR and IX can readily be calculated, and formula (9) can be used to obtain $\tan \varphi$; thence the angle φ and $\cos \varphi$ (the power factor at generating end). Then, by formula (10), the required voltage (V_n) at generating end is easily obtained.

Assume, in the second place, that the size of the conductors has to be determined. The spacing of conductors and the frequency being known, the induced volts IX can be calculated approximately by estimating the value of r for use in formula

(6). The more correct estimate of the size of conductor will be based on the *required regulation*, or total voltage drop. The voltage at the generating end is therefore

$$V_n = E_n + \text{allowable voltage drop per conductor}$$

Now, since IR is not definitely known, formula (7) will have to be used. This gives the value of $\sin \varphi$ with a sufficient degree of accuracy even if quite an appreciable error has been made in estimating the size of conductor for the purpose of calculating the inductive drop IX. Having determined the angle φ, the function $\cos \varphi$ can be obtained from trigonometrical tables; and then, by using formula (11), the ohmic drop can be calculated. Thus the proper size of wire for use under given conditions may be determined.

If the *power loss* in the line is the determining quantity, regardless of the voltage regulation, then, since this loss depends only on the voltage IR (the current, I, being assumed constant), the resistance and size of conductor is readily ascertained, and the unknown quantities would be calculated as in the case first considered.

10. Effect of Capacity on Regulation and Line Losses.—Although the effects of electrostatic capacity will be referred to again in Chapter IV, it will be well to consider briefly how the capacity on long lines may affect the voltage regulation and line losses.

Any arrangement of two conductors of electricity separated by an insulator, forms a condenser, of which the capacity will depend upon the spacing of the conductors, and the nature of the dielectric between them. In the case of overhead conductors running parallel to each other and to the surface of the ground over a considerable distance, the electrostatic capacity between the individual conductors, and between these conductors and earth, becomes a matter of importance.

As in the case of inductance calculations, it is advisable, whenever possible, to consider the capacity of any one conductor as measured between the conductor and the neutral surface or neutral line. Thus the capacity current per conductor can be calculated independently of the current in the other conductors. It is obvious that the potential difference causing the flow of current in and out of the condenser must then be measured between the conductor and the neutral, and not between outgoing and

return conductors. In the case of a transmission line, the capacity is *distributed* over the whole length of the line. It is incorrect to assume that the whole of this capacity is concentrated at either end; but, for the sake of simplicity, the total capacity will be supposed to be concentrated at the receiving end of the line, and a correction will afterward be made in order to conform more nearly to actual conditions.

The following approximate formula may be used for calculating the capacity of overhead lines:

$$\left.\begin{array}{l}\text{Capacity in microfarads per}\\ \quad\text{mile, between conductors}\\ \quad\text{and neutral}\end{array}\right\} = C_m = \frac{0.0388}{\log \dfrac{d}{r}} \qquad (12)$$

where d and r are the spacing between conductors and the radius of cross-section, exactly as in formula (6) for the calculation of the induced volts.

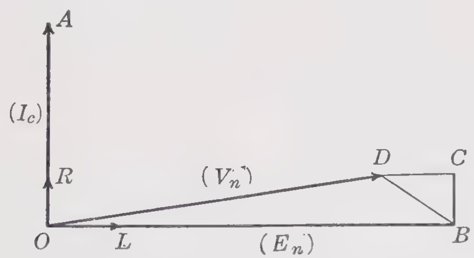


FIG. 11.—Vector diagram showing pressure rise due to capacity.

The charging current, in amperes, on the sine-wave assumption, can be calculated by the formula

$$I_c = 2\pi f C_m L E_n \times 10^{-6} \qquad (13)$$

where L is the distance of transmission in miles, and E_n is the voltage as measured between the conductor and neutral. This charging current (I_c) is always a quarter period in advance of the voltage. This explains why, on a long line open at the distant end, or only very lightly loaded, there can be a *rise of pressure* at the receiving end of the line.

In the diagram Fig. 11, the pressure at the receiving end is represented by the vector OB, while OA, drawn at right angles to OB—in the forward direction—is the capacity current as cal-

culated by formula (13). It is assumed that the load is entirely disconnected, and the current I_c is the total current on the line. The voltage component required at generating end to overcome ohmic resistance is OR, or BC, in phase with I_c, and the component required to balance the e.m.f. of self-induction, as calculated by formula (6), is CD, drawn 90 degrees *in advance* of OA. The pressure required at generating end is OD, which may be *smaller* than OB. It is true that the capacity has been assumed to be concentrated at the receiving end of the line; but with distributed capacity, the same effect of a *rise* in pressure as the distance from generating end increases, will occur. It will be seen that this is due to the e.m.f. of self-induction of the charging current being in phase with the impressed voltage. If the lines were without inductance, there could be no pressure rise.

The effect of capacity on the line when the distant end is closed on the load, will depend upon the amount and nature of the load. If the load is heavy and largely inductive, the current put into the line at the generating end will be *less* than the load current, and the I^2R losses will therefore be smaller than if the line were without capacity.

At light loads, especially if the power factor is high, the line losses will be greater than if capacity were not present. On many overhead lines the effects of capacity are almost negligible; but on long high-voltage lines, these effects become of great importance. As an example of capacity effects on long transmission lines, consider the system of the Southern Power Company which transmits energy at 100,000 volts (three-phase) over a distance of 210 miles, from Great Falls, S. C. to Durham, N. C. Both copper and aluminum conductors are used, the average diameter of which is about 0.4 in. while the spacing between wires is 124 in.; the frequency being 60. The above data and formulas (12) and (13) will enable us to calculate the capacity current. The numerical values for use in the calculation are,

$$d = 124 \text{ in.}$$
$$r = 0.2 \text{ in.}$$
$$f = 60$$
$$L = 210$$
$$E_n = \frac{100{,}000}{\sqrt{3}}$$

The capacity in microfarads per mile (between conductor and neutral) by formula (12) is,

$$C_m = \frac{0.0388}{\log_{10} \frac{121}{0.2}} = 0.0139$$

The charging current, by formula (13) is,

$$I_c = \frac{2\pi \times 60 \times 0.0139 \times 210 \times 100{,}000}{1{,}000{,}000 \times \sqrt{3}}$$
$$= 63.6 \text{ amperes.}$$

It follows that the kilovolt-amperes (or apparent kilowatts) put into the line at the generating station end, *when all the switches at the receiving end are open,* will be of the order of

$$\frac{\sqrt{3} \times 100{,}000 \times 63.6}{1000} = 11{,}000$$

The effects of capacity under various conditions are best studied by constructing vector diagrams.

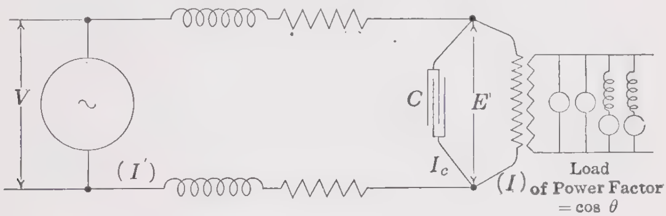


FIG. 12.—Transmission line with concentrated capacity.

In Fig. 12, the current, I, is delivered to the line at the pressure V: each conductor has both resistance and inductive reactance, giving a pressure E at the distant end, where the whole of the capacity, C, is supposed to be shunted across the wires. The load current is I.

In the vector diagram, Fig. 13 (which may with advantage be compared with Fig. 10), OB and OA represent respectively the potential difference and current at the receiving end. The impressed voltage at the terminals of the imaginary condenser C (Fig. 12), will therefore be E volts, and the vector for the condenser current must be drawn 90 degrees in advance of OB; this is the vector ON. The total current put into the line at the generating end will be OM, which is the (vectorial) sum of the currents I and I_c. The pressure at generating end is made up of three components:

OB, the pressure available at receiving end;

BC, the pressure required to overcome resistance (drawn parallel to the total-current vector *OM*);

CD, the pressure required to counteract the e.m.f. of self-induction (drawn at right angles to *OM*).

By varying the angle θ and the length of the current vector *OA*, the effect of the capacity current with different power factors and loads can easily be studied.

This method of correcting the fundamental diagram to take account of capacity, is not theoretically accurate, because the

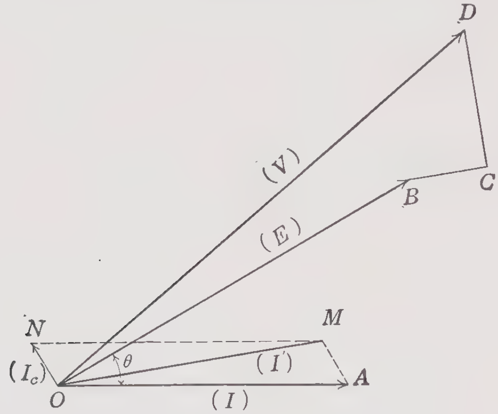


FIG. 13.—Vector diagram for transmission line of appreciable capacity.

capacity is never concentrated at one point of the line; but distributed over the whole distance of transmission.

11. Use of Fundamental Diagram for Three-phase Calculations.—The vector diagram, Fig. 14, shows the relative phases of current and e.m.f. for a three-phase system with balanced load when the power factor is unity. Here the three current vectors are *OA*, *OB*, and *OC*. The "star" voltages are,

$$Oa = Ob = Oc = E_n$$

each being in phase with the corresponding line current; and the voltages measured between the three conductors of the transmission line are,

$$ab = bc = ca = E = \sqrt{3}E_n$$

The total power transmitted is,

$$
\begin{aligned}
W &= \sqrt{3}E \times I \\
&= 3E_n \times I \\
&= 3\ (OA \times Oa)
\end{aligned}
$$

In Fig. 15 the diagram has been drawn for an inductive load. Here there is a certain displacement of the current phases relatively to the e.m.f. phases. It will be noticed that the vertices of the e.m.f. triangle no longer lie on the current lines as in the previous diagram. The three current vectors still subtend the same angle of 120 degrees with each other; but they have been moved bodily round (in the direction of retardation) through

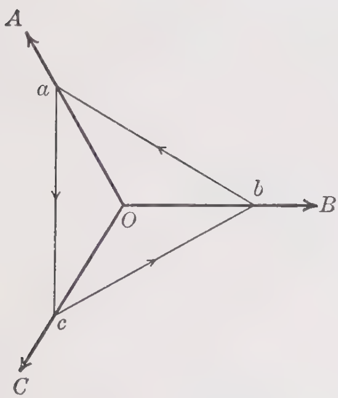


FIG. 14.—Vector diagram for three-phase system on non-inductive load.

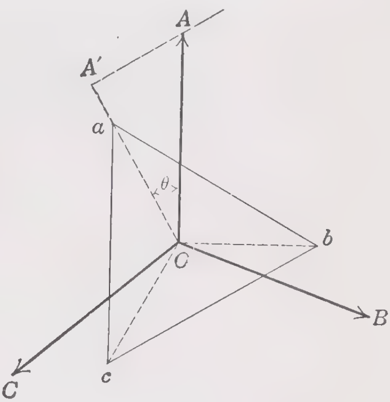


FIG. 15.—Vector diagram for three-phase system on partly inductive load.

an angle θ. The total power is evidently no longer equal to three times $OA \times Oa$, but to $3 \times OA' \times Oa$, where OA' is the projection of OA on Oa; and $\cos \theta$ is the power factor of the three-phase load.

It is a simple matter to complete this diagram by taking into account the effects of resistance and inductance in the line, because when the calculations for resistance drop and induced volts are made *per conductor* as previously explained, the construction can be carried out for each phase exactly as explained when describing the fundamental diagram (Article 9). It is only necessary to bear in mind that OA and Oa, in Fig. 15, correspond to OA and OB in Fig. 10. When this construction

has been carried out for each of the three phases, there will be a new set of star vectors which, when their ends are joined, will form a new e.m.f. triangle representing the necessary pressures at the generating end. This is shown in Fig. 16, where *am* and *md* are the vectors representing the required e.m.f. components to counteract the ohmic drop and reactive voltage respectively, due to the current *OA*. The same construction is supposed to be followed for the other two phases, and the resulting triangle *dcf* indicates not only the magnitude of the potential differences between wires at generating end, but also their phase relations with the other quantities. Thus the power factor at the generating

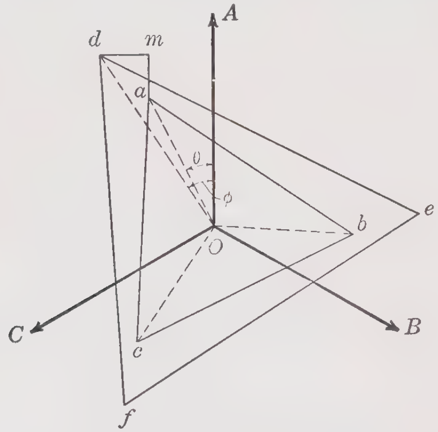


Fig. 16.—Complete vector diagram for three-phase transmission.

end is not cos θ, but cos φ, all as explained in connection with Fig. 10. If it is required to take into account the effects of capacity, the correction *per phase* is made as explained in Article 10.

It is true that a symmetrical arrangement of conductors has been assumed; that is to say, the three conductors are supposed to occupy the vertices of an equilateral triangle, in which case the magnetic flux due to the current in one of the wires will neither increase nor decrease the amount of induction through the loop formed by the other two wires; or, in other words, the whole of the current in any one conductor may be considered as returning at a distance from this conductor equal to the side of the equilateral triangle. As a matter of fact, if the wires are arranged in any other practical manner, the effect of the induction due to any one wire on the loop formed by the other two wires is usually

small; but a method of calculating the induced volts in any one conductor of a system of parallel conductors, whatever may be the arrangement or spacing of these conductors, is explained in Appendix I at the end of this book.

If the conductors of a three-phase transmission are regularly transposed, that is to say, if each of the three wires occupies a particular position relatively to the other wires for one-third of the total length of transmission, then the electrical calculations may be based on an *equivalent* disposition of the wires at the vertices of an equilateral triangle of side

$$d = \sqrt[3]{abc} \tag{14}$$

where a, b and c stand respectively for the actual spacings between the three wires. This is a general statement of the particular problem considered in Appendix I, where the three conductors are assumed to lie in the same plane. A neat proof of formula (14) is given in Prof. H. B. Dwight's book on Transmission Line Formulas.[1]

[1] D. Van Nostrand Company, 1913.

CHAPTER III

ECONOMIC PRINCIPLES AND CALCULATIONS

12. Introductory.—That true engineering is essentially an economic science should be self-evident to every man who lays claim to the title of engineer; yet, there are many engineering undertakings, or portions of such undertakings, in which this fundamental principle has been disregarded. In the case of transmission lines, a certain system, or an exceptionally high pressure, may have been adopted because of its peculiar interest as an engineering problem; or duplicate lines, spare generating plant, and costly automatic gear may have been installed to ensure continuity of supply, apart from the economic value of such increased protection against possible interruption. This, however, is not engineering in the commercial sense. The determination of the economical size of conductors for the transmission of any particular amount of current, in accordance with the principle generally known as Kelvin's law, is but a very small part of the problem to be solved by the transmission-line engineer. An attempt will be made in this chapter to deal with the economics of power transmission lines from a broad practical standpoint. Some approximate figures for use in getting out preliminary estimates will be given, but actual recorded costs of finished work carried out under various conditions can be obtained from other sources; their inclusion in this book might tend to confuse rather than assist the reader.

It is proposed to deal here, in as small a space as possible, with economic principles, and to explain the methods by which the proper size of conductor for a given transmission can be calculated; and it is only when these principles have been grasped, and rightly understood, that the engineer can make the best use of cost data obtained from completed work.

When considering any scheme of power transmission from a generating plant of limited output, it is important to bear in mind that it does not pay to cover a distance greater than that within which there is a reasonable prospect of supplying all the

power available at the generating station. The importance of this principle should be fairly obvious, yet there are instances which prove that it has been disregarded.

On the other hand, where the energy available appears to be in excess of the probable demand within a reasonable radius from the generating station, the possibilities of subsequent requirements greatly exceeding the immediate demand must not be overlooked; and a transmission system on a large scale, designed to satisfy future conditions, may be desirable. Each case must be studied separately, on account of the variable nature of the conditions in different localities. The easiest part of the problem is the designing of an economical transmission line, that is to say, a line that will give the best return for capital invested, on the assumptions of a given amount of energy to be delivered at a given point where a given price will be paid for it. The real difficulty lies in estimating not only the immediate demand, but also the probable future demand and its rate of growth, in order that proper provision may be made to avoid unnecessary waste in remodelling or reconstructing the original transmission line.

13. Choice of System.—On this continent it is usual to transmit electric power by means of three-phase alternating currents, the periodicity being 25 or 60 cycles per second. In Europe the Thury system of continuous current transmission at high voltages has met with success; it has much to recommend it, and there appear to be no reasons why it should not meet with equal success on this continent; but it is probable that three-phase transmission, at pressures even higher than those now in use, will hold its own for a considerable time to come.

14. Type of Transmission Line.—The structures for supporting overhead conductors may be of wood, steel, or reinforced concrete. The wood supports may consist of single poles spaced 120 to 300 ft. apart, or they may be A or H frames built up of two poles suitably braced, and capable of supporting longer spans. The steel poles may be of simple tubular type, or built up of several tubes or angles with the necessary bracing. The more common construction for high-pressure transmission lines consists of light-braced towers with wide rectangular bases, except where the "flexible" type of structure is adopted. These flexible towers are modeled generally on the A and H types of double wood pole supports. Further particulars, with illustrations, of these

different kinds of supports will be found in Chapter X. It is by no means an easy matter to decide upon the most suitable type of supporting structure to be used on any particular transmission scheme. In some cases a composite line including two or more types of support may be found advantageous. Among the factors influencing the choice of the supporting structures may be mentioned the character of the country, the means and facilities of transport, climatic conditions, the nature of the soil, and the scarcity or otherwise of suitable timber in the district through which the line will pass. In undulating or hilly country, advantage may frequently be taken of the heights, by erecting upon them comparatively low and cheap structures and spanning the depressions or valleys without any intermediate supports. The engineering features must, however, be very carefully studied in all such exceptional cases.

When transmission lines, or portions of transmission lines, have to be laid underground, the thickness and nature of the insulation, and the method of laying, will affect not only the first cost, but also the maintenance charges and the life of the cables. All these factors must be taken into account when deciding upon the most economical type of construction under the varying conditions that are likely to occur in practice. It is usual to draw the insulated conductors into some form of duct or conduit; but conditions may be found under which it would be justifiable to use lead-sheathed and steel-armored cable laid directly in the ground without any additional protection. The question of underground cables will be discussed further in Chapter VII.

15. Length of Span.—The type of supporting structure for overhead conductors, together with the height, strength and cost, of the individual pole or tower, will be dependent upon the span or distance between the supports. The determination of the average length of span is indeed a very important economic question. The material of the conductor will, to some extent, influence the choice of span length, because aluminum conductors will usually have a greater summer sag than copper conductors, and this will necessitate higher supports to give the same clearance above ground at the lowest point of the span. In considering span length, the first cost of the individual support is not the only question which has to be taken into account; the cost of maintenance is almost equally important. The longer the span, the fewer will be the points of support; and if the line is well

designed and constructed, there should be less trouble through faults at insulators. Again, where rent has to be paid for poles placed on private property, it is generally the rent per pole apart from the size of pole which has to be considered, and this is another factor in the determination of the best length of span. In level country, the economic span for steel tower construction is usually in the neighborhood of 600 feet. The Sierra & San Francisco Power Co. (104,000 volts) uses spans of 850 feet; while both the Pacific Gas & Electric Co. and Mississippi River Power Co., on their 110,000-volt transmission lines, space the towers 800 feet apart under normal conditions. On short low-vo'tage lines large spans may not prove to be economical. It is sometimes advantageous to increase the number of supports in order that these may be so light in weight as to be easily handled and quickly erected.

When considering the relative advantages of different types of supports for overhead conductors it is obvious that the life and cost of upkeep of the poles or towers must be considered at the same time as the costs of delivery on site and erection, which are less likely to be overlooked. For instance, a transmission line using reinforced concrete poles will usually cost more than one using wood poles; but if we assume the same height of pole (say 35 feet) and a cost per pole of \$10 and \$6.50 respectively for concrete and chestnut poles, the concrete pole line will very probably prove to be the more economical in view of the fact that the chestnut poles will have to be replaced at the end of (say) 13 years, while the concrete should last, without requiring attention or repairs, about twice as long.

16. Effect of Span Variations on Cost of Steel Towers.—The height of towers in level country depends on (1) the minimum clearance between the lowest conductor and ground when the sag is greatest; (2) the voltage, since this has an effect on the spacing of the conductors and also to some extent on the clearance above ground level; and (3) the maximum sag. This last is determined by the length of span, the material and size of the conductors, the range of temperatures, and weather conditions genera'ly. For the purpose of rough approximations suitable for preliminary estimates the writer has made use of the empirical formula

$$H = 34 + \frac{E_k}{4} + \frac{l^2}{20{,}000} \qquad (15)$$

This formula gives the approximate overall height of the tower in feet. By overall height is meant the total height including the portion buried in the gound; the height from ground level to extreme top of tower being therefore from 5 to 8 feet less the dimension given by formula (15). The symbol E_k stands for the working pressure between wires in kilovolts, while l is the distance between towers in feet. The constants have been worked out on the assumption that the tower carries a duplicate three-phase circuit consisting of aluminum conductors of number 0000 B. & S. gauge, and a grounded steel guard-wire joining the tops of the towers. The formula is not supposed to be applicable to spans greater than 650 feet.

The weight of a steel supporting structure will depend not only upon the height, but also upon the stresses the tower has to withstand. These again will be dependent upon the size of the wires, the length of span, and the weather conditions, although none of these factors has perhaps so great a bearing on the weight and cost of the supporting structures as the fancies, prejudices, and idiosyncrasies of the engineer or purchaser. The assumptions made in the matter of probable sleet deposits and wind pressure, together with the factors of safety that are insisted upon, will naturally very greatly influence the cost of the towers. Assuming these figures to be reasonable, the weight in pounds of the complete tower will be approximately $0.85H^2$, although the proportionality between the square of the height and the weight does not always hold in practice.

The cost of galvanized towers, under normal market conditions, varies between 3¢ and 5¢ per pound.[1] For preliminary estimates, the cost may be calculated on a basis of 4¢ per pound.

The weight and cost of towers to carry only one three-phase circuit would be about 25 per cent. less than double-circuit towers; while the so-called flexible "A" frame supports are about 30 per cent. cheaper than the corresponding rigid square-base towers.

It is, however, usual to provide rigid strain towers in place of the flexible type at, say, every mile, and as the cost of such struc-

[1] The increased price of all metals, brought about by the war, has led to the cost of steel structures being, at the time of writing, nearly double what would be indicated by these figures, which are based on conditions existing before 1915. It is not possible to predict metal prices for the future.

tures is about double the cost of the flexible tower, the cost of supports per mile of line may be calculated by assuming $n + 1$ flexible structures per mile, when the actual spacing is n to the mile.

When estimating the cost of a tower line, it is necessary to take into account the special anchor towers required at corners or at the ends of exceptionally long spans. In any case, the practical utility of the approximate formula (15) is somewhat doubtful; and just so soon as the investigation has gone far enough to justify the preparation of a complete estimate of cost, quotations for suitable supporting structures should be obtained from the manufacturers.

Steel structures are usually galvanized; but as an alternative, they may be painted; the extra cost of galvanizing should be compared with the cost of painting periodically, say, every third or fourth year.

The cost of foundations for towers varies greatly. In the case of fairly high steel towers with wide square bases in soil not requiring the use of concrete, the cost of excavating, setting legs, and back filling, not including erection of towers, will generally be between \$10.00 and \$20.00 per tower.

17. Cost of Wood Poles.—The price of wood poles depends upon the kind of wood, the quality, *i.e.*, straightness and freedom from defects, and of course upon the dimensions. As a rough guide for preliminary estimates, average prices for chestnut poles are given below:

Poles 30 ft. long	\$ 4.00 each
Poles 35 ft. long	6.00 each
Poles 40 ft. long	9.00 each
Poles 45 ft. long	10.50 each

The cost of very tall poles may be considerable; but although the cost of a 45-ft. chestnut pole is here given as something over \$10.00, wood poles of this length suitable for transmission lines, although perhaps less durable, can be obtained for \$6.00 or even less. On this basis, a 55-ft. pole would cost about \$9.00, while 70-ft. poles could probably not be obtained for less than double this amount.

An amount varying between \$4.00 and \$6.00 should be allowed to cover unloading, hauling, "framing," digging holes, and erecting a pole of average height.

Some costs of sundry items to be included in an estimate for a wood-pole line are given in the sample estimate which follows; but cost handbooks or manufacturers' catalogues should be consulted for specific information of this kind.

18. Cost of Insulators.—The cost of insulators increases rapidly with the rise of the working voltage. The curve of Fig. 17 gives approximate average prices of insulators complete with pins or suspension links for pressures up to 140,000 volts. The prices are per insulator or per series of insulator units. The

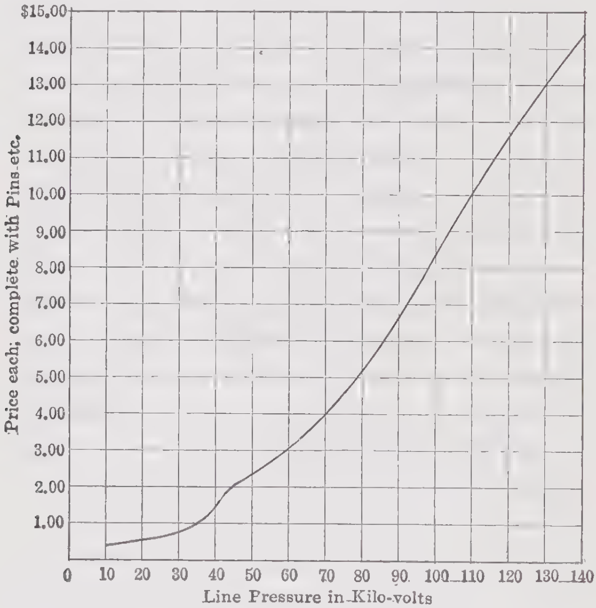


Fig. 17.—Approximate cost of line insulators.

suspension type of insulator consisting of a number of units in series is almost universally used for pressures exceeding 60,000 volts. One golden rule which applies to all overhead transmissions is that it is false economy to reduce first cost by putting in cheap and possibly unreliable insulators.

19. Duplicate Lines.—A point of great importance in connection with power transmission undertakings is the means adopted to guard against interruption of supply. If it is allowed that the least reliable part of a transmission system is the line itself, it is certainly advisable, when circumstances permit, to dupli-

cate the lines, the two sets of conductors being connected in parallel, if desirable, under normal conditions. The best protection against interruption would be afforded by carrying the two sets of conductors on separate poles, preferably by different routes; but this would almost double the cost of the line, and it is usual to carry duplicate lines either on one set of poles, or on two sets of poles erected side by side on the same right of way. As an alternative to the duplication of the lines, the provision of reserve generating plant at the receiving end may be considered, and a comparison should be made between the relative advantages and costs of the various alternatives.

A good example of steam-driven auxiliary plant in connection with hydro-electric power stations, is the oil-burning steam-generating station of the Southern California Edison Co., situated 25 miles from the city of Los Angeles and capable of connecting in parallel with the 60,000-volt and 30,000-volt systems ordinarily supplied by the Kern River and other hydro-electric generating stations of this company.

20. Costs of Typical Transmission Lines.—It would be possible to give a large number of figures relating to material and labor costs of completed transmission lines; but the conditions of transport of materials and quality of labor differ widely, and without complete knowledge of these conditions, such figures are liable to be misleading. For this reason two ideal preliminary estimates, one referring to a wood pole line, and the other to a steel line, are here reproduced, in the hope that they may be useful as a basis on which somewhat similar estimates may be shaped.

Preliminary Estimate No. 1

Wood pole transmission line, 20 miles long, carrying one three-phase line. Line pressure 22,000 volts. Span 130 ft. There is no grounded overhead guard wire; but two telephone wires are carried on the same set of poles. An allowance of 20 per cent. is made for extra insulators and fixtures to permit of doubling these on corner poles and in other selected positions.

Preliminary Estimate No. 2

"Flexible" type steel tower line, 60 miles long, with two sets of three-phase conductors. Line pressure = 80,000 volts.

Average span, 480 ft. Spacing between wires, 8½ ft. A Siemens-Martin steel cable, acting as grounded guard wire, joins the tops of all towers. Insulators of the suspension type. No telephone wires. Cost of right-of-way not included in estimate.

PRELIMINARY ESTIMATE NO. 1

MATERIALS (EXCLUDING CONDUCTORS)

40 creosoted cedar poles, 35 ft. long, 8 in. diam. at top	$400.00	
48 cross-arms, 3½ by 4½ in. by 4 ft. long	20.00	
96 galvanized-iron braces, 1¼ by ¼ by 28 in. long	7.00	
32 galvanized bolts, ⅝ by 12½ in., with washers 8 galvanized bolts, ⅝ by 16 in., with washers 16 galvanized spacing rods, ¾ by 16 in., with nuts and washers 48 galvanized lag screws, ½ by 3½ in. 96 galvanized carriage bolts, ⅜ by 4½ in.	6.50	
1500 ft. galvanized 7-strand 5/16-in. guy wire	12.00	
12 anchor rods with nuts and washers and necessary timber for anchor logs	7.00	
24 galvanized guy clamps with bolts	3.00	
8 galvanized sheet-iron bands to prevent cutting of poles by guy wire	0.50	
12 standard thimbles for guy wire	0.50	
20 galvanized-iron lightning conductors, with bolts	5.50	
20 ground plates or galvanized-iron pipes	8.00	
Staples and sundries, including allowance for breakages and contingencies	20.00	
80 telephone wire insulators (glass) 80 side brackets for same (wood); 5-in. wire nails	10.00	
144 H.T. porcelain insulators	42.00	
96 galvanized-iron insulator pins with porcelain bases	16.00	
48 special pole-top insulator pins, with bolts	21.00	
Total material cost per mile of line		$579.00

Labor

Clearing 50 ft. on each side of pole line @ $30 per acre	363.00		
Distributing poles and other materials along the line	50.00		
Trimming poles, cutting gains, drilling holes, setting cross-arms	40.00		
Digging holes and erecting poles, including the necessary guying	100.00		
Fixing insulators and stringing wires, including telephone line	100.00		
Supervision and sundry small labor items	40.00		
Loss and depreciation of tools	15.00		
Management and preliminary engineering work	35.00		
Total cost per mile for charges other than materials		$743.00	
Total cost per mile, excluding cost of conductor material			$1322.00

Conductors

16,000 ft. No. 1 copper conductors (hard-drawn); 700 ft. No. 4 copper for ties (soft); 10,800 ft. No. 10 copper for telephone circuit; 4550 lb. @ $20 per 100 lb.	910.00
Total cost per mile of finished line not including interest on capital invested during construction period	$2232.00

Preliminary Estimate No. 2

Materials (Excluding Conductors)

10 flexible type, galvanized-steel, A-frame towers @ $90	$900.00	
1 galvanized-steel strain tower	170.00	
concrete foundations where necessary	80.00	
5600 ft. 7/16-in. galvanized Siemens-Martin steel strand cable for guard wire and head guys on half-mile flexible towers	130.00	
4 anchor rods, complete with clamps and thimbles for guy wire	4.00	
90 sets of suspension-type insulators, including strain insulators and small allowance for breakages, complete with clamps	500.00	
Sundry small items or special material	50.00	
Total material cost per mile of line		$1834.00

LABOR

Clearing 60 ft. on each side of line at average cost of $25 per acre	$363.00		
Distributing towers and other materials along the line	90.00		
Foundations for towers	85.00		
Assembly of parts and erection of towers	150.00		
Fixing insulators and stringing wires	160.00		
Supervision and sundry small labor items	50.00		
Allowance for loss and depreciation of tools	20.00		
Allowance for management and preliminary engineering work	50.00		
Total charges other than materials, per mile		968.00	
Total cost per mile not including conductor material			$2802.00

CONDUCTORS

No. 00, hard-drawn, stranded-copper conductors; small amount of No. 2 soft copper for occasional ties; special clamps, shields, jointing materials, etc.; 13,350 lb. @ $20 per 100 lb.	$2670.00
Total cost per mile of finished line, not including right-of-way or interest on capital invested during construction	$5478.00

The curves of Fig. 18 are intended to supplement the figures of the typical estimates. They give approximate costs of transmission poles or towers, with insulators fixed in position. These costs are the averages of many actual figures, and give an approximate idea of the total expenditure per mile of line for various voltages; they do not include any clearing that may be necessary in wooded country, or payments for right-of-way. It is assumed that the conductors are of average size (No. 000 B. & S. gauge), but the actual cost of the conductors, whatever the size, must be added to the costs indicated by the curves in order to arrive at the total cost of the finished line. These curves, however, do include an amount to cover the labor cost of stringing the wires, which will generally lie between $25 and $75 per wire, per mile, depending upon the size and number of the wires and the nature of the ground covered by the transmission line. The lower curve of Fig. 18 refers to wood poles or rigid steel towers (for the higher voltages) carrying three conductors; while the upper curve refers to a single set of poles or towers carrying six conductors. It

should be understood that the curves of Fig. 18 give only an approximate indication of the probable capital expenditure on the line. The actual cost will depend upon the character of the country, the nature of the ground, and other local conditions, such as cost of labor and facilities for transportation. These, together with the weather conditions, force of wind and possible loading of wires with sleet or ice, will determine the most economical span and the average height of pole or tower. The cost, as previously mentioned, will also depend upon the material of the conductors, as a larger or smaller sag will influence spans

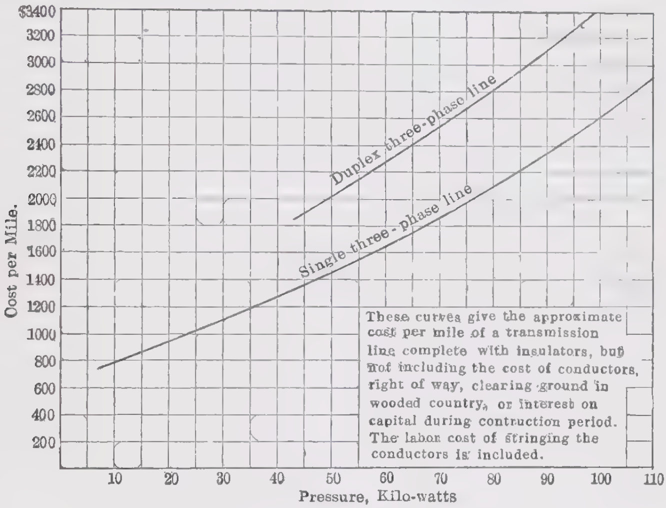


FIG. 18.—Approximate cost of overhead transmission lines.

and height of poles. The weight and diameter of conductors, by affecting the required strength of the supports, will be factors in determining the cost of the complete line, apart from any difference in the value of the conductors themselves. The actual cost of stringing very light or very heavy conductors will also differ from the average amount allowed for the purpose of plotting the curves. The number and style of lightning conductors, if any, and whether or not one or more grounded guard wires are strung above the conductors will obviously modify the average figures. Although steel poles, or steel towers, will generally be found more economical than a wood pole line for voltages above 44,000 on account of the heavier insulators, wider spacing between conductors, and generally greater height of support, it

does not follow that wood poles or wood-pole structures may not prove economical, even for comparatively high voltages, in countries where suitable timber is plentiful and the ready means of transportation and erection of steel towers are wanting. For instance, the cost per mile, as obtained from Fig. 18, is \$2600 for a single 100,000-volt three-phase line supported on rigid square base steel towers, and it is not improbable that an entirely satisfactory wood pole line could be built under favorable conditions at a figure appreciably below \$2000 per mile (not including conductors, right-of-way, or clearing wooded land). But it does not follow that the wood pole line is the most economical, because the probable cost of maintenance, repairs, replacements of decayed or damaged poles, and all charges to be met annually or periodically during future years must be carefully considered before a final decision can be arrived at.

21. Cost of Overhead Conductors.—The capital expenditure on conductors will depend upon the material and the total weight. It is not proposed to discuss, in this place, the relative merits of copper and aluminum as conductor materials, but it may be well to point out that although the market values of these metals may be such that the use of aluminum may lead to some saving on first cost, there are many engineering points to be most carefully considered before definitely adopting either metal. The weight of the conductors necessary to transmit a certain amount of power over a definite distance will obviously depend upon the voltage, but apart from the engineering difficulties encountered at the higher voltages, there are economic considerations which determine the maximum voltage suitable for any given conditions. Among these may be mentioned a possible increase in the cost of generating plant for the higher pressures, the greater cost of step-up and step-down transformers and of the control apparatus, together with the line insulators, entering bushings, etc. The transmission line poles or towers will also, as previously mentioned, cost more for the higher pressures, because of the wider spacing between conductors and the greater length of insulator string. Then again, with the extra high pressures, the increased losses through leakage over insulators and possible corona losses may be quite appreciable.

Given a definite amount of power to be transmitted, and a definite line pressure, the current can be calculated; and the economic conductor cross-section—and therefore the weight and

cost of the conductors—will be directly proportional to this current. It is only of recent years that this fact appears to have been generally recognized, and yet, so long ago as 1885, in his Cantor lectures delivered in London, Prof. George Forbes said: "The most economical section of conductor is independent of e.m.f. and distance, and is proportional to the current." The determination of the current value to be used in the calculation of conductor sections is a real difficulty. It must not be supposed that even a knowledge of the load factor is sufficient by itself. The load factor, being the ratio of average load to maximum load, does not give the relation between the average I^2R loss and the I^2R loss of maximum output. The power lost in the conductors of a constant potential supply is proportional to the square of the power transmitted. On the basis of the average hydro-electric load curve, if the load factor is 50 per cent., the load on which the average transmission line losses should be based—being the square root of the mean of the square of the power—will probably be found to be more nearly 60 per cent. than 50 per cent. of the maximum load.

Although the discussion which follows refers mainly to overhead transmission lines, the same general principles should govern the choice of conductor cross-section in underground cables. As an economic problem, the underground system of transmission differs from the overhead system mainly in the fact that the cost of the insulation in a cable is a function of the conductor diameter, whereas the cost of line insulators is less closely connected with the size or weight of the overhead conductors. The cost of insulation is relatively more important in cable systems than in overhead transmissions. Some reference to the economics of power transmission by underground cables will be made in Chapter VII.

22. Economic Size of Conductor. Kelvin's Law.—Before considering to how great an extent the voltage may be raised, in order to keep down the current, without exceeding the limits determined by economic considerations, it will be well to examine, in some detail, the fundamental principle known as Kelvin's law, by which the proper size of conductor to carry a known current is determined. In this connection it is of no consequence whether the transmission is by direct or alternating currents, single phase or polyphase. If conductors have to be provided to carry a current of known amount, these may be of large cross-

section and therefore of high initial cost, but of so low a resistance that the I^2R losses will be small; or they may be of small cross-section and high resistance, the capital expenditure on which will be small; but in which the I^2R losses will be large. The economical size of conductor for any given transmission will therefore depend on the cost of the conductor material and the cost of the power wasted in transmission losses; and the law of maximum economy may be stated as follows: *The annual cost of the energy wasted per mile of the transmission line, added to the annual allowance (per mile) for depreciation and interest on first cost, shall be a minimum.*

If it is assumed that the cost of poles or towers, insulators and other materials (apart from the conductors themselves) including the labor on erection and stringing of wires, is independent of the actual size of conductor, then the only variable item in the capital expenditure is directly proportional to the cross-section (or weight) of the conductor, and since the I^2R losses (for a given current) are inversely proportional to the conductor cross-section, the law of maximum economy is greatly simplified, and in fact becomes Kelvin's law, which may be expressed as follows:

The most economical section of a conductor is that which makes the annual cost of the I^2R losses equal to the annual interest on the capital cost of the conductor material, plus the necessary annual allowance for depreciation. The cross-section should, therefore, be determined solely by the current which the conductor has to carry, and not by the length of the line or an arbitrary limit of the percentage full-load pressure drop. If there are reasons which make a large pressure drop undesirable, then, if necessary, economy must be sacrificed, and the line calculated on the basis of regulation only. It will, however, generally be found that the economic conductor will give reasonably good regulation.

The diagram, Fig. 19, shows clearly how the minimum total annual cost occurs when the cost per annum of the wasted energy is equal to the capital cost expressed as an annual charge; and if desired a graphical solution of Kelvin's law can readily be obtained by this means. In Fig. 19, the horizontal distances measured to the right of the point O represent increasing conductor resistances; while the vertical distances represent money. The curve A shows how the annual charges depending on capital outlay decrease with increase of conductor resistance; while the straight line B indicates the growth of the cost of wasted power;

this being directly proportional to the resistance. By adding the ordinates of curves A and B, the curve C is obtained, of which the lowest point indicates the resistance per mile of conductor which will be the most economical to use, whatever may be the length of the line, or the pressure required at the receiving end.

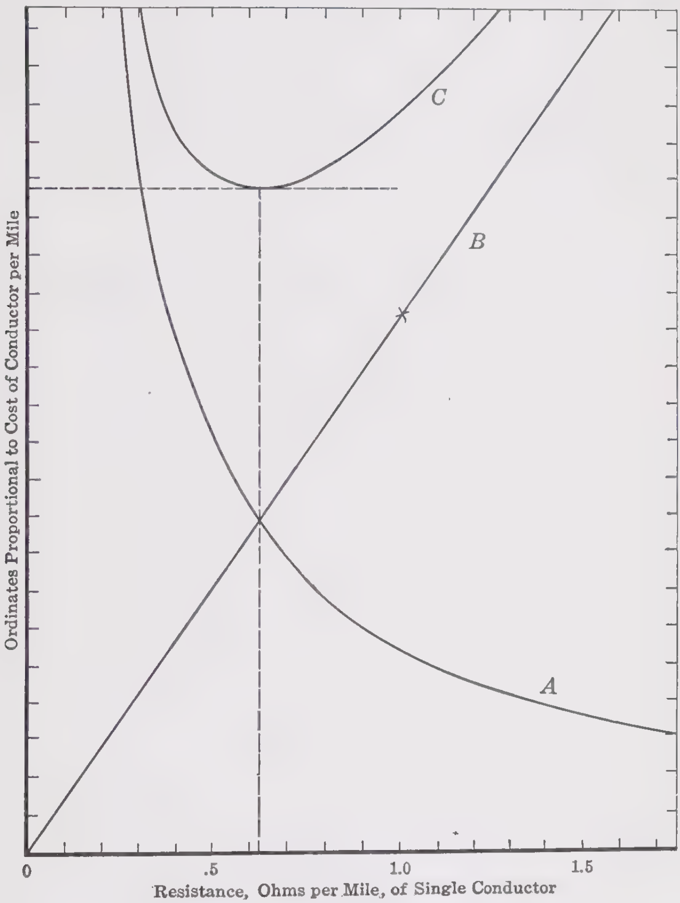


FIG. 19.—Graphical illustration of Kelvin's law.

It will be observed that this minimum occurs where the two curves cross.

23. Practical Method of Applying Kelvin's Law.—The following formulas have been evolved with a view to facilitating the calculation of conductor sizes to give the most economical

results on overhead transmissions. In every case the lesser factors which may, to a small extent, influence the results of the problem will be disregarded, but they may be taken into account when the final details of the transmission line are being considered. On the other hand, it will generally be found that the application of Kelvin's law in its simplicity, without regard to such influences as the possible variations in cost of supports, insulators, etc., depending upon the size of the conductors, will give results sufficiently accurate for practical purposes, and this for two important reasons:

1. A small variation in the diameter of the conductor either on the large or the small side is usually of very little consequence from the economic point of view.

2. As the standard size of conductor nearest in diameter to the theoretically correct size is generally selected, refinements or increased accuracy in the calculations will rarely affect the size of wire which is ultimately decided upon.

24. Economic Ohmic Voltage Drop.—It is not generally realized that when the size of a conductor is determined by the application of Kelvin's law the ohmic drop of pressure per unit length of conductor is independent of the actual voltage or the current to be carried, and therefore bears no definite relation to the total amount of power to be transmitted. The economic data and assumptions alone determine the ohmic drop in volts per unit length of conductor, and this will be a constant quantity whatever the number of conductors or system of electric transmission adopted, the total amount of power to be transmitted, or the voltage ultimately decided upon. This fact very considerably simplifies the problem in its earlier stages.

The formula for the economic voltage drop may be arrived at as follows, bearing in mind that the annual charges to be considered are (1) an annual charge for interest and depreciation on the cost of the line wire; (2) the annual cost of the energy wasted in the conductor in the form of I^2R losses,[1] and that the equality of these two items of cost determines the size of the most economical conductor.

Annual Charges Depending Upon Cost of Conductor.—Let p be the price to be paid for 100 lb. weight of conductor and a the

[1] Other losses due to leakage over insulators and through the air should be taken into account when considering the choice of e.m.f., especially if this should exceed 60,000 volts.

percentage to be taken to cover the annual interest and depreciation, then, if R be the resistance in ohms per mile of the conductor,

$$\text{Annual charge} = \frac{a}{100} \times p \times \frac{1}{R} \times K \tag{16}$$

where K is a constant depending upon the material of the conductor and the temperature.

Annual Cost of Energy Lost.—Let p_1 be the cost per kw.-year of the wasted energy; then,
annual cost per mile of conductor $= p_1 \times$kw. lost per mile.

$$= p_1 \times \frac{I^2R}{1000}$$

$$= p_1 \times \frac{E_r^2}{1000 \times R} \tag{17}$$

where E_r stands for ohmic drop in volts per mile of conductor.

In order to satisfy the condition of equality between the values (16) and (17) we must write

$$\frac{a \times p \times K}{100 \times R} = \frac{p_1 \times E_r^2}{1000 \times R}$$

whence

$$E_r^2 = 10K \frac{a \times p}{p_1} \tag{18}$$

If the temperature is about 20 degrees Centigrade, and the material of the line is copper, the constant K may be taken as 8.76, while for aluminum it works out at 4.32. Inserting these values in the last formula, the economic ohmic voltage drop per mile of copper conductor becomes:

$$E_r = 9.35 \sqrt{\frac{a \times p}{p_1}} \tag{19}$$

and for aluminum:

$$E_r = 6.57 \sqrt{\frac{a \times p}{p_1}} \tag{20}$$

If preferred, these formulas can be put in the form:

$$\text{Circular mils per ampere (copper)} = 5800 \sqrt{\frac{p_1}{a \times p}} \tag{21}$$

and,

$$\text{Circular mils per ampere (aluminum)} = 13{,}400 \sqrt{\frac{p_1}{a \times p}} \tag{22}$$

25. Economic Voltage, and Calculation of Conductor Sizes.—Having ascertained what will be the most economical ohmic

drop of pressure per mile of conductor without reference to the total amount of power to be transmitted, the size of the conductor cannot be determined unless the value of the current is known, and this will depend upon the pressure at which the energy will be transmitted.

If the cost of the conductors forming the transmission line, and the I^2R losses therein, were the only considerations, a high voltage would in all cases be desirable on account of the corresponding reduction of current for a given amount of energy to be transmitted. But, apart from the extra cost of the line due to the better insulation and wider spacing of wires required by the higher pressures, the cost of generation and transformation of high-pressure energy must be taken into account, and as the extra cost per kilowatt of equipment for generating at high pressures will depend largely upon the total output required, it follows that the most economical pressure will bear some relation to the total power to be transmitted. This is apart from the distance of transmission, which is the most important factor governing the choice of voltage. If the distance is great it is obvious that the reduction of material cost and power losses in the line due to the employment of higher pressures will be relatively of far greater importance than the increased cost of plant in generating and transforming stations. On the other hand, the employment of very high pressures even on a comparatively long line might not be justified if the total amount of power to be transmitted were very small.

As a first approximation, the writer has found the following formula useful in getting out preliminary estimates; the line voltage given by the formula agrees generally with modern practice.

$$\text{Line pressure (kilovolts)} = 5.5\sqrt{L + \frac{\text{k.w.}}{100}} \qquad (23)$$

This empirical formula may be used for estimating the probable economical transmission voltage on lines over 20 miles in length. The symbol L stands for the distance of transmission in miles, while k.w. stands for the estimated maximum number of kilowatts that will have to be transmitted over one pole- or tower-line.

Given the amount of power to be transmitted and the length of line, one can with the aid of formula (23) decide upon a stand-

ard voltage and proceed with the calculations for current and size of conductor; but it is necessary always to bear in mind that a transmission line cannot be considered by itself; it must be treated as part of a complete scheme of transmission and distribution, and the best voltage to use on any given system can generally be arrived at only by a method of trial and error, taking into account the cost of the various parts of the complete system as influenced by alterations in the transmission voltage. No accurate formula can be evolved which would be applicable to all the varied conditions encountered in actual work; but a practical method of attaining the required end will be explained later.

26. Example Illustrating Quick Method of Determining Economic Size of Conductors.—For the purpose of working out a practical example the following assumptions have been made:

Total kilowatts to be transmitted, $P = 12{,}000$.

System, three-phase.

Power-factor = 0.8.

Distance of transmission = 120 miles.

Copper conductors to be used, the cost p being \$20 per 100 lb.

Percentage to be taken to cover depreciation and annual interest on cost of copper, $a = 12.5$.

Estimated cost of wasted power per kilowatt-year, $p_1 = \$22$.

The economic voltage drop per mile of single conductor will be, by formula (19),

$$E_r = 9.35\sqrt{\frac{12.5 \times 20}{22}}$$
$$= 31.5 \text{ volts}$$

The transmission voltage as given by formula (23) is:

$$\text{Kilovolts} = 5.5\sqrt{120 + \frac{12{,}000}{100}}$$
$$= 86$$

or, say, 88,000 volts at the receiving end.

The current per conductor will be:

$$I = \frac{\text{Watts}}{\sqrt{3} \times E \times \cos\theta}$$
$$= \frac{12{,}000{,}000}{\sqrt{3} \times 88{,}000 \times 0.8}$$
$$= 98.4 \text{ amp.}$$

Resistance of conductor per mile $= \frac{E}{I} = \frac{31.5}{98.4} = 0.32$ ohm, and since No. 3-0 B. & S. wire has a resistance of 0.326 ohm per mile, that is the standard size which should be adopted unless a more careful study of the complete scheme should lead to a different decision in regard to the pressure of transmission.

Since, for a given amount of power to be transmitted, the current will vary inversely as the pressure, it follows that the resistance per mile of conductor to give the economic voltage drop per mile (31.5 volts in this particular example) will be directly proportional to the pressure at which the power is transmitted. Thus if 110,000 volts were found to be a more economical pressure than 88,000, the ohms per mile of conductor would be $\frac{0.32 \times 110}{88} = 0.4$, the nearest standard size being No. 2-0 (ohms per mile = 0.41).

Power Lost in Line.—If w stands for the total I^2R watts lost in the three conductors, based on the calculated value of the resistance, then

$$\begin{aligned} w &= 3 \times \text{length of line} \times I \times E_r \\ &= 3 \times 120 \times 98.4 \times 31.5 \\ &= 1115 \text{ k.w.} \end{aligned}$$

on the assumption that a transmission pressure of 88,000 volts is adopted; and since the total kilowatts transmitted are 12,000, the *percentage* power loss is:

$$\frac{1115 \times 100}{12,000} = 9.3 \text{ per cent.}$$

Voltage Regulation.—The drop in pressure per conductor, due to ohmic resistance only, will be:

$$E_r \times \text{length of line} = 31.5 \times 120 = 3780 \text{ volts}$$

or $3780 \times \sqrt{3} = 6550$ volts between wires, since the system is three-phase and the volts E_r refer to a single conductor only. The *percentage* ohmic drop is, therefore:

$$\frac{6550}{88} = 7.44 \text{ per cent.}$$

This figure alone does not, however, give much indication as to what will be the actual regulation of the line, as the effects of inductance and electrostatic capacity must be taken into account and the resultant difference of pressure between the transmitting and receiving ends of the line calculated by any

one of the usual methods. The resultant pressure drop may be found to be excessive; it may be such as cannot readily be dealt with in a practical scheme, and in such a case the economy of the line may have to be sacrificed by putting in larger conductors.

It is obvious that other conditions may render it inexpedient or impossible to adopt the most economical size of conductor as calculated by the application of Kelvin's law, but in such cases experience and common sense will usually indicate the proper course to follow. If the economic size of wire is small it is possible, but not probable, that there may be trouble due to excessive heating. A want of mechanical strength, or loss of power due to corona formation, are more likely to lead to the selection of a conductor diameter larger than the "economic size." If, on the other hand, the conductor diameter is very large, there may be difficulties in handling and in taking the strain on the individual insulators. The remedy in this case is obviously to subdivide the single circuit into two or more parallel circuits, and, in fact, there are many advantages in doing so rather than running very heavy single conductors. One particular aspect of the question of subdivision of transmission lines is dealt with in Appendix I.

Again, even from the economic point of view, the case might arise of a temporary installation intended to give a quick return on capital invested, and an exceptionally small size of wire giving a large I^2R loss might produce the best results. This, however, leads to the consideration of the most important factor in the whole problem, namely, the correctness of the estimates of costs, depreciation allowances, and power transmitted, upon which the value of the calculated results will mainly depend. It is here that the experience, foresight and sound judgment of the engineer must necessarily play an important part, and it is not possible in this chapter to do more than draw attention to some considerations which must not be overlooked.

27. Estimation of Amount and Cost of Energy Wasted in Conductors.—The correct value of the power (P) from which the value of the current (I) is determined is frequently very difficult to estimate. This is a point which is best considered when determining the cost of the wasted energy. It is, however, clear that the annual amount of energy wasted will depend not only on the average value of I^2, but also on the *time* during

which the average amount of power may be considered as being transmitted by the wires. If, therefore, it is desired to estimate accurately the amount of energy wasted annually in the lines, a probable load curve for the year should be drawn and the average I^2 calculated therefrom. This will give a value for I which, if considered as flowing in the wires continuously throughout the year, will lead to a certain watt-hour or yearly energy loss, the cost of which it is desired to know.

Now, the annual cost of production of an additional electrical horse-power, considered apart from the total cost of production, is always difficult, if not impossible, to estimate accurately, but where coal is the source of energy there is at least the extra cost of coal consumed to be taken into account when estimating the production cost of the lost energy. The case is different in a water-power generating station, where the cost of running the station at full output is very little in excess of the cost of running at one-quarter or one-tenth of maximum output, and it is even more difficult to decide upon a figure which shall represent the cost of wasted energy (p_1 in the calculations) with sufficient accuracy to make the calculations of the economic conductor of real practical value.

There are two points in connection with water-power propositions which must never be lost sight of:

(1) If the amount of water-power available is limited, while the demand for power is unlimited, the cost (p_1), of the wasted energy may be taken at the price which the user would actually be prepared to pay for it were it available for useful purposes.

(2) If the water-power is unlimited as compared with the demand for power, the cost of wasted energy is practically *nil*, except for the fact that a generating plant has to be installed of a somewhat larger capacity than would otherwise be necessary; and the works cost of the wasted energy must, of course, include a reasonable percentage to cover interest and depreciation on this extra plant.

28. Estimation of Percentage to Cover Annual Interest and Depreciation on Conductors.—So far as interest is concerned, if cash is to be paid for the conductors, the figure to be taken for interest on capital should be on a par with the expected percentage profit on the complete undertaking; but if the conductors are mortgaged, it is the annual amount of the mortgage which should be taken.

In regard to depreciation, the probable life of the conductor must be estimated, and this, to a certain extent, may depend upon the life of the transmission line considered as a whole.

29. Economic Voltage.—It should be clearly understood that the foregoing articles deal only with the determination of the correct size of conductors based on certain assumptions as regards voltage and power to be transmitted. The cost of generating and transforming plant and buildings, *as influenced by the voltage,* must be carefully considered, together with the type and cost of pole line, so far as these are influenced by the size of the conductors. The character of the country, too, will have some bearing on the design of the transmission line, and the final choice of voltage may depend to some extent upon whether a wood pole line with comparatively short spans and (preferably) small spacing between wires is likely to be more economical than a line with steel towers which will permit of longer spans with wider spacing between wires. In other words, the total cost of the whole undertaking and the total annual losses of energy from all sources, as influenced by any change of voltage, must be considered before the line pressure as given by formula (23) can be definitely adopted as being the most economical for the undertaking considered as a whole.

Clearly, the choice of the transmission voltage is a very important matter; and since it is possible to determine the proper voltage on purely economic grounds, the use of exceptionally high pressures merely because of their interest from an engineering standpoint, should be discouraged. On the other hand, it would appear that most transmission-line troubles occur on lines working at pressures between 30,000 and 80,000 volts; and an important consideration to bear in mind is that more trouble may be experienced with heavy currents than with high voltages, owing to the more serious effects of interruptions or transient disturbances when the current is large, so that greater security may sometimes be obtained by increasing the voltage with a view to reducing maintenance and operating costs.

When figuring on the best voltage for any particular scheme, the capital cost of all works, buildings, or apparatus, which is liable to be influenced by the transmission-line pressure, together with all operating and maintenance charges which may be similarly influenced, must be taken into account. It will usually

be found convenient to reduce all such costs or differences of cost to the basis of annual charges.

30. Costs Other Than Transmission Line, Liable to be Influenced by Voltage Variations.—The cost of a generating station complete with all plant and machinery, but not including transmission line, may be anything from \$25 to \$200 per kilowatt installed. It will depend on total output, that is, on the size of the station, on location, and transport and labor facilities. The cost of a hydro-electric station will depend on the head of water, the amount of rock excavation, the size of dam, length of tunnels and penstocks, etc.

The figures given in the accompanying table are approximate costs per kilowatt (not including the transmission line) for a medium-head hydro-electric development suitable for a total output in the neighborhood of 10,000 k.w. to be transmitted over

	Transmission-line voltage		
	30,000	60,000	100,000
Hydraulic works outside power-station buildings	\$15.00	\$15.00	\$15.00
Power-station building, including excavations	5.00	5.06	5.10
Receiving-station building	1.00	1.03	1.05
Switch-gear (both ends)	1.20	1.35	1.70
Electrolytic lightning arresters	0.34	0.66	1.20
Transformers (both ends)	2.50	2.90	3.50
Generators and exciters	8.00	8.00	8.00
Cables in buildings, entering bushings, etc	0.40	0.40	0.50
Crane, sundries, and accessories, including preliminary work	2.00	2.10	2.20
Turbines and hydraulic equipment	10.00	10.00	10.00
Total cost per kilowatt	\$45.44	\$46.50	\$48.25

two outgoing three-phase feeders. The usefulness of these figures lies mainly in the indication they give of the probable differences in cost with the variation of transmission-line pressure.

31. Annual Charges Depending on Voltage.—These charges may be summarized as follows:

1. A percentage on all capital expenditure, whether for generating station, transmission line, or receiving stations, which is not constant irrespective of voltage.

2. The yearly cost of the power lost in the transmission line.

3. The yearly cost of power lost in generators and transformers (the efficiency of the electrical plant will not necessarily be the same for all voltages).

4. The yearly cost of maintenance and operation. This may depend upon length of spans in transmission line, and on the necessary plant, switch-gear, etc., to be attended to, and kept in working order.

The percentages referred to under item (1) must include interest on capital invested and depreciation.

32. Depreciation.—Depreciation is the loss of value or commercial utility due to deterioration with age. The term may be used to cover loss of value resulting from very different causes. A distinction should be made between *natural* and *functional* depreciation.

Natural or physical depreciation is the loss of value due to physical or chemical changes which, in time, will render the machine or plant practically useless. Atmospheric changes, alternations of heat and cold, wear and tear, erosion, rust, decay, electrolysis, are causes of natural depreciation.

Functional depreciation is the loss of value due to the fact that, with the lapse of time, the machine, plant, or structure under consideration does not function as efficiently as when it was first put into use, or as efficiently as it should function to compete with improved methods or apparatus. It may become inadequate owing to rapid growth in the demand for the service which it is intended to render; or it may become obsolete. Thus, functional depreciation may be due to either *inadequacy* or to *obsolescence.* A machine or structure becomes obsolete owing to scientific or artistic developments, *i.e.*, inventions. It is practically impossible to predict a future state of development in any branch of engineering, and the proper amount to allow for depreciation is largely a matter of guesswork based upon previous experience. A sinking fund for the creation of a depreciation reserve should be formed by placing annually at compound interest a certain sum of money which, at the end of the estimated life of the structure or plant, will reproduce the sum originally invested. The accompanying table has been worked out on a basis of 5 per cent. compound interest. It gives the amount, in dollars, which must be put aside each year in order to provide a fund of $100 at the end of a term of years after which the value of the works or materials under consideration is assumed to be *nil.*

Depreciation Table

(On basis of 5 per cent. compound interest earned by money put aside annually)

Life, years	Depreciation, per cent.	Life, years	Depreciation, per cent.
2	48.70	28	1.710
4	23.20	30	1.505
6	14.70	32	1.325
8	10.50	34	1.175
10	7.95	36	1.045
12	6.28	38	0.928
14	5.10	40	0.828
16	4.23	42	0.740
18	3.55	44	0.662
20	3.03	46	0.593
22	2.60	48	0.532
24	2.25	50	0.477
26	1.96		

Although it is rarely necessary to consider scrap values, an exception should be made in the case of the copper conductors of transmission lines. If D is the price originally paid for the material, and S is the estimated scrap value at the end of n years, the percentage of the original sum D to be put aside annually to cover depreciation is not r per cent.—as calculated for zero value at the end of n years—but r' per cent. of which the value is

$$r' = r\left(\frac{D - S}{D}\right)$$

The "life," in years, of any part of a machine or structure is very difficult to estimate. It is here that the distinction between natural and functional depreciation becomes important, because whichever one appears to indicate the shortest life, should be considered to the exclusion of the other. Thus, the life of wood poles—liable to decay and attacks by insects—will lead to the allowance for natural depreciation being larger than for functional depreciation, and the latter can therefore be ignored. But there are many kinds of plants, such as generators of inefficient design or insufficient capacity, of which the life determined on a basis of functional depreciation is shorter than their probable

wearing possibilities, and it is then the natural depreciation which should be ignored.

33. Example: Method of Determining Most Economical Voltage.—Consider the case of a typical medium-head hydroelectric power station:

Distance of transmission = 50 miles.

Duplicate three-phase line with copper conductors.

Cost of copper conductors = $20 per 100 lb.

Power demanded = 15,000 hp. or 11,200 k.w. (It is assumed that this power will be required continuously day and night for industrial purposes, and that it is the probable limit of the water-power available.)

Power factor = 0.8.

Selling price of power = $21 per horsepower-year.

Interest on capital invested; allow 6 per cent.

The economic drop of voltage per mile of single conductor as given by formula (19) is:

$$E_r = 9.35\sqrt{\frac{a \times p}{p_1}}$$

Where p is the price in dollars of 100 lb. weight of conductor (in this example $p = 20$), a is the percentage to cover annual depreciation and interest on cost of conductors, and p_1 is the cost per kilowatt-year of the wasted power. The proper value for a may be arrived at by estimating the term of years corresponding to the life of the conductors, at the end of which they are supposed to be of no value. Taking 16 years as the life of the conductors, the depreciation to be allowed according to the table is 4.23 which would be the proper value to take on the basis of 5 per cent. compound interest if the copper wire has no scrap value at the end of this time. It is very difficult to estimate the scrap value of conductors 16 years ahead. Apart from the market quotations which may then determine the price per pound of the metal, the fact that the transmission line is very likely a long way from the place where there is a demand for the copper must not be overlooked. Not only must the labor cost of removing the wires from the poles, together with the transportation charges, be deducted from the price obtainable for the copper, but a further deduction should usually be made to cover, in whole or in part, the cost of stringing the new conductors. Assuming the net amount likely to be obtained from the sale of the scrap copper to be $6

per 100 lb. the proper allowance for depreciation will be

$$\frac{4.23 \times 14}{20} = 2.96$$

which makes

$$a = 6 + 2.96 = \text{(say) } 9 \text{ per cent.}$$

With regard to p_1, if the demand for power were equal to the available supply from the time of the power-plant being put into operation, the works cost of waste power would be the same as the selling price; but, if we assume that the supply exceeds the demand during the first four years of operation, and that the cost of waste power during this period is only $7 per horsepower-year,[1] the average cost of wasted power during the 16 years life of the conductors should be arrived at by estimating the current and power loss for each year that the plant is in operation.

A first approximation to the required line voltage may be obtained by formula (23):

$$\text{Kilovolts} = 5.5\sqrt{\text{distance} + \frac{\text{kilowatts}}{100}}$$
$$= 5.5\sqrt{50 + \frac{11,200}{100}}$$
$$= 70$$

The current will be,

$$I = \frac{\text{kilowatts transmitted}}{\sqrt{3} \times 70 \times 0.8}$$
$$= \frac{\text{kilowatts}}{97}$$

The line losses will be proportional to I^2, and in order to arrive at a suitable value for p_1 for use in formula (19), the demand for power during the first four years of operation (before the hydraulic plant is utilized to its possible limit) should be estimated, and a table constructed as below. An average figure may be assumed for power supplied during any given period of twelve months.

[1] The actual works cost of the wasted power is always difficult to determine exactly. It must, however, be remembered that even with unlimited power, and no appreciable increase in maintenance and operating charges with increase of losses, the greater capital cost of the plant installed to provide this waste power has to be taken into account and expressed in the form of an annual charge per kilowatt wasted, whether this waste occurs in the generating and transforming plant or the line itself.

1	2	3	4
Period	Estimated kilowatts demanded	Current I	$I^2 \times$ years
1st year	4,000	41.3	1,710
2d year	5,000	51.6	2,660
3d year	6,000	61.8	3,820
4th year	8,000	82.5	6,800
5th to 16th year	11,200	115.5	160,000

Total = 174,990 or, say, 175,000

The total of the figures in the last column covering the four years during which the cost of waste power is estimated at $7 per horsepower-year, is 14,990, or say 15,000, as compared with 160,000 for the period of 12 years during which the cost of the wasted power will be $21 per horsepower-year. A reasonable value to take for p_1 is, therefore,

$$p_1 = \frac{(15 \times 7) + (160 \times 21)}{175 \times 0.746} = \$19.8$$

where the figure 0.746 is merely for the purpose of converting cost per horsepower into cost per kilowatt.

The economic resistance pressure drop, by formula (19) is therefore

$$= 9.35 \sqrt{\frac{9 \times 20}{19.8}}$$

$$= 28.2 \text{ volts per mile}$$

It is well to note that the economic voltage drop does not correspond, in this particular example, to the full load ohmic drop of pressure. The current which causes the ohmic drop of 28.2 volts per mile may be calculated as follows. The average value of I^2 is the total of column 4 in the above table, divided by the number of years, namely, $\frac{175,000}{16} = 10,930$, the square root of which is 104.5, and this is the figure for current to be used in the preliminary power-loss calculations, instead of 115.5 which is the full load current. The line may therefore be considered as transmitting continuously $\sqrt{3} \times 70 \times 0.8 \times 104.5 = 10,130$ or, say, 10,000 kilowatts.

When the section of the conductors is such as to satisfy Kelvin's law of economy, the yearly cost of the I^2R losses is equal to the amount representing annual depreciation and interest on first cost of conductors; and the total annual charges on active line material, for a three-phase line, will therefore be:

$$2 \times \frac{p_1 \times I^2RL}{1000} \times 3$$

where R is the resistance per mile of conductor. But

$$I = \frac{P \times 1000}{\sqrt{3} \times E \times \cos\theta}$$

where P stands for the kilowatts transmitted.

Also: IR = voltage loss per mile = E_r. So that the formula for the total yearly charges on conductors may be written

$$\frac{2 \times \sqrt{3} \times E_r \times P \times p_1 \times L}{E \times \cos\theta} \tag{24}$$

which in this example becomes

$$\frac{2 \times \sqrt{3} \times 28.2 \times 10{,}000 \times 19.8 \times 50}{70{,}000 \times 0.8} = \$17{,}300$$

An amount which is independent of the fact that the transmission, in this particular instance, is by two three-phase lines ordinarily connected in parallel.

34. Closer Estimate of Economical Voltage.—In order to take into account first cost, life, annual maintenance, and operating charges of every portion of the complete undertaking which may be affected by a change in the transmission voltage, the costs, worked out on an annual basis, may be arranged in tabular form as here shown, where the total charges for the 70,000-volt scheme are compared with the estimated charges for an 88,000-volt transmission. In this particular example, the figures are favorable to the lower voltage; but the difference is very small.

By increasing the voltage of transmission from 70,000 to 88,000 volts, a saving of $3690 is effected on the annual charges if the copper conductors alone are considered; but the increased cost of other portions of the complete plant, *due to the raising of the line pressure*, results in an actual increase of the *total* annual charges thus showing the pressure first chosen to be preferable

COMPARISON OF COSTS AT DIFFERENT VOLTAGES

Portion of complete undertaking affected by change of voltage	Estimated life, yr.	Depreciation (from tables)	Depreciation plus 6 per cent. interest	Total cost		Annual charges	
				70,000 volts	88,000 volts	70,000 volts	88,000 volts
Line conductors (copper) of most economic section $\left(\text{annual cost varies as } \frac{1}{\text{voltage}}\right)$	16	....	....	....	....	\$18,050	\$14,360
Steel tower transmission line, without conductors, but otherwise complete (from curves, Fig. 18)	18	3.55	9.55	\$126,000	\$153,000	12,030	14,600
Generating-station buildings	40	0.828	6.828	56,600	57,000	3,870	3,895
Substation buildings	30	1.505	7.505	11,500	11,900	863	893
Transformers (both ends of line)	18	3.55	9.55	85,000	91,000	8,120	8,680
Switch-gear, including lightning arresters, cables in buildings, and entering bushings	14	5.10	11.10	30,000	38,000	3,330	4,220
Assume unaltered:							
Yearly cost of power lost in generators and transformers.							
Yearly cost of operation and maintenance.							
Right-of-way and clearing.							
Difference in favor of 70,000 volts = \$385.						\$46,268	\$46,648

to a higher pressure. The process could be repeated for a voltage lower than 70,000; but the very small difference in favor of this pressure as compared with 88,000 volts indicates that any further appreciable reduction of the transmission voltage would almost certainly lead to a higher annual cost.

It will be understood that the accompanying estimate of total annual charges of the two selected voltages does not include any items other than those that are liable to vary with changes in the line voltage. An estimate covering the complete undertaking would, in addition to the items named, have to take account of riparian rights for dam, reservoir, etc., preliminary legal and other expenses; cost of providing proper access for materials to site of works; dam and hydraulic works outside station building; turbines; electric generators and exciters; auxiliary plant; sundries and contingencies.

In the case of a short distance transmission with a line pressure not exceeding 11,000 volts, and the possibility of winding the generators for the full pressure, the relative costs and efficiencies of generators wound for different voltages should be taken into account.

CHAPTER IV

ELECTRICAL PRINCIPLES AND CALCULATIONS

35. Materials.—Under ordinary circumstances, the choice of material for the conductors of an overhead H. T. transmission line lies between copper and aluminum. Under certain conditions, as for the transmission of continuous currents or when the price of more suitable materials is abnormally high, galvanized iron or steel may prove satisfactory and economical; and compound wires or cables such as copper-clad steel, and aluminum cables with galvanized steel core, are used where great mechanical strength is of more importance than high conductivity. Much has been written on the relative advantages of copper and aluminum for transmission-line conductors, and some writers, who have not been interested in the sale or manufacture of conductor materials, have no doubt treated the subject impartially, and stated the case for either metal with clearness and ability; but there is usually a tendency to give too much weight to the question of first cost. It is very difficult to make a comparison which shall be of general utility, between various metals, because not only the electrical, but also the mechanical, properties have to be taken into account, and the requirements in the latter respect will depend largely on local conditions. Then again, with market fluctuations, and tariffs controlling the prices of raw materials in different countries, together with varying costs of freight from manufacturers' works, a comparison of costs, except when based on current quotations, is of little value. For these reasons no direct comparison between conductors of different materials will be made here, but leading particulars will be given, together with such notes as the writer's experience may suggest, which it is hoped will be helpful to the transmission-line engineer in deciding upon the right material to use under given circumstances. Tables of resistances, sizes and weights, and other physical properties of the materials will be found in the various engineering handbooks and manufacturers' catalogues; and only such particulars will be given here as may be useful for preliminary calculations.

36. Copper.—It is probably safe to assert that, *apart from the question of cost,* the high conductivity combined with the great strength and elasticity of hard-drawn copper, give this material the advantage over all others for use on the average high-tension electric transmission line.

The ultimate tensile strength of hard-drawn copper is greater per square inch of section in the smaller wires, being approximately as follows:

Gauge No., B. & S.	Diameter, inches	Breaking stress, lb. per sq. in.
000	0.410	52,000
0	0.325	55,000
2	0.258	58,000
4	0.204	60,000
6	0.162	62,000
8	0.128	64,000
10	0.102	65,000
12	0.081	66,000
14	0.064	67,000
16	0.051	67,500
18	0.040	68,000

A stranded cable, in which the pitch is usually between 12 and 16 diameters of the cable, will generally break under a load slightly smaller than the combined breaking loads of the individual wires. The tensile strength of a stranded cable should, however, not be less than 90 per cent. of the combined strengths of the single wires.

The elastic limit of hard-drawn copper wires is about 60 per cent. of the breaking stress; but it may be as high as 70 per cent. and even 75 per cent. of the ultimate stress.

37. Aluminum.[1]—The conductivity of hard-drawn aluminum wire is between 60 per cent. and 61½ per cent. by Matthiessen's standard; pure copper being 100 per cent. The weight of an aluminum conductor is almost exactly half that of the copper conductor of equal resistance, and it is about 77 per cent. as strong as the equivalent copper cable (safe working stress).

[1] Valuable information regarding the properties and uses of Aluminum wire will be found in the publication entitled "*From the Falls to the Factory,*" issued by the British Aluminium Company, Ltd. of London, England, and Toronto, Canada.

Comparing aluminum of 61 per cent. conductivity with copper of 97 per cent. conductivity, the diameter of the equivalent aluminum cable would be 1.26 times the diameter of the copper cable.

The ultimate tensile stress of hard-drawn aluminum wire usually lies between 24,000 and 32,000 lb. per sq. in., depending upon the size of wire and hardness; if carried beyond a certain point, high tensile strength is a disadvantage, because the conductivity is lowered and the wire becomes "short." Some recent tests made on the strands composing an aluminum cable of 61 per cent. conductivity gave the following results:

Diameter of wire (inch)	0.1092	0.116	0.138
Number of tests	8	.....	33
Breaking stress, highest	34,500	.....	28,900
Breaking stress, lowest	28,200	.....	24,200
Breaking stress, average	32,100	29,300	26,100

The elastic limit of hard-drawn aluminum wire is from 50 to 60 per cent. of the breaking stress.

Aluminum is readily attacked by alkaline substances, and coils of cable should not be left lying on wet marshy ground liable to contain alkalies, or in old stables where ammonia may be present.

Aluminum is not easily soldered, because of the thin film of oxide which quickly forms on the surface exposed to the atmosphere. The tin must be *mechanically* worked through the oxide coating with the aid of an old file or, preferably, a scratch brush with bristles of 0.01 in. diameter steel not more than 1 in. long. A little experience is needed for neatly soldering aluminum into cable sockets, etc., partly for the above reason and also because the metal is a good conductor of heat, and the parts to be tinned will cool rapidly unless special precautions are taken.

38. Iron and Steel.—The ordinary commercial galvanized steel strand cable, as used for guy wires, has a breaking strength averaging 70,000 lb. per sq. in., and a conductivity of about 11½ per cent. by Matthiessen's scale. When used to convey alternating currents, the high permeability of iron increases the so-called skin effect, with the result that the resistance to the flow of current may be greatly increased, depending upon the size of the cable and the frequency. Apart from the greater

loss of voltage due to apparent increase of *resistance* when iron wires are used with alternating currents, the loss of pressure due to increased *reactance* must also be taken into account. The *external* reactance is the same for a given diameter and spacing of wires whatever may be the material; but the *internal* reactance will obviously be much greater for a "magnetic" than for a "non-magnetic" conductor. This point will be taken up again in a later article. When comparing iron with copper or aluminum as a possible material for conductors; the shorter "life" of the iron wire must not be overlooked. A good quality of galvanized wire or stranded conductor should be used, such as the E. B. B. (Extra Best Best) grade of which particulars are given in the following tables.

The weight of an iron or steel cable will be at least 5 times that of the copper cable of equal resistance, and with the higher grade (and stronger) steels, this multiplier may be as high as 10. High-grade steel conductors can be used to advantage for very long spans, or where the climatic conditions are such as to subject the cables to abnormally great stresses. Extra high strength steel wire can be obtained with an ultimate strength of 180,000 lb. per sq. in. and an elastic limit of 110,000 lb. per sq. in. A possible maximum working stress for this material would be about 80,000 lb. per sq. in.

Whatever may be the material of the conductor, a stranded cable made up of a large number of small wires will be stronger than a cable of the same sectional area made up of fewer large wires.

39. Copper-clad Steel.—By welding a coating of copper on a steel wire, a compound wire known as hard-drawn copper-clad steel wire is produced. This has been well tested, and experience has shown it to be an excellent material for many purposes. The wire can be made up in the form of cables if desired, which, when used as conductors for overhead transmissions, will have greater strength than cables made entirely of copper, and lower resistance than cables made entirely of steel. The two metals are intimately and permanently welded together by means of a special copper-iron alloy, and the relative quantities so adjusted that the finished wire has a conductivity of 30 per cent. to 40 per cent. of a copper wire of the same diameter. The ultimate tensile strength of commercial copper-clad wire of various sizes is approximately as below:

Gauge No., B. & S.	Diameter, in.	Breaking weight, lb.	Number of times stronger than copper of same diameter
000	0.410	7600	1.15
0	0.325	5400	1.20
2	0.258	3700	1.23
4	0.204	2700	1.38
6	0.162	1750	
8	0.128	1200	1.47
10	0.102	780	

40. Stranded Cables with Steel Wire Core.—The central wire of a stranded conductor may be galvanized steel, or a small diameter steel cable may be used for the core. This increases the strength, especially in the case of aluminum cables, and a compound conductor of this sort is useful for long spans on an aluminum wire transmission line.

It is usual to neglect the current-carrying capacity of the steel core, and calculate the conductivity on the assumption that all the current is carried by the strands of the higher conductivity metal. Composite cables can be made of steel and copper wires, but the strength of hard-drawn copper is so great that the gain due to the addition of the steel core is comparatively small.

The impedance of a steel core conductor will be higher than that of a conductor made entirely of non-magnetic material, but experiment has shown that the increase of impedance is practically negligible when there are two layers of copper wires spiralled in reverse directions on a central steel core; it would seem as if the current divided itself in two equal parts circulating in opposite directions, thus neutralizing any tendency to magnetize the steel core.

Hemp Core Cables.—When a stranded conductor is made up of one material only, the central wire is subjected to a greater strain than the wires that are spiralled around it. This difficulty can be overcome by using hemp for the central core. A hemp core cable will have slightly increased diameter for the same conductivity and greater smoothness of surface than a metal core cable, and these features will raise the critical voltage at which corona will form.

For particulars relating to underground cables, the reader is referred to Chapter VII.

41. Physical Constants and Sizes of Commercial Conductors.—The accompanying table gives the most important physical constants for various conductor materials. It will be noticed that aluminum has a larger temperature coefficient than copper. This has an important bearing on the economic length of span; the difference in sag between summer and winter temperatures is often considerable with aluminum conductors, but this difference is, of course, more noticeable on the shorter spans such as occur with a wood pole construction: on long spans, the difference in sag due to temperature changes is very small, whatever metal is used.

An argument often advanced in favor of aluminum conductors is that the weight of these, for any given transmission scheme, is only about half that of copper. This is certainly an advantage in the handling of the wire, but otherwise it is at least counterbalanced by the fact that the wind effect is greater on the increased diameter and that the towers must often be higher than if copper is used, partly on account of the higher coefficient of expansion of aluminum, but mainly because of the lower permissible stress. The advantage of lighter weight is largely discounted by the fact that the equivalent aluminum conductor can only be drawn up to a tension equal to about three-quarters of the permissible maximum tension of the copper cable. On the other hand, the larger diameter of the aluminum cable may be an advantage on very high-pressure transmissions, because it raises the critical voltage at which corona losses become appreciable.

It has been customary in the United States for those in control of the metal markets to regulate the price of aluminum so that there shall be no economic advantage in using it to replace copper; but it is used by several power companies, among which may be mentioned the Pacific Light and Power Co. and the Southern Sierras Power Co. On the continent of Europe and in Canada aluminum has found favor and is much used.

The accompanying wire table gives the approximate resistances and weights of the usual sizes of cable, whether of copper or aluminum, but makers' lists should be consulted for exact particulars, as the method of building up the stranded conductors necessarily modifies to a small extent the average figures here given. The figures in the table are intended for quick slide rule calculations, and the resistances are approximately correct for

PHYSICAL CONSTANTS OF CONDUCTOR MATERIALS

Properties of conductor materials	Copper, hard-drawn solid or stranded	Aluminum, hard-drawn stranded	Copper-clad steel, 40 per cent.	Copper-clad steel, 30 per cent.	Siemens-Martin steel, galvanized	Iron, E. B. B. grade, galvanized
Breaking stress, lb. per sq. in. of cross-section	50,000 to 65,000	22,000 to 30,000	60,000 to 100,000	60,000 to 100,000	60,000 to 80,000	
Breaking stress (average)	57,000	26,000	80,000	80,000	70,000	55,000
Maximum working stress (average)	28,000	13,000	40,000	40,000	33,000	26,000
Elastic limit (lb. per sq. in.)	30,000 to 35,000	14,000 to 17,000	50,000	50,000	38,000	30,000
[1]Modulus or coefficient of elasticity (Young's modulus)	15×10^6	9×10^6	20×10^6	20×10^6	29×10^6	25×10^6
Coefficient of linear expansion of wire per degree Fahrenheit = a	9.6×10^{-6}	1.28×10^{-5}	6.7×10^{-6}	6.7×10^{-6}	6.6×10^{-6}	6.6×10^{-6}
Weight per cubic inch	0.323 lb.	0.097 lb.	0.298 lb.	0.298 lb.	0.285 lb.	0.282 lb.
[2]Weight per mile per circular mil	0.016 lb.	0.0048 lb.	0.015 lb.	0.015 lb.	0.014 lb.	0.014 lb.
[3]Coefficient k	0.485	0.146	0.447	0.447	0.427	0.423
[4]Resistance; ohms per mile per circular mil at 68° F.	54,700	89,600	140,000	187,000	630,000	330,000
Relative resistance	1.00	1.64	2.56	3.41	11.5	6.00
Relative conductivity	1.00	0.61	0.39	0.293	0.087	0.167

[1] Being ratio $\dfrac{\text{stress in lb. per sq. in.}}{\text{extension per unit length}}$.

[2] To obtain weight per mile of any size of wire, multiply these figures by the cross-sectional area expressed in circular mils.

[3] Used in sag calculations.

[4] To obtain resistance per mile of any size of wire, divide these figures by the number of circular mils in the cross-section.

a temperature of 60° F. The sizes of the smaller conductors are given in the B. & S. gauge because this is generally used on this continent. With this system, when the area of any particular gauge number is known, it is only necessary to double this in order to get the area of the third size larger; or if instead of multiplying by two, the multiplier 1.261 is used, this will give the area of the next size larger in the B. & S. series. It is convenient to remember that No. 10 B. & S. copper wire measures almost exactly 1/10 in. in diameter, and has a resistance of 1 ohm per 1000 ft.

When the resistance, R, per mile of a stranded conductor is known, the *weight* per mile is approximately:

$$\text{For Copper; pounds per mile} = \frac{885}{R}$$

$$\text{For Aluminum; pounds per mile} = \frac{440}{R}$$

RESISTANCE AND WEIGHT OF STRANDED CONDUCTORS

Size, cir. mils and B. & S. gauge	Diameter, inches, approx.	Circular mils, nominal	Area, sq. in., approx.	Copper		Aluminum	
				Ohms per mile	Weight per mile, lb.	Ohms per mile	Weight per mile, lb.
600,000	0.89	600,000	0.472	0.0920	9750	0.153	2920
500,000	0.81	500,000	0.393	0.1095	8100	0.182	2430
450,000	0.77	450,000	0.354	0.1210	7300	0.202	2187
400,000	0.73	400,000	0.314	0.1363	6500	0.227	1944
350,000	0.68	350,000	0.275	0.1566	5650	0.260	1701
300,000	0.63	300,000	0.236	0.1818	4880	0.303	1458
250,000	0.58	250,000	0.1965	0.2192	4060	0.364	1215
4/0	0.53	211,600	0.1661	0.260	3448	0.430	1028
3/0	0.47	167,800	0.1317	0.326	2730	0.542	816
2/0	0.42	133,100	0.1045	0.410	2165	0.684	647
0	0.37	105,600	0.0830	0.518	1705	0.862	513
1	0.33	83,700	0.0657	0.655	1346	1.085	407
2	0.29	66,400	0.0521	0.826	1067	1.370	323
3	0.26	52,600	0.0413	1.040	850	1.728	256
4	0.23	41,700	0.0327	1.313	675	2.185	203
5	0.207	33,090	0.0260	1.635	540	2.720	162
6	0.183	26,250	0.0206	2.091	422	3.470	126

42. Skin Effect.—Imagine a straight length of cable of fairly large cross-section, through which a steady continuous current is flowing, the return circuit being a considerable distance away. The magnetic induction due to this current will not be only in the non-conducting medium surrounding the wire, but a certain amount—due to the current in the central portions of the cable—will be in the substance of the conductor itself. In other words, the magnetic flux surrounding one of the central strands of the cable will be greater than that which surrounds a strand of equal length situated near the surface. It follows that, if the circuit be now broken, the current will die away more quickly near the surface of the conductor than at the center; and, for the same reason, on again closing the circuit, the current will spread from the surface inward.

If, now, the conductor be supposed to convey an alternating current, it is evident that, with a sufficiently high frequency (or even with a low frequency if the conductor be of large cross-section), the current will not have time to penetrate to the interior, but will reside chiefly near the surface. This crowding of the current toward the outside portions of the conductor has the effect of apparently increasing the resistance; and it follows that if I is the total current in a cable of ohmic resistance R, the power lost in watts would no longer be I^2R, as in the case of a steady current, but I^2R', where R'—which stands for the apparent resistance of the conductor—is k times greater than R, its true resistance. The multiplier k may be read off the diagram Fig. 20, or if preferred, it can be calculated by means of the formula:

$$k = \frac{1 + \sqrt{1 + F^2}}{2} \tag{24}$$

where F is a factor proportional to the vertical distances on the diagram, that is to say, to the quantity *area of cross-section* $\times$ *frequency*. The value of F for copper is:

$$F = 0.0105d^2f$$

and for aluminum,

$$F = 0.0063d^2f$$

where d is the diameter of the conductor in inches, and f is the frequency in periods per second. This formula and the curves of Fig. 20 are based on the assumption that the return current is at an infinite distance; but this assumption introduces

no appreciable error when dealing with overhead transmission lines.

It will be observed that, so long as the product d^2f remains unaltered, the multiplier k is constant provided the material

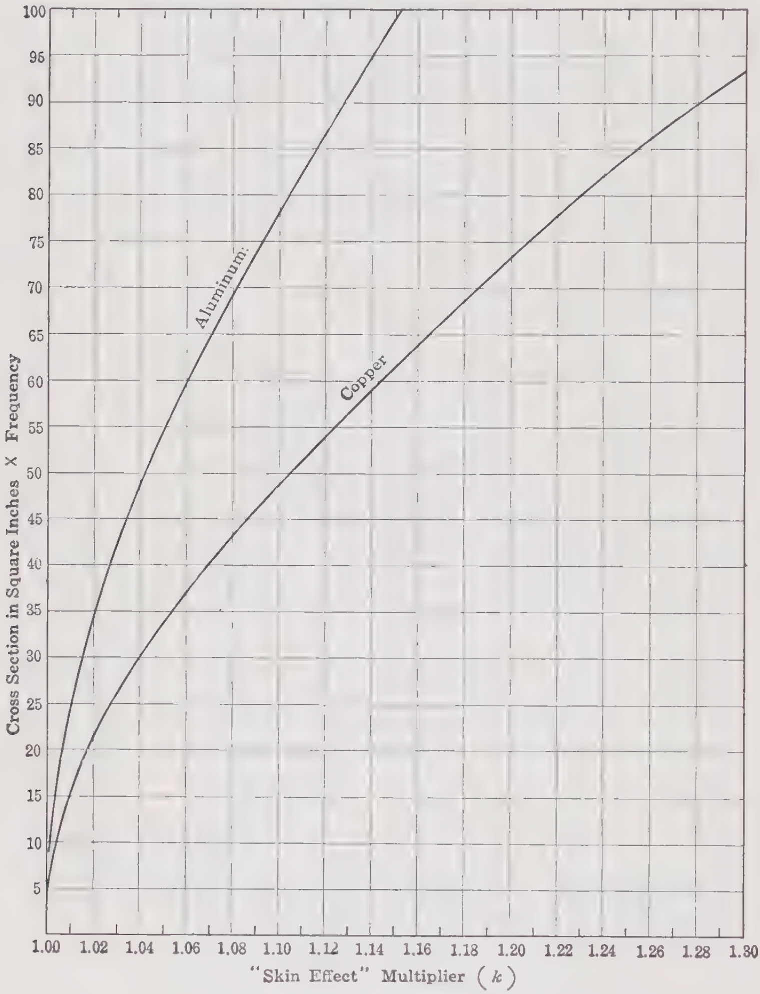


Fig. 20.—Diagram giving "skin effect" coefficient.

remains the same. Thus if, when doubling the frequency, the sectional area of the (circular) conductor is halved, the ratio $\frac{\text{resistance to alternating currents}}{\text{resistance to continuous currents}}$ remains unaltered.

In regard to the *material* of the conductor, the value of F in the formula is directly proportional to the specific conductivity of the metal so long as the frequency remains constant. Thus if F (or the value of the ordinates in the diagram, Fig. 20) is known for a conductor of given diameter, made of copper, its value for any other "non-magnetic" material is given by the ratio:

$$\frac{\text{conductivity of metal of conductor}}{\text{conductivity of copper}}$$

If the conductor is of iron (or other "magnetic" material), the value of k may be much greater than this ratio would indicate. This point will be taken up again in Article 44.

It is a not uncommon belief that when aluminum conductors are used in place of copper, the larger diameter necessary to give the same conductivity will lead to a greater loss through "skin effect;" but the above multiplying ratio makes it clear that the percentage increase of losses with alternating currents of the same frequency will be independent of the material of the conductor (iron excepted), because the greater sectional area necessary to maintain the same ohmic resistance of the lines when a wire of lower conductivity is used, is evidently exactly balanced by the higher specific resistance of the metal.

The increased pressure drop and I^2R loss on overhead lines at normal frequencies and with conductors of average size are usually very little greater with alternating than with continuous currents; but when the material is iron or steel the difference may be very noticeable, and in such cases as the rail return of an alternating current traction system, it should be taken into account.

43. Inductance of Transmission Lines.—For the purpose of calculating the flux of induction outside a straight cylindrical conductor, it is permissible to assume that the current is concentrated on the center line of the wire. The lines of magnetic induction surrounding a long straight wire carrying an electric current of which the return path is at a considerable distance, will be in the form of circles concentric with the conductor. The number of lines, or flux in maxwells, contained between any two imaginary concentric cylinders, of average radius x centimeters,

and axial length l centimeters (see Fig. 21) will be the product of the magnetomotive force by the permeance, or

$$d\Phi = \frac{4\pi}{10}I \times \frac{l \times dx \times \mu}{2\pi x}$$
$$= \frac{2Il\mu}{10} \times \frac{dx}{x}$$

where I is the current in the wire, μ is the permeability, and dx is the separation between the cylinders, in centimeters.

Assuming dx to become smaller and smaller without limit, and putting $\mu = 1$ (for the condition of flux lines in air), the ex-

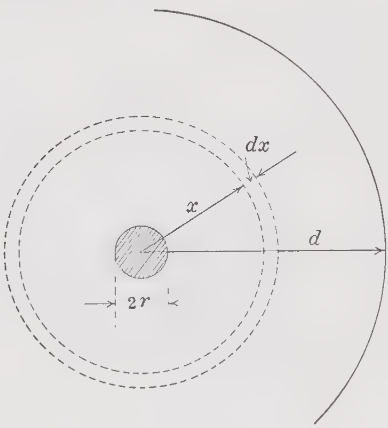


FIG. 21.

pression for the total flux *outside* the conductor, up to a limiting distance of d centimeters, is

$$\Phi = \frac{2Il}{10}\int_r^d \frac{dx}{x}$$
$$= \frac{2Il}{10} \log_\epsilon \left(\frac{d}{r}\right) \qquad (25)$$

where r is the radius of the conductor, in centimeters.

44. Effect of Taking into Account the Return Conductor.—The effective flux surrounding any single conductor of a transmission system will depend upon the distance of the parallel return conductor or conductors.

Consider, first, the loop formed by two parallel conductors of circular cross-section, one carrying the outgoing current I and the other carrying the return current $-I$ (see Fig. 22). The flux due to the current I in the conductor A may be considered as extending indefinitely throughout space, with ever-weakening intensity as the distance from the conductor increases, and the same argument applies to the flux surrounding the return conductor B, the only difference being that, if the direction of the flux round A be considered positive, that which surrounds B will be in a negative direction. It follows that the whole of the magnetic flux due to the current I in A, which is situated at a distance greater than the distance d between centers of the outgoing and return conductors, is exactly neutralized by the flux

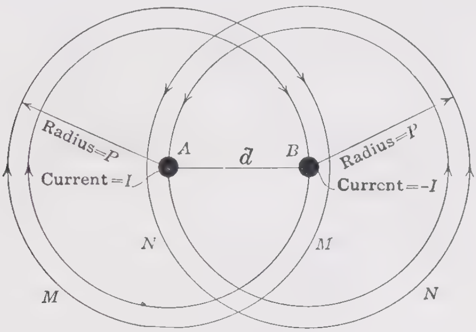


FIG. 22.—Magnetic lines of force around two parallel conductors.

due to the current $-I$ in B. Thus, in Fig. 22, it will be seen that the flux of induction surrounding A up to a distance d, is not neutralized by the current $-I$ in the conductor B; but any magnetic line, such as M, situated at a greater distance, P, from the center of the conductor A, is exactly neutralized by the magnetic line N, due to the return current in conductor B, since it also surrounds the conductor A, but in a direction opposite to that of the line M. It follows that the total *effective flux* surrounding A—that is, the resultant flux which will give rise to an induced e.m.f. in the conductor when carrying an alternating current—is merely that portion of the total self-produced flux included between the surface of the conductor and the surface of an imaginary cylinder, concentric with the conductor, and of radius d, equal to the distance between the centers of the outgoing and return conductors. The formula (25) may, there-

fore, be used for calculating the flux which is effective in producing an e.m.f. of self-induction in a straight conductor when the whole of the return current is situated at a distance d from the center of the conductor.

45. Effect of Flux Lines in the Material of the Conductor.—The flux as calculated by formula (25) is that which surrounds the wire, and when the current to which it owes its existence alternates in direction, an e.m.f. of self-induction will be induced in the conductor.

Since 100,000,000 lines cut per second generate one volt, and the total effective flux surrounding the conductor is twice created and twice destroyed in the time of one complete period, the *mean* value of the e.m.f. of self-induction will be $\frac{4\Phi f}{10^8}$ volts. The virtual or r.m.s. value, on the sine-wave assumption, is $\frac{\pi}{2\sqrt{2}}$ or 1.11 times this quantity, whence

$$\text{Induced volts} = \frac{4.44\Phi f}{10^8} \tag{26}$$

If I stands for the virtual value of the alternating current, the maximum value of Φ, by formula (25), will be

$$\Phi = \frac{2(\sqrt{2}I)l}{10}\log_\epsilon \left(\frac{d}{r}\right)$$

Substituting in formula (26) after replacing l by the number of centimeters in a mile, and converting the Napierian logarithms into common logarithms, we get

$$\text{Volts induced per mile of conductor} = 0.00466 fI \log\left(\frac{d}{r}\right) \tag{27}$$

This formula is approximately correct for conductors of overhead transmission lines when these are of "non-magnetic" material; but it should be slightly modified to take into account the effect of the flux lines *within* the material of the conductor. This additional drop of pressure is not easily calculated because different amounts of flux link with different portions of the conductor. It is obvious that a portion of the conductor near the surface is surrounded by fewer flux lines than a portion near the center of the cross-section. The result is that the e.m.f.

induced per unit length of conductor is not the same throughout the cross-section. This suggests the possibility of eddy currents in the wire; but what actually takes place is a distribution of the current density over the cross-section such that the total impedance drop—or the IR drop added (vectorially) to the IX drop—will have the same value at all parts of the conductor cross-section. The correct calculation of the internal reactance drop for "non-magnetic" cylindrical conductors is given in Prof. H. B. Dwight's book on Transmission Line Formulas.

The result is that the inductive pressure drop is actually somewhat greater than as indicated by formula (27), which neglects the internal flux. The corrected formula is

$$\left.\begin{array}{l}\text{Volts induced per mile}\\ \quad\text{of single conductor}\end{array}\right\} = fI\left[0.00466 \log\frac{d}{r} + 0.000506\right] \quad (28)$$

$$= 0.00466fI\left[\log\frac{d}{r} + \frac{0.000506}{0.00466}\right]$$

The antilogarithm of the constant in the brackets is 1.285, and it is more convenient to write the formula

$$\left.\begin{array}{l}\text{Reactive voltage drop per mile}\\ \quad\text{of single conductor}\end{array}\right\} = 0.00466fI \log\left(1.285\frac{d}{r}\right) \quad (29)$$

which is the same as formula (6) already given in Chapter II. The reactance of stranded cables is slightly less than that of solid conductors of the same cross-section, owing to the fact that the overall diameter of the cable is greater than that of the solid wire.

Excellent *tables* giving inductive reactance in ohms per mile for different spacings and sizes of wires are given in the Standard Handbook for Electrical Engineers; these figures, when multiplied by the value of the current flowing in the conductor, give the induced volts as calculated by formula (29).

46. Iron as a Material for Transmission Line Conductors.—The European war, by limiting the supply of copper and aluminum available in Germany, and by causing an abnormal increase in the price of these metals all over the world, has led electrical engineers to consider the possibility of using other metals as conductors of electricity. Zinc has been used in Germany for insulated wires and cables; but it is mechanically weak, and generally unsuitable for overhead transmission lines.

When considering the economic advantages of using iron or steel conductors, it is necessary to take into account: (*a*) the cost of the material at the place where it is to be used; (*b*) the

"life" of galvanized iron wires or cables as compared with that of copper and aluminum; (*c*) the energy losses in transmission; (*d*) the voltage regulation, and the increased cost (if any) of maintaining the pressure within specified limits at the receiving end of the line.

Under item (*c*) the greatly increased "skin effect" with alternating currents must be taken into account as well as the higher specific resistance which requires a larger cross-section of iron than of copper wire even when the transmission is by continuous currents.

Under item (*d*) the *internal inductance* of the wire—which is almost negligible with copper or aluminum—becomes a matter of considerable importance owing to the greatly increased magnetic flux in the material of the conductor when iron or steel is used.

Although cables of extra high strength steel wire are occasionally used for long spans—such as river crossings—on important overhead lines transmitting large amounts of energy, this material would not be satisfactory as a substitute for copper or aluminum except on comparatively short sections of the entire line. It seems, however, that iron or steel conductors may be used to advantage on short-distance small-power transmissions when the price per pound of copper wire has been forced up to 30 cents or more.

On account of the wide variations in the electric and magnetic qualities of the different grades of iron and steel wire, it is practically impossible to predetermine losses and pressure regulation with a high degree of accuracy. The particulars and data, together with the numerical example, in the following articles should, however, be helpful to the reader when making preliminary calculations on iron wire transmission lines.

47. Apparent Resistance of Iron and Steel Conductors.—The relative resistances of iron and copper wires were given in Article 41, and wire tables will be found in the Handbooks for Electrical Engineers;[1] but the accompanying table includes the sizes likely to be used in practice. The figures give the approximate resistance to *continuous currents* and must be multiplied by the skin effect factor when the current is alternating.

[1] Very complete particulars relating to conductor materials will be found in the Handbook on Overhead Line Construction published by the National Electric Light Association.

Approximate Resistance per Mile of Solid Galvanized Iron Wire at 68° F.

Gauge No., B. W. G.	Diameter, in.	Ohms per mile		Weight, lb. per mile
		E. B. B.	B. B.	
2	0.284	4.1	4.9	1160
3	0.259	4.9	5.8	960
4	0.238	5.8	6.9	810
5	0.220	6.8	8.1	690
6	0.203	8.0	9.5	590
7	0.180	10.2	12.1	460
8	0.165	12.1	14.4	390
9	0.148	15.0	17.9	315
10	0.134	18.2	21.7	260

7-strand 5/16-in. galv. steel (ordinary). 5.4 ohms per mile. . 1110 lb. per mile
7-strand 1/4-in. galv. steel (ordinary). . 8.6 ohms per mile. . 660 lb. per mile
7-strand 5/16-in. Siemens-Martin steel. 7.4 ohms per mile. . 1110 lb. per mile
7-strand 1/4-in. Siemens-Martin steel. . 9.6 ohms per mile. . 660 lb. per mile
7-strand 1/4-in. E. E. B. iron. . . . . 7.8 ohms per mile. . 660 lb. per mile

The resistance of ordinary steel wire is about 30 per cent. higher than that of the E. B. B. iron.

The skin effect coefficient will depend not only upon the diameter of the wire and the frequency, but also upon the resistivity and magnetic properties of the iron or steel. The magnetic permeability will, in its turn, be some function of the current in the wire, and it is not possible to express the skin effect coefficient (k) by means of a simple formula as was done in Article 42 in connection with "non-magnetic" conductors. The coefficient k, for a frequency $f = 60$, as calculated from tests on certain samples of iron and steel conductors, may be obtained from Fig. 23.

When the true ohmic resistance, R, of the iron conductor, is multiplied by the skin effect factor (k), the product, R', will be the effective resistance of the wire to an alternating current of the given frequency (in this case 60 cycles per second). In other words, if the power wasted in heating the wire with continuous currents is I^2R, it will $I^2\,(kR)$ when carrying an alternating current of virtual value I.

48. Internal Reactance of Iron and Steel Conductors.—The formula (28) in Article 45 gives the total inductive voltage drop

in a mile of "non-magnetic" conductor; the term $0.000506fI$ being the pressure drop due to the flux lines within the material of the conductor. Obviously, if the permeability is no longer $\mu = 1$, but a larger number, the loss of pressure will be greater, and this is what occurs with iron conductors.

It is a very simple matter to write

$$\left.\begin{array}{c}\text{Voltage drop per mile of single conductor}\\ \text{due to internal reactance}\end{array}\right\} = 0.000506fI \times \mu$$

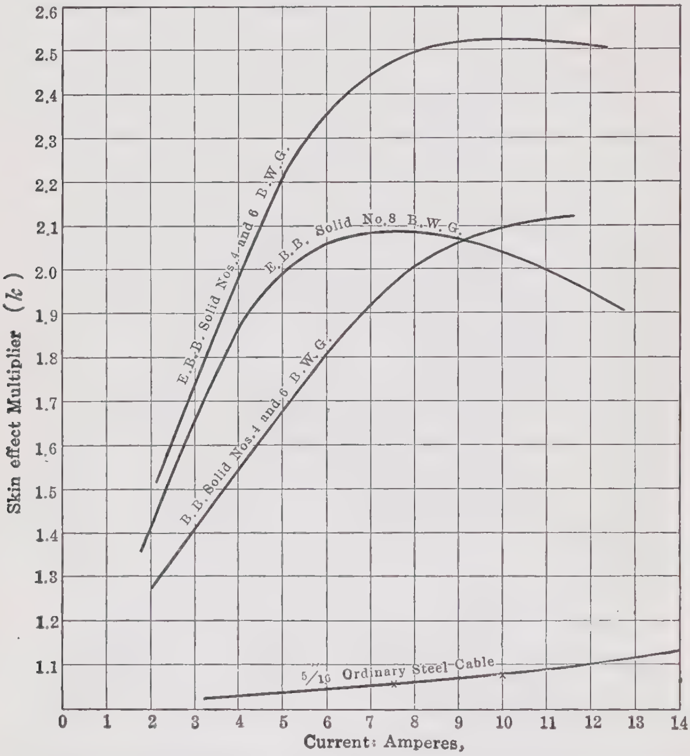


FIG. 23.—Curves giving skin effect coefficient for iron conductors for a frequency $f = 60$.

but in this connection μ is a purely imaginary number representing an "equivalent permeability" which cannot be calculated, and which is, in any case, some function of the current (I) and the frequency (f). A formula in this shape is therefore practically worthless, and it is necessary to rely on test data obtained from sizes and grades of wire approximating to those of the conductor it is proposed to use.

One of the most valuable contributions available for the use of those desiring to calculate the probable regulation and losses in lines using iron or bi-metallic conductors, is the Paper No. 252 by J. M. Miller issued by the Bureau of Standards at Washington, D. C. Additional data will be found in the article by Messrs. C. E. Oakes and W. Eckley published in the *Electrical World* of Oct. 14, 1916, in the article by L. W. W. Morrow in the *Electrical World* of July 14, 1917, and in the article by C. E. Oakes and P.

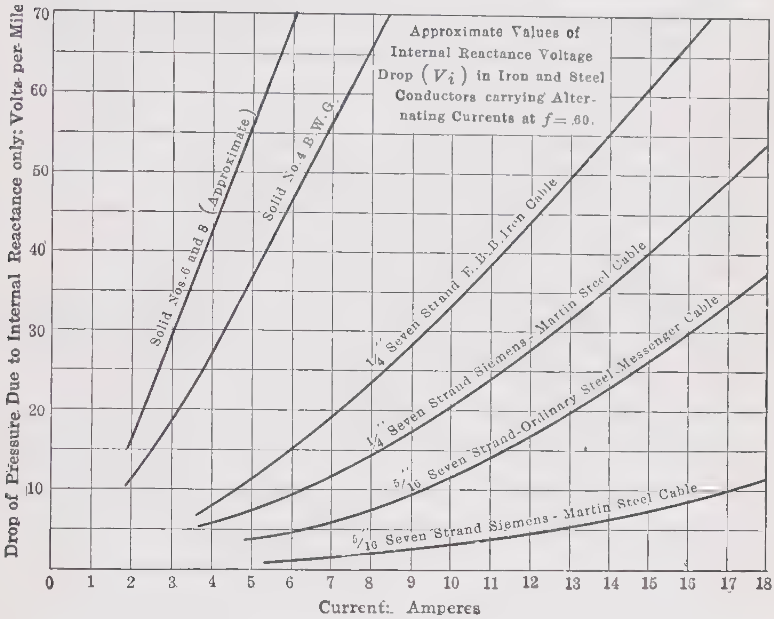


FIG. 24.—Internal reactive voltage drop in iron conductors.

A. B. Sahm in the *Electrical World* of July 27, 1918. Data from these sources have been used in preparing the curves of Figs. 23 and 24.[1]

For calculating the total inductive voltage drop in an iron wire transmission line, a modification of formula (28) may conveniently be used, because it is desirable to distinguish between the *external* reactance—which depends only upon the size and spacing of the conductors, apart from the *material*—and the *internal*

[1] A valuable collection of data referring to iron wires for transmission lines will be found in Prof. W. T. Ryan's article "Iron wire for short high-voltage lines" in the *Electrical Review* (Chicago) Sept. 22, 1917, Vol. 71, p. 496.

reactance which will be greater with "magnetic" than with "non-magnetic" materials.

If **L** is the *external inductance* (coefficient of self-induction), in henrys, of the conductor due to the flux of induction outside the wire, and $\mathbf{L}_i$ is the *internal inductance* due to the flux of induction inside the wire, the total reactance (in ohms) being

$$X\ (\text{total}) = X(\text{external}) + X(\text{internal})$$

may, on the sine wave assumption, be written

$$X\ (\text{total}) = 2\pi f\mathbf{L} + 2\pi f\mathbf{L}_i$$

and the reactive drop (in volts) when the circuit is I amperes is

$$IX = 2\pi fI\mathbf{L} + 2\pi fI\mathbf{L}_i$$

The value of **L** per mile of conductor is $0.000741 \log \frac{d}{r}$, whence

$$\left.\begin{array}{l}\text{Total reactive drop in volts}\\ \quad\text{per mile of single iron or}\\ \quad\text{steel conductor}\end{array}\right\} = 0.00466fI \log \frac{d}{r} + V_i \qquad (30)$$

where $V_i = 2\pi fI\mathbf{L}_i$ and has to be determined experimentally. Its value, for a frequency $f = 60$, may be read off Fig. 24 which, however, refers only to a limited number of sizes and kinds of wire.

49. Example of Calculations for Iron Wire Conductors.—Given a transmission line 10 miles long consisting of No. 4 E. B. B. galvanized iron wires spaced 3 feet apart, carrying a current of 5 amperes at a frequency of 60; calculate (*a*) the loss of power, and (*b*) the loss of pressure, in each wire.

(*a*) The D.C. resistance (from wire table) is $R = 5.8 \times 10 = 58$ ohms.

The skin effect factor (from Fig. 23) is $k = 2.2$

$$\text{whence } R' = 58 \times 2.2 = 127.5 \text{ ohms}$$

The watts lost $= I^2R' = 5 \times 5 \times 127.5 = 3.19$ k.w. per wire.

(*b*) The internal reactive pressure drop per mile (from Fig. 24) is $V_i = 37$ volts, whence IX (total) for 10 miles of wire (by formula (30) is

$$10 \times 0.00466 \times 60 \times 5 \times \log \frac{36}{0.119} + 370 = \text{(say) } 405 \text{ volts}$$

The IR' drop being $5 \times 27.5 = 638$ volts, it follows that the impedance drop is

$$IZ = \sqrt{(638)^2 + (405)^2}$$
$$= 755 \text{ volts}$$

This figure does not, however, necessarily represent the difference in pressure between the generating and receiving ends of the line; but this point will be taken up in the following article.

50. Inherent Regulation of Transmission Line. Regulation Diagrams.—The fundamental vector diagram, Fig. 10, which was described in Article 9 of Chapter II, is reproduced here for convenience. The resistance drop *CB* (of which the numerical value was 638 volts in the foregoing example) is drawn parallel to the current vector, while the reactance drop *DC* (of which the numerical value was 405 volts in the example) is drawn at right angles to the current vector. The impedance drop is *DB*,

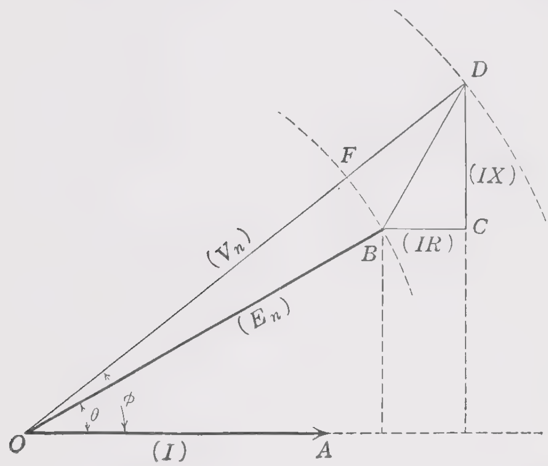


FIG. 10.—Vector diagram for line calculations—capacity neglected.

but this does not correspond with the loss of pressure in transmission except when the angle ϕ happens to have the same value as the angle θ. The difference in pressure between the generating end and receiving end voltages may be calculated as explained in Article 9, and it will depend not only on the resistance and size and spacing of the conductors, but also on the power factor of the load, since this will determine the position of the point B on the dotted circle. It is the position of the point *B* on the circle that modifies the ratio of the length *FD* to the length *BD*, even if the proportions of the impedence triangle *BCD* remain unaltered. Problems can be solved graphically by drawing the diagram, Fig. 10, to the proper scale; but the objection to this method is that the radius *OB* is generally large in proportion

to the quantities represented by the impedance triangle, and the process is either tedious or the results are unsatisfactory. The field for ingenuity in the construction of practical charts based

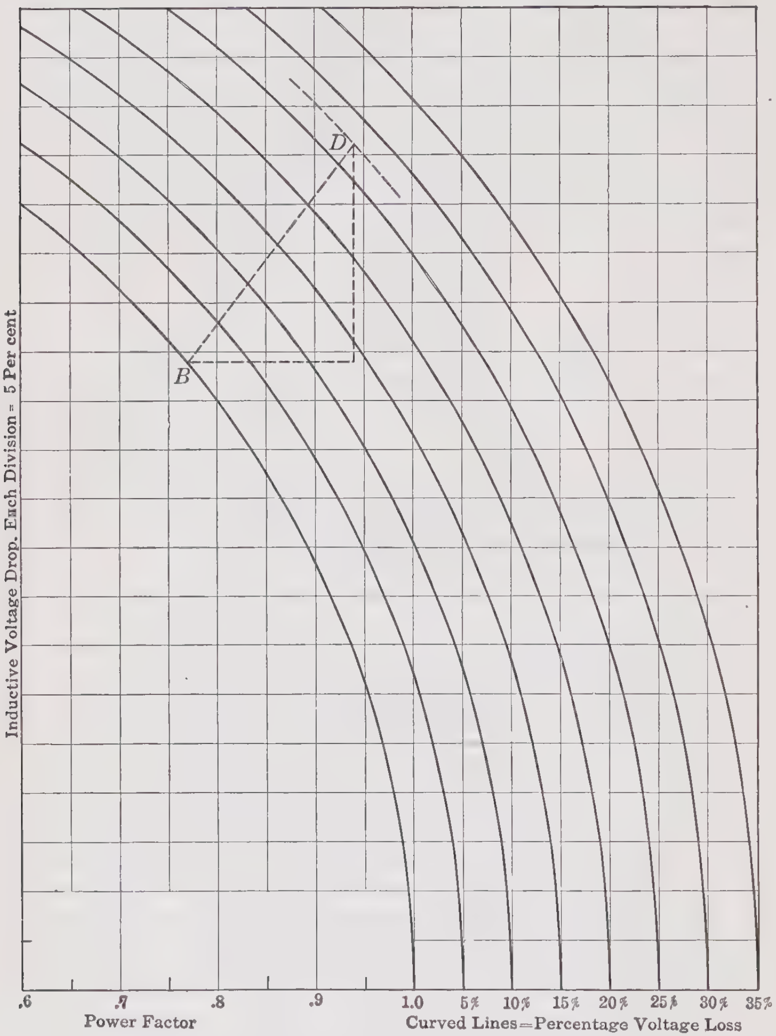


FIG. 25.—Mershon diagram for determining voltage regulation.

on the fundamental diagram (Fig. 10) is very great. One of the methods of obtaining graphical solutions is with the aid of the Mershon diagram.

In Fig. 25 curves concentric with the dotted circles of Fig. 10 are drawn on a piece of squared paper from a center which lies on the prolongation of the base line, but at a considerable distance outside the diagram. The radius of the inner circle is 10 (or 100) divisions in length, and the projection on the horizontal axis of any point B is therefore the cosine of the angle BOA of Fig. 10 and it indicates directly the power factor at the receiving end. By expressing the calculated resistance and reactive voltage drops as *percentages* of the receiving end pressure, the impedance triangle can readily be drawn to the proper scale, and by making the space between the circles equal to the side of the squares on the divided paper, the regulation, or difference

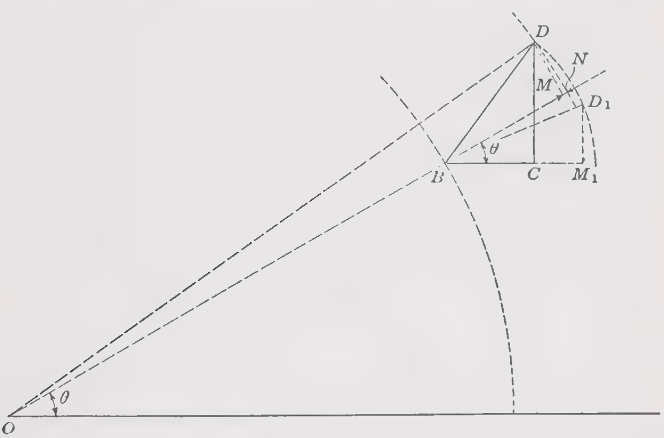


Fig. 26.—Vector diagram illustrating approximate method of determining regulation.

between generating and receiving end pressures (FD in Fig. 10), can be read off the diagram as a percentage of the receiving end pressure.

As an example of the use of the diagram, suppose the power factor of the load is 0.77, and that the calculated components of the pressure drop are,

Resistance volts = 17 per cent. of receiving end pressure.

Reactance volts = 22 per cent. of receiving end pressure.

From the division on the horizontal axis corresponding to power factor 0.77 follow the vertical ordinate until it meets the inner circle at B; then measure horizontally 17 divisions, and vertically 22 divisions, and the point D which lies on the dotted

circle 27.5 divisions larger in radius than the inner circle (which is described with a radius equal to 100 divisions) indicates that the difference in pressure between generating and receiving ends of the line is 27.5 per cent. of the receiving end pressure.

Consider now Fig. 26, which is merely a repetition of the fundamental diagram, Fig. 10, with the addition of a few lines. Drop the perpendicular DM on the radius OB extended beyond the point B. It will be seen that when the angle DOB is small, that is to say, when there is little difference between the power factors at the receiving and generating ends of the line, the distance MN will be very small, and for nearly all practical purposes the voltage regulation may be expressed by the ratio $\frac{BM}{OB}$ instead of $\frac{BN}{OB}$, this last being theoretically correct and as given by the Mershon diagram. By adopting the alternative construction, and replacing the arc DN by a straight line perpendicular to either OD or OB, the necessity for drawing circles from a center outside the limits of a practical diagram is avoided.

The method used by Professor L. A. Herdt for the calculation of transmission lines (originally described in the *Electrical World* of Jan. 2, 1909) employs this approximation; and it is also employed in the method about to be described, which the writer has found very quick and convenient for practical calculations.

It will be observed that if the impedance triangle BCD (Fig. 26) be moved round on the point B through an angle θ, so that the hypotenuse BD now occupies the position BD_1, the perpendicular dropped from D_1 on the extension to the horizontal line BC, meets this line at the point M_1, the distance BM_1 being obviously equal to BM. Thus, by revolving the impedance triangle through an angle θ such that $\cos \theta =$ the power factor of the load, the projection of the hypotenuse on any line parallel to the current vector will be a measure of the volts lost in transmission.

To apply this method in practice, nothing more is required than a piece of squared paper and a piece of tracing paper. The squared paper is divided into any convenient number of equal parts to represent, horizontally, the percentage ohmic drop of voltage, and, vertically, the percentage reactive voltage drop, as indicated in Fig. 27. On the vertical axis on the left-hand side of the diagram, a power factor scale is provided. This is merely an arbitrary length divided into ten equal parts with suitable sub-

divisions so chosen as to make use of the horizontal ruling of the squared paper. This scale is used for turning the hypotenuse of the impedence triangle through the proper angle, as will be explained shortly.

The method of using the diagram is best explained by working

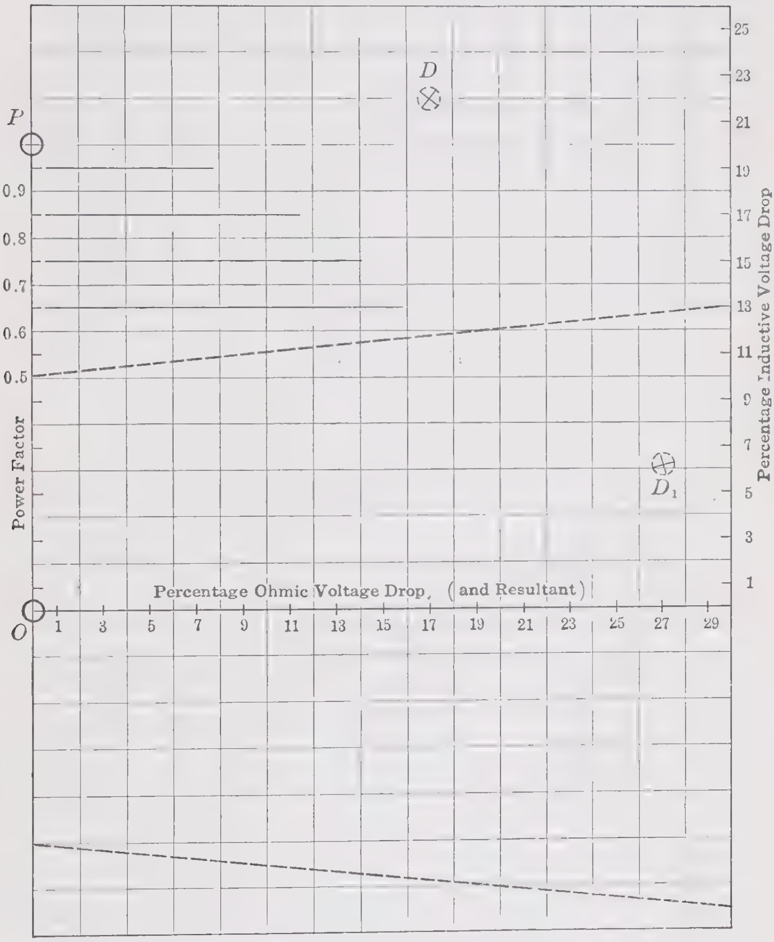


FIG. 27.—Author's diagram for determining voltage regulation.

out an example. The data used for illustrating the Mershon chart will be suitable:

$$\begin{aligned} \text{Power factor of load} &= \cos\theta = 0.77 \\ \text{Ohmic volts} &= 17 \text{ per cent.} \\ \text{Reactive volts} &= 22 \text{ per cent.} \end{aligned}$$

Place a piece of tracing paper over the diagram (Fig. 27) and mark upon it the point D, 17 divisions to the right of the vertical axis, and 22 divisions above the horizontal axis. Then, with a pin or pencil point held at the point O, move the tracing paper through an angle of 39 degrees 40 minutes ($\cos 39° 40' = 0.77$), bringing the point D to D_1, and read on the horizontal axis the distance 27.1, which is the difference between the pressures at the two ends of the line, expressed as a percentage of the receiving-end pressure. The result, as read off the Mershon diagram was 27.5, which might at first sight be thought to be more nearly correct; but, as a matter of fact, the writer's method will usually give more accurate results notwithstanding that the solution is not theoretically correct. This is because the impedance triangle is very much larger for the same size of chart than in the Mershon diagram; and the subdivisions are more easily read. When the point D_1 falls between the two inclined dotted lines drawn on the diagram (Fig. 27), this is an indication that the error introduced by substituting the chord for the arc is less than half of 1 per cent.

The use of the power factor scale will now be explained. It is not necessary, as suggested in working out the example, to calculate the angle θ from the value of the power factor and then set out this angle on the diagram. If in addition to marking the point D on the tracing paper, the position of the point P is also marked, it is merely necessary to move the tracing paper round (on the center O) until the point P falls on the horizontal line representing the required power factor, as this will ensure that the point D has been moved through the proper angle. The reason of this will be obvious to anyone possessing even the most elementary knowledge of trigonometry.

If it is preferred to work with trigonometrical tables, the formulas (7) to (11) of Article 9, Chapter II may be used instead of the diagram Fig. 25 or 27.

51. Pressure Available at Intermediate Points on a Transmission Line.—Referring again to Fig. 10, the volts per phase at generating end are V_n and at receiving end, E_n, the total drop being $(V_n - E_n)$ volts. It does not follow, however, that the pressure available at a point half way along the line will be $V_n - \left(\frac{V_n - E_n}{2}\right)$ because the power factor is rarely the same at all points.

The method of calculating the pressure available at some inter-

mediate point L' miles from the supply station on a line of total length L miles, when the effects of capacity are negligible, is illustrated in Fig. 28 where $C'C = BC\left(\frac{L'}{L}\right)$ and $OD - OD'$ is a measure of the voltage drop between the supply end and the point considered. The power factor angle at this point will be ψ which can be calculated by making the required changes in the formulas of Article 9. Thus, formula (10) would be written

$$OD' = \frac{BC' + E_n \cos \theta}{\cos \psi}$$

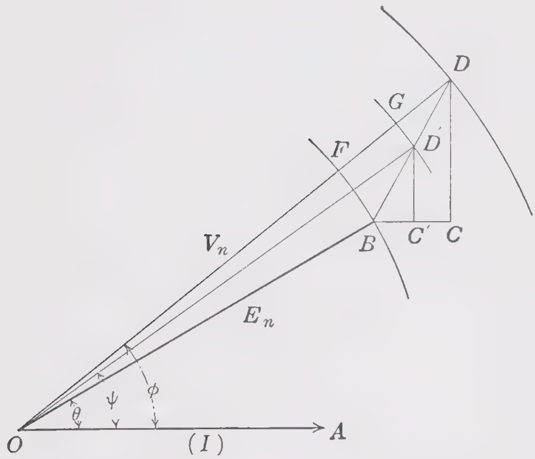


FIG. 28.—Vector diagram showing pressure at intermediate point on transmission line.

and the procedure throughout is exactly the same as if calculating the required generating end voltage on a line $(L - L')$ miles in length, to give E_n volts at the receiving end when delivering I amperes at a power factor $\cos \theta$.

Such calculations are usually unnecessary refinements, and the error introduced by assuming a uniform drop of pressure along the line is rarely of any practical importance.

The manner in which the values of pressure drop, as calculated for a single conductor, are used in determining the inherent regulation of three-phase lines was explained in Article 11 of Chapter II. The principles governing the inductive effects with any number and arrangement of parallel conductors, are discussed in Appendix I.

52. Capacity of Transmission Lines.—The formula given in Chapter II (Article 10) for the capacity in microfarads per mile of conductor, as measured between wire and neutral was:

$$C_m = \frac{0.0388}{\log \frac{d}{r}} \qquad (12)$$

This formula is not theoretically correct and would not be applicable if the distance d were very small in relation to the diameter of the wire (or the radius r); but for overhead transmissions it is a serviceable formula, and, in the writer's opinion, it may be used in all practical calculations. The question of capacity on overhead lines is, however, one of very great importance, especially in view of the increasing pressures and distances of transmission; and it is felt that some space should be devoted to it, even if it be only to sum up our present knowledge on this subject, and refer the reader to sources from which he can obtain more complete information.

The exact formula,[1] which gives the linear capacity in microfarads per mile between two cylindrical parallel wires is

$$C_m = \frac{0.0194}{\log \left[a + \sqrt{a^2 - 1}\right]} \qquad (31)$$

where $a = \frac{d}{2r}$; but it is more generally useful to consider the capacity as being measured between one wire and the neutral potential surface. This will be *twice* the value of the capacity as measured between the two wires; but, when calculating the charging current, it is the voltage *between wire and neutral surface* that must be taken, if this latter value of the capacity is used.

The formula (31) may be put into another form which is very convenient if tables of hyperbolic functions are available.

In the formula (31) common logarithms are referred to in the denominator; but by making the proper correction to the numerator and substituting Napierian logs, the denominator becomes $\log_\epsilon (a + \sqrt{a^2 - 1})$ which is the quantity of which the hyperbolic cosine is a. Thus, the inverse hyperbolic cosine of a, or $\cosh^{-1} a$, is the equivalent of $\log_\epsilon (a + \sqrt{a^2 - 1})$; and with the corrected numerator, the formula (31) becomes,

$$C_m = \frac{0.0447}{\cosh^{-1} a} \qquad (32)$$

[1] H. Pender and H. S. Osborne in *Electrical World*, Sept. 22, 1910, p. 667.

If the capacity per mile of single conductor, measured between wire and neutral, is required, the numerators of these formulas must be doubled, and the correct formula may be written either

$$C_m = \frac{0.0388}{\log\left(a + \sqrt{a^2 - 1}\right)} \tag{33}$$

or,

$$C_m = \frac{0.0895}{\cosh^{-1} a} \tag{34}$$

Some excellent practical diagrams based on these formulas are to be found in an article by Dr. A. E. Kennelly which appeared in the *Electrical World* of Oct. 27, 1910.

The approximate formula (12) given in Chapter II may be written

$$C_m = \frac{0.0388}{\log 2a}$$

and, by comparing this with the correct formula (33), it will be seen that the first gives results slightly smaller than the true values; but when a is large, that is to say, when the distance between wires is many times the diameter, the error is negligible. The error only becomes appreciable if a is less than 10, and even if a is as small as 4 (a quite impossible state of things on an overhead transmission with bare wires), the error would be only 0.8 per cent.

53. Capacity of Three-phase Lines.—The formulas in the last article give the capacity between two parallel wires as measured from wire to neutral, and in the case of a single-phase transmission, the capacity between the two wires would, as it were, consist of two such capacities in series, and would therefore measure *half* the value given by these formulas, all as previously mentioned. It should, however, be noted that it makes no difference which value of the capacity is taken for the purpose of calculating the charging current, provided proper attention is paid to the potential difference available for charging the condenser. In the case of the single-phase transmission, the pressure available for charging the two imaginary condensers in series, is exactly twice the pressure between one wire and neutral.

Consider, now, a three-phase transmission with the conductors occupying the vertices of an equilateral triangle, as indicated in

Fig. 29. If the radius r of the conductors and the distance d between them are the same as in the case of a single-phase transmission, then the capacity *as measured between the wire and neutral* is the same for the three-phase as for the single-phase transmission; but the charging current is different because the potential difference across each imaginary condenser is no longer $\frac{E}{2}$ but $\frac{E}{\sqrt{3}}$, where E stands for the voltage between wires. By treating the three-phase system—or indeed any polyphase system—as a combination of several single-phase systems each having a condenser connected between conductor and ground,

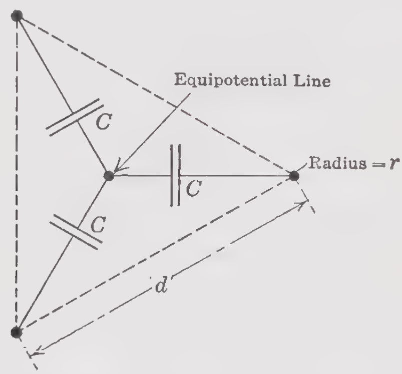


FIG. 29.—Distribution of capacity. Three-phase transmission.

the calculation of capacity currents becomes a comparatively simple matter, unless great refinements and scientific accuracy are aimed at.

It has been shown by Mr. Frank F. Fowle[1] and other careful investigators in this field, that the presence of the conducting ground or other neighboring circuits affects only very slightly the capacity between the conductors of an overhead transmission. By systematic transposition of wires on a long transmission, so that each conductor occupies the same position relatively to ground and neighboring parallel wires over the same portion of the total distance, even these slight unbalancings of the charging currents can be corrected if desired.

The electrostatic capacity of underground cables—in which the conductors are not only very close together, but are separated by

[1] "The Calculation of Capacity Coefficients for Parallel Suspended Wires," *Electrical World*, Aug. 12, 19, and 26, 1911.

insulating materials of which the dielectric constant is no longer unity as in the case of air—will be considered in Chapter VII.

54. Charging Current Due to Capacity of Transmission Lines. —Although on short, low-voltage, lines the charging current (or capacity current) is so small as to be negligible, this current becomes a matter of considerable importance on long-distance high-voltage transmission lines. In Article 10 of Chapter II the charging current as calculated for an existing overhead transmission amounted to 42.4 amperes, representing an output of 7350 apparent kilowatts from the generating station with the line entirely disconnected from the load at the receiving end.

Assuming a sine wave of impressed e.m.f., it is easy to calculate the charging current of a condenser of known capacity. The fundamental law of the dielectric circuit is

$$\Psi = E_{(\text{max.})} \times C \tag{35}$$

where Ψ is the maximum value of the dielectric flux expressed in coulombs; $E_{(\text{max.})}$ is the maximum value of the alternating voltage; and C is the capacity (or *permittance*) of the condenser, expressed in farads.

The charge, or quantity, of electricity—*i.e.*, the dielectric flux —will reach its maximum value (Ψ) at the instant when the charging current is changing its direction, that is to say, when the current is zero, and since *quantity of electricity* = *current* $\times$ *time*, we may write Ψ = average value of charging current (in amperes) during one quarter period $\times$ time (in seconds) of one quarter period

$$= \left(\frac{2\sqrt{2}}{\pi}\right) I_c \times \frac{1}{4f}$$

where I_c stands for the virtual or r.m.s. value of the charging current, on the sine wave assumption. Let E stand for the virtual value of the voltage across the condenser of capacity C farads, then $E_{(\text{max.})} = \sqrt{2}\, E$, and formula (35) becomes

$$\frac{2\sqrt{2}\, I_c}{4\pi f} = \sqrt{2} E \times C$$

whence

$$I_c = 2\pi f E C \tag{36}$$

which is the well-known formula for calculating capacity current when sinusoidal wave shapes are assumed. This is the same as formula (13) of Chapter II, which was given without proof.

It is possible to express the charging current on overhead lines in terms of the external inductance, or of the external reactive voltage drop. This is due to the fact that there is a constant relation between the inductance and the capacity, which is independent of the size and spacing of the conductors. Thus, the formula for capacity in microfarads per mile (page 29) is

$$C_m = \frac{0.0388}{\log \frac{d}{r}} \tag{12}$$

while the formula for the external inductance per mile (page 88) is

$$\mathbf{L} = 0.000741 \log \frac{d}{r}$$

giving a constant product

$$C_m\mathbf{L} = \frac{1}{34{,}700} \tag{37}$$

The external reactive pressure drop is

$$(IX) = 2\pi f\mathbf{L}$$

whence a value for **L** in terms of reactive drop is obtained. This value, substituted in (37) gives

$$C_m = \frac{2\pi f}{34{,}700\ (IX)} \tag{38}$$

By putting this value of C_m in formula (36), we get,

$$I_c = \frac{(2\pi f)^2 E}{34{,}700\ (IX) \times 10^6}$$

$$= \frac{f^2 E}{8.78\ (IX) \times 10^8} \text{ amperes} \tag{39}$$

These formulas for calculating the magnitude of the charging current, when multiplied by the length of the line in miles, will give the charging current entering the line at the generating end. The result is usually smaller than the value obtained by measurement on actual lines. The reason is that the assumption of sinusoidal impressed e.m.f. is rarely justified, and the irregularities and peaks in the actual pressure wave may cause an increase of charging current amounting to 20 or even 40 per cent. of the calculated value. These considerations emphasize the absurdity of devoting a considerable amount of time to

mathematical refinements, or of using complicated formulas of which the increased accuracy is of no practical value seeing that they are based upon assumptions that are never realized.

55. Effect of Distributed Capacity and Inductance.—A long transmission line of resistance R ohms, reactance X ohms, and capacity C farads, may be thought of as consisting of n sections, of resistance R/n ohms and reactance X/n ohms, with a condenser of capacity C/n farads shunting the wires at the end of each section. The charging current will fall off in amount as the distance from the generating end increases, and the total current in the conductor—being the (vectorial) sum of the charging current and load current—will be different in each section. The problem is further complicated by the fact that the voltage will not necessarily be the same across all the imaginary condensers, so that the reduction of the charging current component will not even follow a "straight line" law. By dividing a long line into a large number of sections, and calculating the pressure drop and power factor at the end of each section, the voltage drop and power losses of the complete line could be estimated accurately; but the work would be tedious and, indeed, unnecessary. By imagining the number of sections, n, to become larger and larger without limit, we approach the condition of *distributed capacity* for which accurate mathematical formulas are available. A great deal of excellent work has been done by Dr. A. E. Kennelly, Dr. Harold Pender, Dr. J. A. Fleming, Prof. H. B. Dwight, and others[1] in the matter of simplifying the exact methods of calculation for long lines in which the effects of capacity are not negligible; but these methods nearly all include the use—and knowledge—of hyperbolic functions in place of the trigonometric tables with which all engineers are familiar. It is true that Prof. Dwight, following the lead of Prof. T. R. Rosebrugh, has evolved a fairly simple means of computing line voltages[2] by substituting the method of convergent series, and using complex quantities, thus dispensing with the necessity for tables or charts of hyperbolic functions of angles; but, for the solution of prac-

[1] Harold Pender, *Electrical World* June 8, 1909.
A. E. Kennelly and Harold Pender, *Electrical World*, Aug. 8, 1914.
J. A. Fleming, *Jour.* Inst. E. E., p. 717, Vol. 52, June 15, 1914.
A. E. Kennelly, *Jour.* Franklin Inst., Sept., 1914.
H. B. Dwight, Book "Transmission Line Formulas," D. Van Nostrand Co.

[2] *Electrical World*, Sept. 5, 1914.

tical power transmission line problems, all of these refined methods of calculation are unnecessary. From the academic point of view, the exact solution of all engineering problems is attractive and occasionally desirable, and when the mathematical work can be so simplified as to be available for use by the average engineer, there is no objection to his using it. On the other hand if the engineer has at his disposal simpler and shorter methods of working which yield results *within the required practical limits of accuracy*, he should not be criticised for preferring them. He may base his calculations on an assumed sine wave of e.m.f., on a maximum (sine wave) current of 100 amperes, and on an estimated power factor of 0.9; but he would expect his line to give satisfaction with an actual full load current which might be anything between (say) 90 and 110 amperes, and he would consider himself lucky if the actual power factor of the load proved to be within 5 per cent. of the value he had guessed at. It is for such reasons that the practicing engineer—whose time is valuable, and who has a habit of using factors of safety somewhat freely—rarely evinces a fervent interest in mathematical refinements whereby the (theoretical) accuracy of his results may be increased by a small fraction of 1 per cent.

On lines over 50 miles in length, the effects of capacity on voltage regulation may be appreciable, and some practical methods of taking into account the capacity current on long transmission lines will be explained in the following article.

56. Electrical Calculation of Lines with Appreciable Capacity.—In Article 10 of Chapter II, an example was worked out showing how the charging current at the supply end of a long transmission line could be calculated, and the effect of this current in modifying the fundamental vector diagram was illustrated in Fig. 13. Thus, when we imagine a condenser to be connected across the lines at a point where the load current is I amperes on a power factor $\cos\theta$, the conditions on the supply side of the condenser will be a current of I' amperes and a power factor of $\cos\theta'$, these modified values being calculated as follows.

The vector diagram Fig. 30 is the same as Fig. 13 except for a few additions. The charging current can be calculated by means of the formula (36), which is

$$I_c = 2\pi f E_n C \tag{36}$$

This component of the total line current is drawn 90° in advance

of the vector $E_n = OB$ which is the pressure across the condenser terminals. By dropping the perpendicular AN on OB and making $AM = I_c$, we obtain $OM = I'$ the resultant or total current in the line on the supply side of the condenser.

In order to calculate I' and the new power factor angle θ', we may write,

$$ON = I \cos \theta$$
$$NA = I \sin \theta$$
$$NM = I \sin \theta - I_c$$

$$\text{Tan } \theta' = \frac{NM}{ON} = \frac{I \sin \theta - I_c}{I \cos \theta} \tag{40}$$

$$= \tan \theta - \frac{I_c}{I \cos \theta}$$

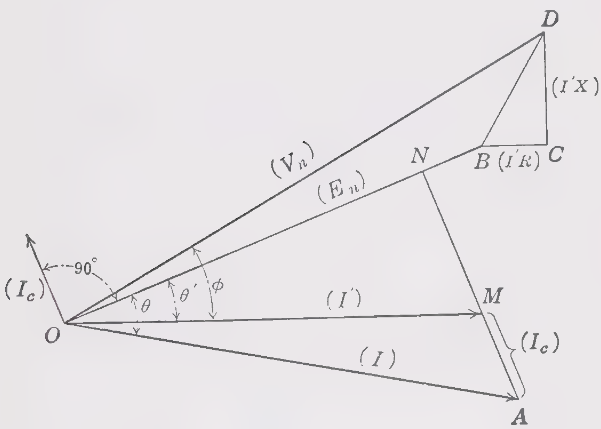


Fig. 30.

whence θ' and the other trigonometrical functions such as $\sin \theta'$ and $\cos \theta'$ can be read off the slide rule or obtained from tables.

The line current is

$$I' = \frac{ON}{\cos \theta'} = I\left(\frac{\cos \theta}{\cos \theta'}\right) \tag{41}$$

Using this value of current, the sides $I'R$ and $I'X$ of the line impedance triangle can now be calculated, and the procedure for calculating the line losses and line drop will be as described in Article 9 (where the line was assumed to be without capacity) except that I' and θ' must be substituted for I and θ in the formulas.

It will perhaps simplify matters and prevent confusion if some of these formulas are reproduced here with the necessary changes to make them applicable to Fig. 30.

The functions of the angle ϕ are

$$\sin \phi = \frac{I'X + E_n \sin \theta'}{V_n} \tag{42}$$

$$\cos \phi = \frac{I'R + E_n \cos \theta'}{V_n} \tag{43}$$

$$\tan \phi = \frac{I'X + E_n \sin \theta'}{I'R + E_n \cos \theta'} \tag{44}$$

From this last formula (44) we obtain ϕ and therefore $\cos \phi$ (the power factor at the sending end of the line), whence V_n of which the value, from (43), is

$$V_n = \frac{I'R + E_n \cos \theta'}{\cos \phi} \tag{45}$$

57. Numerical Examples Illustrating Use of Formulas for the Calculation of Power Factor and Voltage Drop.—The data for use in the calculations is as follows:

System; three-phase.
Line pressure at receiving end, $E = 66{,}000$ volts.
"Star" voltage, $E_n = \dfrac{66{,}000}{\sqrt{3}} = 38{,}100$ volts.
Frequency, $f = 60$.
Load $= 3000$ k.v.a.
Power factor of load, $\cos \theta = 0.9$.
Length of line, $L = 100$ miles.
Conductors of No. 3 copper cable (radius $r = 0.13$).
Spacing, 8 feet ($d = 96$).

From wire table, the resistance is found to be $R = 1.04$ ohms per mile. By formula (29) the reactance per wire is

$$X = 0.00466 \times 60 \times \log \left(1.285 \times \frac{96}{0.13}\right)$$

$$= 0.832 \text{ ohms per mile.}$$

By formula (12) the capacity (wire to neutral) is

$$C_m = \frac{0.0388}{\log \frac{d}{r}} = 0.0135 \text{ microfarads per mile.}$$

The load current is $I = \dfrac{3{,}000{,}000}{\sqrt{3} \times 66{,}000} = 26.2$ amperes.

By way of illustration, we shall calculate the line drop by imagining (1) the whole of the line capacity to be concentrated at the center of the line, and (2) one-half of the total capacity to be concentrated at each end of the line.

Case (1), *capacity concentrated half way between sending and receiving ends* (Fig. 31).

With the star voltage $E_n = 38{,}100$ at the receiving end, we will first calculate the voltage E'_n at the point where the condenser is supposed to be connected. We could if desired use one of the charts, Fig. 25 or 27; but it may be advisable to use, throughout these examples, the trigonometrical formulas which were developed from the fundamental vector diagram.

By formula (9),

$$\tan\phi = \frac{(26.2 \times 50 \times 0.832) + (38{,}100 \times 0.436)}{(26.2 \times 50 \times 1.04) + (38{,}100 \times 0.9)}$$

$$= \frac{17{,}690}{35{,}660} = 0.496$$

whence $\cos\phi = 0.896$ and $\sin\phi = 0.444$.

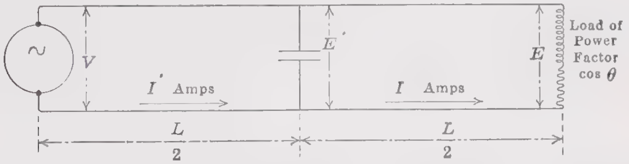


FIG. 31.—Diagram showing total capacity concentrated at center of line.

By formula (10),

$$E'_n = \frac{35{,}660}{0.896} = 39{,}800 \text{ volts.}$$

For the other half of the line we must use the formulas developed from Fig. 30. By formula (36), the charging current is

$$I_c = 2\pi \times 60 \times 39{,}800\,(100 \times 0.0135 \times 10^{-6})$$
$$= 20.25 \text{ amperes.}$$

By formula (40),

$$\tan\theta' = \frac{(26.2 \times 0.444) - 20.25}{26.2 \times 0.896}$$
$$= -\,0.367$$

From tables, $\theta' = 20°\,9'$, $\cos\theta' = 0.939$ and $\sin\theta' = -\,0.344$ whence, by (41), the line current in this section is

$$I' = 26.2\left(\frac{896}{939}\right) = 25 \text{ amperes,}$$

which is in *advance* of the pressure across the condenser because of the negative sign resulting from the solution of formula (40).

The sides of the impedance triangle BCD (Fig. 30) are

$$I'R = 25 \times 50 \times 1.04 = 1300 \text{ volts}$$

and

$$I'X = 25 \times 50 \times 0.832 = 1040 \text{ volts.}$$

The procedure is now as for the 50 miles of line already calculated. Putting ϕ' for the power factor angle at the sending end of the line (the angle $\theta' = 20° \ 9'$ being the power factor angle at the other end of this section), we have by formula (9) or (44)

$$\tan \phi' = \frac{1040 - (39{,}800 \times 0.344)}{1300 + (39{,}800 \times 0.939)}$$

$$= \frac{-\ 12{,}660}{38{,}680} = -\ 0.325$$

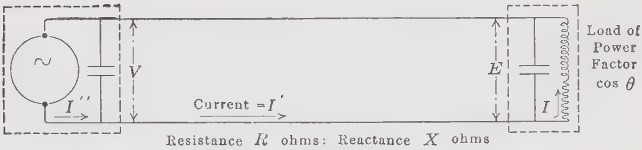


FIG. 32.—Diagram showing one half of total capacity concentrated at each end of the line.

whence $\phi' = 18$ degrees; $\cos \phi' = 0.951$; and $\sin \phi' = -\ 0.309$; the fact of $\sin \phi'$ being negative indicating a leading current. By formula (10) or (45),

$$V_n = \frac{38{,}680}{0.951} = 40{,}700 \text{ volts.}$$

The pressure drop per phase is 40,700 − 38,100 = 2600 volts, or 6.82 per cent. of the receiving end pressure.

The current entering the line at the generator end under full load conditions is $I' = 25$ amperes, on a leading power factor ($\cos \phi'$) of 0.951.

It should be mentioned that no high degree of accuracy is claimed for these or any numerical examples worked out in this book. A 10-inch slide rule is used for all calculations.

Case (2), *half the total capacity concentrated at each end of the line* (Fig. 32).

The effect of connecting a condenser of capacity $C/2$ between each wire and neutral at the receiving end, is to modify the power

factor of the load, the line calculations being based on an imaginary load power factor $\cos\theta'$ instead of the actual power factor $\cos\theta$. The condensers at the sending end have obviously no effect on the line drop or line losses, but they will modify the power factor of the load at generator terminals.

The calculations of line drop are exactly as carried out for the supply end of the line in Case (1). The charging current is now about half the value previously calculated,

$$I_c = 2\pi \times 60 \times 38{,}100\ (50 \times 0.0135 \times 10^{-6}) = 9.7 \text{ amperes.}$$

Referring to Fig. 30, we have, by formula (40),

$$\tan\theta' = \frac{(26.2 \times 0.436) - 9.7}{26.2 \times 0.9} = 0.073$$

whence $\theta' = 4°\ 10'$; $\cos\theta' = 0.997$, and $\sin\theta' = 0.073$.

By formula (41) the line current is $26.2 \times \dfrac{0.9}{0.997} = 23.65$ amperes.

The resistance drop is $I'R = 23.65 \times 100 \times 1.04 = 2460$ ohms.
The reactance drop is $I'X = 23.65 \times 100 \times 0\ 832 = 1970$ ohms.
By formula (44),

$$\tan\phi = \frac{1970 + (38{,}100 \times 0.073)}{2460 + (38{,}100 \times 0.997)} = 0.1175$$

whence $\cos\phi = 0.994$, and $\sin\phi = 0.110$.
Thus, by formula (45),

$$V_n = \frac{2460 + (38{,}100 \times 0.997)}{0.994} = 40{,}700 \text{ volts}$$

which is the same as the figure obtained by assuming the whole of the capacity to be concentrated at the center of the line.

If it is desired to calculate the current and power factor at the generator terminals, we have merely to repeat the process by putting the new numerical values in the formulas. Thus

$$I_c = 9.7\,\frac{407}{381} = 10.35 \text{ amperes,}$$

and the formula (40) when re-written, becomes

$$\begin{aligned}\tan\phi' &= \frac{I'\sin\phi - I_c}{I'\cos\phi}\\ &= \frac{(23.65 \times 0.110) - 10.35}{23.65 \times 0.994} = -\,0.331\end{aligned}$$

whence $\cos \phi' = 0.95$ (leading), and by formula (41), the line current is

$$23.65 \frac{0.994}{0.95} = 24.8 \text{ amperes.}$$

In this example, the capacity current is relatively large because the maximum load on a line 100 miles long would generally be more than the assumed value of 3000 k.v.a.; but nevertheless, both methods give approximately the same results. Comparing the figures, and bearing in mind that no high degree of accuracy is claimed, we have,

	Approximation (1) (Fig. 31)	Approximation (2) (Fig. 32)
Percentage line drop	6.82	6.82
Line current at generating end	25.00 amperes	24.80 amperes
Power factor at generating end	0.951 (leading)	0.95 (leading)

Either method gives results that are sufficiently accurate for practical purposes, even in the case of long high-voltage lines.[1]

58. Distinction Between Regulation and Line Drop.—The percentage drop of pressure on a transmission line may be defined as the difference between the sending end and receiving end pressures expressed as a percentage of the receiving end pressure. Thus,

$$\text{Per cent. pressure drop} = \frac{V - E}{E} \times 100$$

and this is the same as the *regulation* when the capacity current is so small as to be negligible. When the capacity current is appreciable, it will cause the voltage of the unloaded line to be *greater* at the receiving than at the generating end, as explained in Article 10, Chapter II (Fig. 11); and since the *regulation* is defined as the change of pressure at the receiving end when the load is thrown off (the supply voltage remaining constant), the regulation of a long high-voltage transmission line will usually be greater than the pressure drop.

[1] The manner in which the total current in a long line of appreciable capacity changes both in magnitude and phase, may be illustrated graphically by means of models or diagrams involving the idea of two planes perpendicular to each other. The writer has in mind diagrams similar to those used by Prof. D. D. Ewing in the *Electrical World* of Dec. 29, 1917, Vol. 70, p. 1252.

The rise of pressure at the end of a long transmission line is independent of the size and spacing of the wires. It may be calculated approximately as follows.

In Fig. 33, the IR drop (CB) due to the charging current I_c, may be neglected as it has no appreciable effect on the pressure rise ($E_n - V_n$) which we shall therefore consider as being equal to the induced volts DC. By formula (36) the charging current is

$$I_c = 2\pi f E_n C_m L \times 10^{-6}$$

where C_m is the capacity in microfarads per mile, and L is the length of the line in miles. The induced volts are,

$$IX = 2\pi f \mathbf{L} I_c L$$

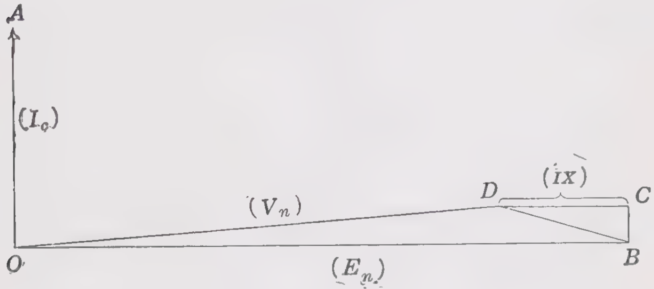


FIG. 33.—Vector diagram showing pressure rise at end of long unloaded line.

which, after substituting the above value of I_c, become

$$IX = (2\pi f)^2 E_n (C_m \mathbf{L}) L^2 \times 10^{-6}$$

but, by formula (37), the product $C_m\mathbf{L}$ has the constant value $\frac{1}{34{,}700}$, whence the pressure rise (conductor to neutral) of the unloaded line is

$$IX = (E_n - V_n) = (2\pi f)^2 E_n L^2 \times 2.88 \times 10^{-11}$$
$$= 1.14 E_n f^2 L^2 \times 10^{-9} \text{ volts} \qquad (46)$$

which assumes sinusoidal wave shapes.

This pressure rise, expressed as a percentage of the line voltage is

$$\left.\begin{array}{r}\text{Per cent. pressure rise}\\ \text{due to capacity and}\\ \text{inductance of line}\end{array}\right\} = 1.14 f^2 L^2 \times 10^{-7} \qquad (47)$$

The regulation, on the sine wave assumption, is therefore equal to *Percentage line drop* $+ 1.14f^2L^2 \times 10^{-7}$; but the last term is negligible unless the distance of transmission (L) is great.

As an example of the application of formula (47), the percentage line drop in the numerical problem of the last article was 6.82 and the percentage rise at the end of the unloaded line is approximately $1.14 \times (60)^2 \times (100)^2 \times 10^{-7} = 4.11$, whence the regulation is $6.82 + 4.11 = 10.93$ per cent.

59. Line Losses.—Apart from leakage and corona losses, which will be considered in Chapter V, the watts lost in transmission are the I^2R losses. On a three-phase line these will be

$$w = 3I^2RL$$

where R is the resistance per mile of conductor—corrected if necessary for skin effect—and L is the distance of transmission, in miles.

When the current entering the line at the sending end is equal to the current leaving the line at the receiving end, the value to take for I in the above formula is simply the load current; but when the effect of the distributed capacity becomes important—as on a long-distance high-voltage transmission—the question arises as to what particular value of the current should be used in the calculations for line loss.

As an alternative to dividing the line into a large number of sections and calculating the current in each section—which would involve a considerable amount of tedious work—we can calculate the average value of the square of the current over the whole distance of transmission. This calculation is easily made if the effect of voltage drop is neglected, or, in other words, if the amount of the charging current is supposed to diminish in direct proportion to the distance from the supply end of the line. Thus, if I_c is the value of the charging current at the sending end, the value of the charging current component of the total current at a point x miles from the generating end of a line L miles long will be

$$I_x = \frac{I_c}{L}(L - x) \tag{48}$$

Referring, now, to the vector diagram Fig. 34, where I is the load current and $\cos\theta$ is the power factor of the load, the total line current (I_l) at a point x miles from the generating end is the sum, or resultant, of I and I_x. This resultant can be ex-

pressed in terms of its "in-phase" and "wattless" components thus:

$$I_l = \sqrt{(a - I_x)^2 + b^2}$$

and the average value of the *square* of this quantity is b^2 + *average value of* $(a - I_x)^2$ as x increases from zero to its maximum value L. Average of

$$(a - I_x)^2 = \frac{1}{L}\int_0^L (a - I_x)^2\,dx$$

$$= \frac{1}{L}\int_0^L \left[a^2 - 2a\frac{I_c}{L}(L-x) + \frac{I_c^2}{L^2}(L-x)^2\right]dx$$

$$= \frac{a^2}{L}\int_0^L dx - \frac{2aI_c}{L^2}\int_0^L (L-x)dx + \frac{I_c^2}{L^3}\int_0^L (L-x)^2\,dx$$

$$= a^2 - aI_c + \frac{I_c^2}{3}.$$

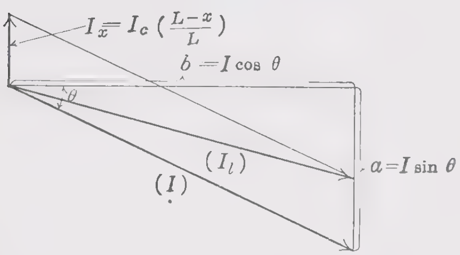


FIG. 34.—Vector diagram showing total current at any distance from end of line.

Adding b^2 to this quantity, we get for the average value of the square of the line current

$$(I_l^2)_{\text{average}} = (b^2 + a^2) - aI_c + \frac{1}{3}I_c^2$$

$$= I^2 - I_cI \sin\theta + \frac{1}{3}I_c^2 \qquad (49)$$

and the watts lost in a three-phase line, neglecting corona and leakage losses, are approximately

$$w = 3RL\ (I_l^2)_{\text{average}} \qquad (50)$$

If the current waves are not sinusoidal, and if the pressure at the sending end is appreciably higher than the receiving end pressure, the average square of the current will not be quite correct'y obtained from formula (49).

Example of Line Loss Calculations.—Instead of assuming entirely new conditions, we shall use the data of the numerical example in Article 10 of Chapter II, and calculate the line losses (1) when there is no load at the receiving end, and (2) when the load at the receiving end is 10,000 k.w. with a power factor of 0.9.

Assuming the conductors to be No. 00 copper throughout, the resistance per mile of wire (neglecting skin effect) will be 0.41 ohm, and the known quantities are therefore:

Load at receiving end = $\sqrt{3}\ EI \cos\theta$ = 10,000,000 watts.
Line voltage, E = 100,000.
Load power factor, $\cos\theta$ = 0.9.
Load current, I = 64.2 amperes.
Resistance of conductors, R = 0.41 ohms per mile.
Distance of transmission, L = 210 miles.
Capacity current at generating end (as previously calculated) = 63.6 amperes.

(1) By formula (49) the average value of the square of the charging current when $I = 0$ is

$$(I_c{}^2)_{\text{average}} = \frac{1}{3} I_c{}^2 = 1345.$$

By formula (50) the total line loss is $3 \times 1345 \times 0.41 \times 210 \times 10^{-3} = 348$ k.w. This is the true output of the generating station (neglecting corona and leakage losses) when the working voltage is applied to the unloaded line; but the k.v.a. or *apparent kilowatt* output is $\sqrt{3} \times 100{,}000 \times 63.6 \times 10^{-3} = 11{,}000$ k.v.a.

(2) By formula (49) the average value of the square of the line current when the load current is 64.2 amperes on a power factor $\cos\theta = 0.9$ ($\sin\theta = 0.436$) will be

$$(I_l{}^2)_{\text{average}} = (64.2)^2 \times 1345 - (63.6 \times 64.2 \times 0.436) = 3680$$

whence the total line loss at full load is $3 \times 3680 \times 0.41 \times 210 \times 10^{-3} = 952$ k.w.

60. Control of Voltage on Transmission Lines.—The pressure drop at the receiving end of a transmission line may be compensated for by raising the voltage at the generating end as the load increases. There are obvious disadvantages to such a method of operation, and it is better, if possible, to regulate the voltage at the point, or points, where constant pressure is required. This is

especially true of transmission lines on which there are substations or branch lines at intermediate points.

The necessary steady voltage at the receiving points may be obtained by installing hand operated or automatically controlled variable-ratio transformers or "boosters," which may be either

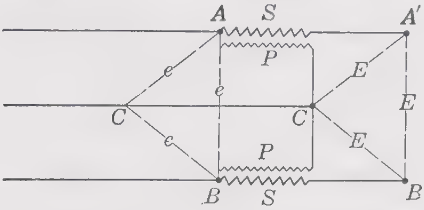


Fig. 35.—"Boosters" on three-phase system.

of the type with movable iron core, or with tappings from the windings taken out to a multiple-contact regulating switch.

On a delta-connected three-phase system it is not necessary to provide more than two single-phase regulators as these may be connected up as indicated in Fig. 35. Here P and S represent

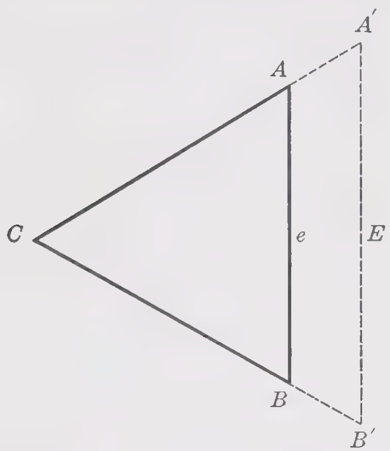


Fig. 36.—Vector diagram—two "boosters" on three-phase system.

respectively the primary and secondary windings of the variable-ratio transformers. That these are capable of raising the voltage equally on all three phases to the extent of the volts induced in the secondary coil S will be clear from an inspection of Fig. 36. In this diagram, AB, BC, and CA are the three vectors representing the pressures e before boosting up: AA' and BB' repre-

sent the added volts between the terminals A and A' or B and B' (Fig. 35). These added volts are evidently in phase with the pressures indicated by the vectors CA and BC respectively, because the potential difference at the secondary terminals of a well-designed transformer is always in phase with the primary impressed e.m.f. It is only necessary to complete the triangle $CA'B'$ to see that the two transformers connected up in the manner described will do all that is required in the way of raising the pressure on the three-phase circuit.

61. Effect of Boosting Voltage at Intervals Along a Transmission Line.—If a long transmission line, insulated for a maximum working pressure of (say) 100,000 volts, can be worked as a 100,000-volt line at all times through its entire length, it will be more efficient than if only a portion of it is working as a 100,000-volt transmission while portions farther from the generating end

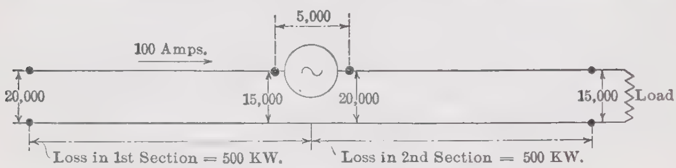


FIG. 37.—Method of maintaining pressure on long line.

are working at (say) 80,000 volts. By installing boosters along the line to maintain the pressure at or near the maximum working value, whatever the load may be, economies may frequently be effected. It is true that the energy put into the line at intermediate points cannot be cheaper—and, indeed, is usually more costly—than the energy supplied to the line at the generating end; but the booster system allows of the pressure being kept up all along the line, thus effecting economy; provided always that the losses in the boosters themselves, their maintenance, and the necessary allowances for interest and depreciation, do not counterbalance the saving.

As an example, consider a single-phase line conveying a current of 100 amperes at an initial pressure of 20,000 volts. Suppose the drop in pressure in the whole length of line to be as great as 10,000 volts; this will leave only 10,000 volts at the receiving end.

The power put into the line at generating end = 2000 k.w.
The loss in the line = 1000 k.w.
The power available at receiving end = 1000 k.w.
Hence, *efficiency of line* = 50 per cent.

Now imagine a booster to be introduced at a point half-way along the line. This booster may be considered as a suitably insulated alternator of 500 k.w. capacity, capable of generating 100 amperes at 5000 volts, the arrangement being as shown in Fig. 37. The drop in the first section of the line is, as before, 5000 volts; and the drop in the second section is evidently similar—namely, 5000 volts—which means that the total amount of power dissipated in the line is the same as it was before the booster was introduced. But by providing this booster at the middle point of the line, it has been possible to raise the pressure at this point up to the initial value of 20,000 volts, with the result that 15,000 volts (1500 k.w.) are available at the receiving end. The additional power available for useful purposes has, of course, cost something to produce; but the point to be noted is this: by keeping up the pressure, it has been possible to transmit a greater

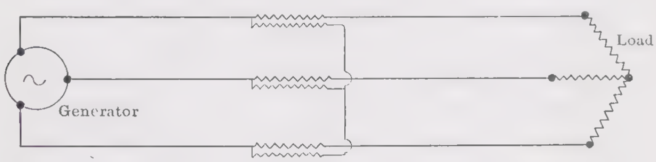


Fig. 38.—Transformers connected as "boosters" on transmission line.

amount of energy to the receiving end of the line *without increasing the losses in the conductors.* If, for the sake of simplicity, the losses in the booster are neglected, the line efficiency is arrived at thus:

Power supplied to the line = 2000 + 500 = 2500 k.w.
Power lost in the line = 1000 k.w.
Power available at receiving end = 1500 k.w.

$$\text{Line efficiency} = \frac{1500}{2500} = 60 \text{ per cent.}$$

Boosters may be arranged to take their power from the generating end of the line; that is to say, they may take the form of variable-ratio transformers, with hand or automatic regulation, connected up as indicated in Fig. 38. Transformers so connected will provide the additional volts at the cost of a corresponding loss of current.

62. Control of Power Factor.—The advantages of operating alternating machinery and systems on unity power factor, when possible, are so well known that it will not be necessary to discuss the matter here. The line losses alone—as shown in Article

6 of Chapter II—are inversely proportional to the *square* of the power factor. Thus, if the total I^2R loss in the line were 200 k.w. with a power factor of 0.707 (a by no means impossible figure in practice), this loss would be reduced to 100 k.w. if the *same total power* could be transmitted at unity power factor.

The control of power factor is obtained by balancing any excess of inductive reactance with condensive reactance, or *vice versa.* With a changing load at the end of a long transmission line, the power factor at any given point on the line is continually changing, even if the power factor of the load remains constant; and the most convenient means of providing the reactance necessary to maintain a constant and improved power factor is to install synchronous motors which can be made to draw

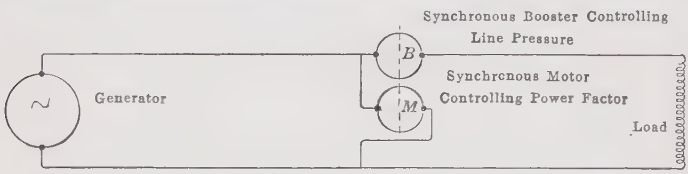


FIG. 39.

leading or lagging currents from the line by over- or under-exciting their field magnets.

The principles underlying the fact that an alternating current synchronous motor can be made to act either as a rotary condenser or a rotary reactor have been dealt with by the writer elsewhere,[1] and they are, moreover, so generally understood that we shall assume the power factor to be capable of control by merely providing one or more synchronous motors of sufficient capacity and with the necessary equipment for varying the field current, at the desired point on the transmission line.

Bearing in mind that power factor control is quite as important, if not more important, than voltage control so far as the losses and efficiency of transmission are concerned, the arrangement shown diagrammatically in Fig. 39 should be preferable to the arrangements of Figs. 37 and 38.

Here B is a synchronous generator connected as a "booster" and provided with field regulation in order to control the *voltage.* It is direct-coupled to the synchronous motor, M, which is provided with field regulation in order to control the *power factor.*

[1] *Polyphase Currents,* Whittaker and Co., London.

Before deciding to install synchronous machinery to control the voltage and power factor of a transmission line, it is necessary to consider the increased cost due to such machinery over alternative methods of pressure control, and compare this with the capitalized cost of the saving in transmission losses due to the improved power factor. A point which should not be overlooked is the possibility of synchronous machinery falling out of step, and so causing troubles and interruptions to supply which are best avoided by installing no auxiliary apparatus which it is possible to do without.

63. Use of Rotary Reactors to Control the Voltage.—Although Fig. 39 shows two synchronous machines, each with a particular function to perform, it is not always necessary to maintain the power factor at a constant value, and if the machine M is of

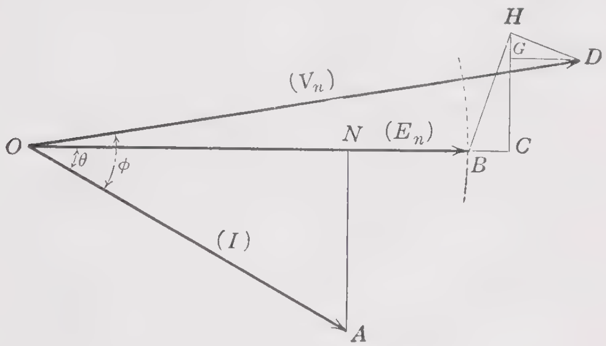


Fig. 40.—Vector diagram showing components of voltage which cause pressure drop.

sufficient capacity and properly designed, it may be used to maintain constant voltage without the addition of the second machine (B) connected in series with the line. The reason for this is that by changing the power factor—or the phase difference between the line current and e.m.f.—the inductance of the line and of such apparatus as transformers connected thereto, can be utilized to provide the required voltage. This is best explained with the aid of vector diagrams.

Fig. 40 is similar to the fundamental regulation diagrams (Figs. 10 and 26) except that the ohmic and reactive voltage components of the total pressure drop are shown separately for the "in-phase" and "wattless" components of the total line current. Thus, if E_n, I, and cos θ, stand respectively for the

"star" voltage, the line current, and the power factor at the receiving end, the IR and IX drops due to the "in-phase" component, ON, of the current are CB and HC respectively, while the IR and IX drops due to the "wattless" component, NA, of the current are GH and DG respectively. The pressure necessary at the generating end is $V_n = OD$, and the pressure drop is $DF = V_n - E_n$. If, now, we can—by means of overexcited synchronous machinery—so change the power factor that the point D will fall on the dotted circle of radius OB, we shall obtain the condition of constant voltage, *i.e.*, the same voltage at the generating, as at the receiving, end of the line.

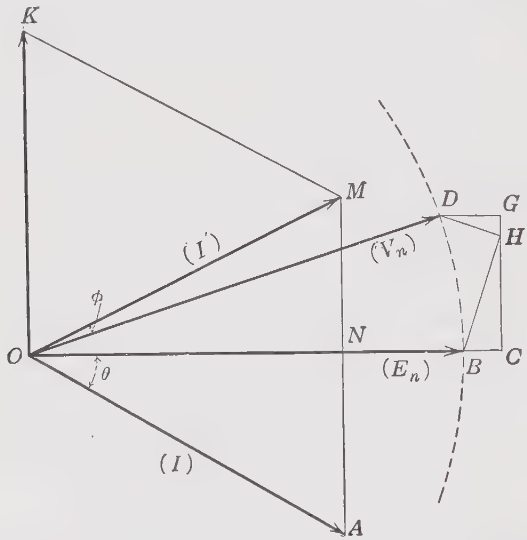


FIG. 41.—Vector diagram illustrating effect of reactors in maintaining constant voltage.

Assuming no change in the load current I, the impedance triangle BCA will also remain unaltered; but, by drawing a leading "wattless" current from the line, the current component NA can be not only annulled, but actually reversed, thus making the line current *lead* the receiving end voltage, thereby changing the voltage *drop* due to the reactive component of the current into a voltage *rise*. This is shown in Fig. 41 where the excitation of synchronous reactors, connected across the line at the receiving end, has been increased until the leading

component of the line current is equal to OK. The resultant current is OM with a leading "wattless" component NM, giving the impedance triangle HGD which throws the point D on the dotted circle and makes $V_n = E_n$.

This is the principle of constant voltage transmission with the pressure regulation obtained by providing the necessary number of variable-field synchronous motors at suitable points on the transmission line, but mainly at the receiving end where the heavy load is taken off. By this method, the reactance of a long line—usually an objectionable feature tending to limit the size of individual conductors—is actually necessary to the proper regulation of the voltage. The machines used as "rotary reactors" may be synchronous motors from which mechanical power is obtained, or rotary converters, or again, machines specially designed for no other purpose than to regulate the amount and direction of the "wattless" current, in which case they would be installed in the receiving point substations and would be run "idle." The power factor will not necessarily be unity, but the improvement in the power factor will generally permit of more energy being transmitted along a given line than would otherwise be permissible or economical. The most economical cross section of conductor may be used without regard to pressure drop, because where a drop of 10 to 15 per cent. would be about the upper limit with the older systems of regulation, a drop of 25 per cent. (due mainly to the inductance of the line) can be taken care of by synchronous reactors.

One of the most ardent advocates of this system of regulation is Prof. H. B. Dwight, whose book[1] should be consulted by those desiring further information on this subject.

The charging current of the line was not referred to in connection with the diagram Fig. 41, but it is evident that it must to some extent be helpful in reducing the necessary size of the synchronous motors, of which the capacity is determined by the amount of "wattless" current that they are able to provide.

The regulation of power factor (and incidentally of the voltage) by means of synchronous motors is not applicable to short-distance small-power transmissions, and even on long-distance lines transmitting large amounts of energy, the engineer should be careful to consider the whole problem from the *economic point of view:* there are a great many factors to be taken into account,

[1] "Constant Voltage Transmission," by H. B. Dwight, John Wiley & Sons.

among which reliability of service and maintenance costs are not the least important.

The space taken up by this discussion of a particular system of control may seem excessive in view of the limitations of this book; but with the improvements in design of electrical machinery and the increasing magnitude of power transmission schemes, there is a possibility of the system being used extensively in the future. It is advocated not only by the manufacturers of synchronous alternating-current machinery, but by engineers who have satisfied themselves that economy and good service can, under favorable conditions, be obtained thereby. A notable instance is the transmission line from the hydroelectric plant at Point du Bois in Canada to the city of Winnipeg where two 6000-k.v.a. synchronous motors with automatic regulation are installed for the sole purpose of regulating the voltage by power factor control; the line pressure at the generating station remaining constant.

64. Power Factor of Load.—The power factor of the load is not always easy to estimate; it may consist of induction motors of various sizes, together with lighting circuits, all having different power factors. If several circuits of different power factors are connected in parallel, the joint power factor may be calculated by the formula:

$$\text{Cos } \theta = \frac{1}{\sqrt{1 + \left(\dfrac{I_1 \sin \theta_1 + I_2 \sin \theta_2 + \; . \; . \; .}{I_1 \cos \theta_1 + I_2 \cos \theta_2 + \; . \; . \; .}\right)^2}} \qquad (51)$$

where I_1, I_2 . . . are the currents taken by the various circuits of power factors $\cos \theta_1$, $\cos \theta_2$, . . . etc.

The formula (51) is easily developed by summing up the "wattless" and "in-phase" components of the various currents separately. The quantity in brackets is thus seen to be $\tan \theta$, while the complete formula is derived from the well-known relation

$$\cos^2 \theta = \frac{1}{1 + \tan^2 \theta}$$

65. Grounded versus Isolated Transmission Systems.—Whether or not it is advisable, on three-phase transmissions, to use the star connection with grounded neutral, or a system—either star or delta—without any connection to ground, is not a matter of very great importance; and since no theoretical con-

clusions based on general principles have been arrived at, the engineer is compelled to consider each particular case on its own merits, and be guided by practical results obtained under similar conditions.

With a view to eliminating the third harmonic and its multiples, and so obtaining as nearly as possible a sine wave of e.m.f., the generators are usually Y connected, a practice which has the further advantage that the neutral point can be readily grounded if desired. The low-tension windings of the transformers, both at generating and receiving ends of the line are generally delta connected; but so far as the high-tension windings are concerned, these may be star at both ends, or delta at both ends, or star at one end and delta at the other. Then again, the neutral point of a high-tension system may be connected to ground either directly or through a resistance, the results being by no means the same in the two cases.

The object of grounding the neutral of a high-tension system is mainly to protect the insulation from abnormally high pressures which might aggravate the trouble in the event of a ground occurring on one wire, and so lead to serious interruption of service. It is, in fact, the question of line insulation considered in connection with continuity of service which is generally the determining factor in deciding whether or not the neutral shall be grounded, and whether the grounding, if adopted, shall be with or without the intervention of a resistance.

Other considerations, such as the effect on neighboring telephone lines, both under normal conditions of working and when a breakdown occurs, will also influence the decision; and it is hardly possible in this place to add much to what has already been said in Article 7 of Chapter II; there is no general rule to be followed seeing that the circumstances of each individual case will have a bearing on the settlement of this question.

There would appear to be no particular object in grounding well-insulated systems of moderate pressures—up to, say, 60,000 volts; but, in the large high-pressure transmissions, especially with an extended system of branch circuits and tie lines, a dead-grounded neutral is usually desirable. Any three-phase transmission system on which the insulation is likely to give trouble when the pressure to ground is raised in the proportion of $\sqrt{3} : 1$ should have the neutral grounded, either directly or through a suitably proportioned resistance.

66. Interference Between Power and Telephone Lines.—Although the question of interference between power lines and neighboring telephone lines is a very important one, it is also a difficult one to settle satisfactorily or even discuss adequately in a book dealing primarily with the design of high tension transmission lines. The problem is of particular interest to the telephone engineer who will no doubt ultimately find a satisfactory remedy for the very real troubles which are liable to occur—especially when abnormal conditions lead to unbalancing of the power load—when telephone wires run parallel to alternating-current power lines for a considerable distance.

It is a comparatively easy matter to calculate the flux of induction which the current in the power conductors will set up in the loop formed by the telephone wires, and by carefully planned and frequent transpositions, this effect can be greatly reduced if not entirely overcome; but the electrostatic effects are probably of greater importance because they are less easily dealt with.

67. Insulation of Telephone Lines.—It has only lately been realized that one of the essential requirements for telephone lines strung on the same supports as, or very close to, high-pressure power conductors, is high insulation. This good insulation is necessary to prevent puncture of the insulators when high potentials are induced on the telephone wires at times of abnormal conditions—such as intermittent short circuits, or lightning disturbances—on the power lines. As an example of good practice in this respect, the Georgia Railway & Power Co. have provided insulators suitable for a working pressure of 22,000 volts to carry the telephone wires which parallel their 110,000-volt power lines from Atlanta to Tellulah, Ga.

68. Electrostatic Induction.—The dielectric field due to the alternating voltage of the power conductors induces a varying charge upon the neighboring telephone wires. If each wire of the telephone line were at an equal average distance from each conductor of the power line, there would be no difference of potential created between the two sides of the telephone receiver, and there should be no buzzing, etc., due to this cause. In other words, with adequate, properly worked out transpositions, the capacity currents passing between the power line and the telephone line will not pass through the telephone receiver. But, even if the electrostatic flux is at all times of the same kind and

amount for both wires of a telephone circuit, this does not prevent the telephone circuit as a whole being subject to alternating pressures relatively to ground, and these pressures may reach high values, depending upon the voltage of the power line and the proximity of the two (parallel) circuits. If the telephone circuit is grounded, the charging current passing between the power conductors and the telephone line will find its way to ground by flowing *along* the telephone wires, and since this current may amount to several amperes, trouble is almost certain to occur unless what are known as "drainage coils" are provided. If a choke coil with an iron core—such as the primary of an ordinary lighting transformer—is connected across the two wires of the telephone circuit, and then has its middle point connected to ground, the electrostatic charge will be "drained" off the line without interfering with the operation of the telephone. The telephone line paralleling the 110,000-volt transmission of the Georgia Railway and Power Co.,[1] is provided with 15-k.w. standard 2200-volt distribution transformers (with open secondaries) for this purpose.

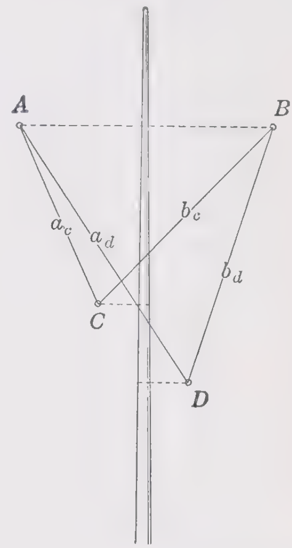


Fig. 42.—Telephone wires on same pole as single phase power circuit.

69. Magnetic Induction.—Referring to Fig. 42, the voltage induced by the single-phase power circuit AB in the loop formed by the wires C and D of the parallel telephone circuit may be calculated as follows if we assume that there are no transpositions and that the current wave is a pure sine curve.

By formula 25 (Art. 43) the flux in the loop CD due to the conductor A carrying a current I is

$$\Phi_A = \frac{2Il}{10}\left[\log_\epsilon\left(\frac{a_d}{r}\right) - \log_\epsilon\left(\frac{a_c}{r}\right)\right]$$

and the flux due to the current $-I$ in the conductor B is

[1] Refer to interesting article by E. P. Peck in *Electrical World*, Sept. 9, 1916, Vol. 68, p. 515.

$$\Phi_B = -\frac{2Il}{10}\left[\log_\epsilon\left(\frac{b_d}{r}\right) - \log_\epsilon\left(\frac{b_c}{r}\right)\right]$$

the total flux being,

$$\Phi = \frac{2Il}{10}\log_\epsilon\left(\frac{a_d b_c}{a_c b_d}\right) \tag{52}$$

By making the substitutions and alterations as in obtaining formula (27) we get,

$$\left.\begin{array}{c}\text{Volts induced per mile run of the}\\ \text{two parallel circuits}\end{array}\right\} = 0.00466fI \log\left(\frac{a_d b_c}{a_c b_d}\right) \tag{53}$$

Since the value of log 1 is zero, there will be no flux linkages with the telephone circuit when the condition $a_d b_c = a_c b_d$ is

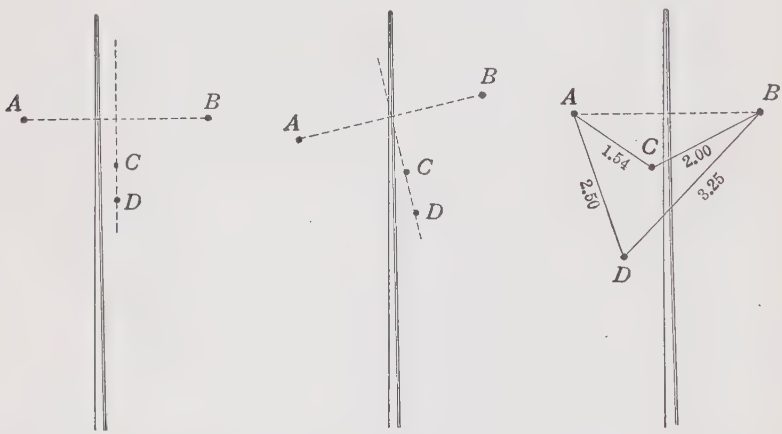


Fig. 43.—Arrangement of wires which eliminates magnetic induction from telephone circuit.

satisfied (see Fig. 42). This would be the case in either of the arrangements shown in Fig. 43. It is true that these arrangements refer to a telephone line running parallel to a single-phase power circuit, and that in any case transpositions would probably be necessary in practice; at the same time more attention might, in the writer's opinion, be given to the relative positions of power and telephone lines, apart from the question of transpositions. Practical methods of transposing both power and telephone wires will not be considered here, as the number of possible conditions to be remedied is almost unlimited; but once the principle is understood, some means of obtaining the required result can generally be found. Transpositions on the high tension power

lines should be avoided as far as possible, and it should not be necessary to transpose the power conductors of a high voltage transmission line at more frequent intervals than every mile.

Although formulas (52) and (53) were worked out, for simplicity, by taking the case of a single-phase transmission, the inductive effects of any number of power conductors can be calculated in a similar manner, proper attention being paid not only to the magnitude but also to the direction, or phase relation, of the currents in the several conductors. The general problem of inductive effects between parallel wires is taken up in Appendix I.

The fundamental frequency (f) occurs in the formulas, because pure sine waves are assumed; but it should be pointed out that disturbances due to voltages and currents of the fundamental frequency have little effect on the telephone, the chief trouble being due to the fact that in practice the pure sine waves are not obtained, and the higher harmonics, even when of small magnitude, are liable to produce noises in so sensitive an instrument as the telephone receiver, which may render conversation impossible.

It is not proposed to discuss here the protective apparatus as used on telephone circuits paralleling power lines, partly because the subject is somewhat beyond the scope of this book, but also because the troubles of the telephone engineer in this connection have not been entirely overcome, and he is still working on the problem. The reader who desires to pursue the subject further is therefore referred to other sources of information among which may be mentioned the interesting Report of the Joint Committee on Inductive Interference to the Railroad Commission of the State of California, which will be found on page 1441 of Vol. XXXIII (Part II) of the *Transactions* of the American Inst. E. E., and the paper by H. S. Warren entitled "Inductive Effects of Alternating Current Railroads on Communication Circuits" in the *Proceedings* A. I. E. E. of Aug., 1918.[1]

70. Fault Localizing.—The location of broken insulators, crossed or fallen wires, or any fault leading to unsatisfactory or interrupted service, must be dealt with by the operating staff;

[1] Refer also to the recent paper by William W. Crawford, "Telephone circuits with zero mutual induction" in the *Proceedings* A. I. E. E., p. 377, Vol. XXXVIII, No. 3, March, 1919.

and any but the briefest reference to these matters would be beyond the scope of this book.

The usual methods of testing are explained in many text-books, and in the electrical pocket-books and hand-books; but the well-known Varley and Murray loop tests are not always satisfactory on high-voltage transmission lines. Then, again, the conditions in regard to grounded or ungrounded neutrals, transformer connections, positions of section switches, telephonic facilities, etc., are so variable that it would be a difficult matter to lay down rules to be followed in emergencies, except in connection with a particular system; but such rules should be laid down by the chief operating engineer, and rigidly adhered to. By giving careful attention to the position of patrolmen's houses, switching and telephone stations, as referred to in Chapter I, much may be done toward preventing long interruption of service in the event of accidents.

CHAPTER V

INSULATION OF OVERHEAD TRANSMISSION LINES

71. Insulator Materials.—The material most commonly used for insulators on high-tension overhead lines is porcelain; but glass, which is cheaper than porcelain, may sometimes be used to advantage on the lower-voltage lines. Insulators made of special moulded materials such as "Electrose" have the advantage over both glass and porcelain in that they are lighter in weight and less liable to fracture from mechanical shocks or high-pressure discharges. This material is used in preference to porcelain by the Canadian Niagara Power Co. on its high-voltage transmission lines. Glass is a material of high resistance and dielectric

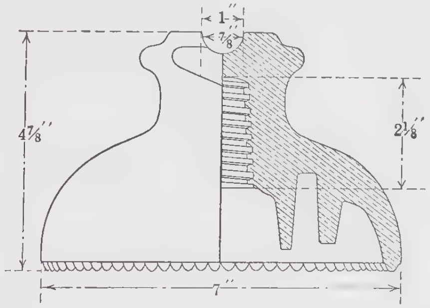


FIG. 44.—Single piece glass insulator. Average weight per piece, $4\frac{3}{4}$ lb. Made for standard 1 in. and special $1\frac{3}{8}$ in. pins. Voltages—Test—Dry 86,200 Wet 50,100, Line 17,000.

strength and makes excellent insulators for pressures up to about 25,000 volts. The fact that it is pervious to light tends to discourage spiders and cocoon-spinning insects, which ordinarily find their way to the interior of insulators; but, on the other hand, more moisture will condense on the inside surfaces of glass than on porcelain insulators, thus attracting dust and dirt.

Figs. 44 and 45 are from drawings supplied by the Hemingray Glass Co. of Muncie, Ind. Some interesting particulars relating to the 9-in. glass insulator with sleeve (Fig. 45) have been commu-

nicated to the writer by Mr. M. H. Gerry, Jr., General Superintendent of the Missouri River Electric and Power Co. of Helena, Montana. Mr. Gerry, who has something like 50,000 of these insulators on his transmission lines, is actually using them for working pressures up to 70,000 volts, where they have given entire satisfaction for the last 13 years. The wooden supporting pins are treated with paraffin in a vacuum, and moreover, the climate is dry and the air free from dust or salt which might cause trouble with so high a voltage under less favorable conditions. Mr.

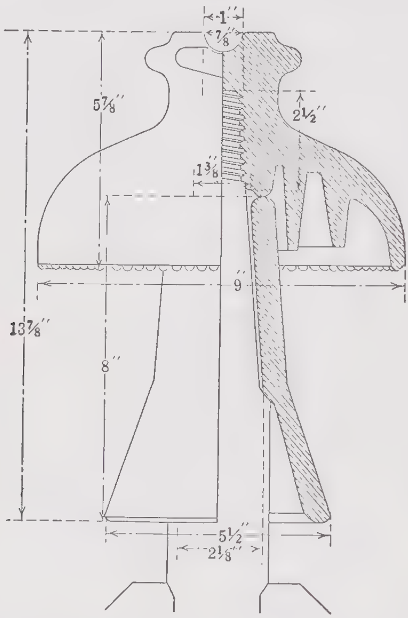


FIG. 45.—Two-piece glass insulator. Average weight, 12¾ lb. Voltages—Test—Dry 110,000, Wet 95,000, Line 33,000.

Gerry sums up his opinion of glass insulators for low and medium voltages by stating that notwithstanding their mechanical weakness relatively to porcelain, they are entirely satisfactory when intelligently used under suitable conditions.

For the suspension type of insulator, as used for the higher pressures, porcelain appears to be the best material available at present, although it is far from being ideally suited to the requirements. An enormous amount of study and research has lately been devoted to the development and improvement of

porcelain insulators, and it is not improbable that if as much engineering skill had been devoted to the improvement of glass insulators, these might now be in more general use, even in the form of suspension insulators for the highest voltages.

72. Design of Insulators.—The design of an insulator, to comply with any given specification, is a matter which concerns the manufacturer, who has been compelled of late years, owing to the rapid increase of working pressures, to devote his attention to the principles underlying the correct and economical design of insulators for high-pressure lines, and who is therefore something of a specialist on this particular subject. The transmission-line engineer should understand the principles underlying the correct design of overhead insulators; but it is suggested that, however great his knowledge of the subject, he will be well advised to leave details of design to the manufacturer, and to use standard types when possible.

The design of insulators for the lower voltages is a comparatively simple matter, the difficulties becoming greater with the increase of pressure, although the introduction of the suspension type, which permits of many units being connected in series, has considerably simplified the problem.

It is important to bear in mind that every insulator is necessarily a more or less complicated condenser, and it can generally be thought of as consisting of a number of separate condensers in series, the dielectric being alternately air and porcelain. The current passing from line to ground is partly a leakage current over the surfaces (the leakage *through* the porcelain being generally negligible), and partly a capacity current. This capacity current spreads itself over the high resistance surfaces of the insulator material in a way which will depend upon the surface conductivity and on the spacing and disposition of the various parts. It is well to keep the electrostatic capacity as low as possible, but it is of equal if not greater importance so to distribute it by a scientific arrangement of the component parts that abnormal stresses will not occur locally, as these may puncture or damage the insulator at one particular part, while a more carefully designed insulator of lighter weight may withstand a greater total breakdown pressure, because proper attention has been given to this important matter of capacity distribution. The effect of rain on the exposed surfaces of an insulator is to increase the capacity, and this will generally lower the flash-over

point; but the increased surface conductivity has the effect of equalizing the potential distribution; and in the case of a large number of condensers in series, such as occurs especially with the suspension type of insulator, it has actually been observed that this equalizing of the potential distribution may cause the flash-over pressure of the wet insulator to be no lower than the flash-over pressure of the same insulator when dry.

If the distribution of dielectric flux could be easily determined in the case of the rather complicated shapes and varying thicknesses of dielectric which occur in high tension line insulators, it would be an easy matter to predict the performance of new types and sizes under specified conditions; but although the dielectric circuit can conveniently be treated in a manner analogous to the engineer's treatment of the magnetic circuit, there is always difficulty (except in the simplest cases) in predetermining the amount and direction of the lines of flux—or stress.

The fundamental law of the dielectric circuit is

$$\Psi = EC \tag{54}$$

where Ψ is the total dielectric flux (in coulombs) in a space of which the *permittance* or *capacity* is C farads, when the potential difference producing this flux is E volts.

If we consider a small element of flux, *i.e.*, a "tube" of induction, of length l cm. and cross-section A sq. cm. over which the flux Ψ is evenly distributed, the flux density, in coulombs per square centimeter, is

$$D = \Psi/A \tag{55}$$

If the difference of potential between the two ends of the path l cm. long is E volts, the capacity (or permittance) is proportional to A/l—exactly as in the analogous case of the magnetic circuit of which the *permeance* is directly proportional to A and inversely proportional to l.

With the introduction of the proper constants, we have,

$$C = K\frac{kA}{l} \tag{56}$$

where K has the numerical value 8.84×10^{-14} and k is the *specific inductive capacity* or *dielectric constant* of the material. Approximate values for k are given in the accompanying table.

DIELECTRIC CONSTANTS AND DISRUPTIVE VOLTAGES

Material	Dielectric constant k	Dielectric strength, k.v. per cm.
Air	1	22
Porcelain	4.4 to 6	100
Glass	5 to 10	90
Transformer oil	2.2 to 2.5	
Paraffin	1.9 to 2.3	100

The figures in the last column of the Table are approximate only: they indicate the virtual or r.m.s. value of the (sinusoidal) alternating voltage which may be expected to break down a slab of the material 1 cm. thick placed between two large flat metal electrodes. What is usually understood by the expression "disruptive gradient" is obtained by multiplying the values in the Table by $\sqrt{2}$.

A more useful expression for the flux density, D, can now be obtained by putting the value of C from (56) in formula (54) and substituting the value of Ψ so obtained in formula (55). Thus

$$D = Kk\left(\frac{E}{l}\right)$$

but E/l is the potential gradient, or volts per centimeter—usually denoted by the letter G—whence

$$D = Kk \times G \tag{57}$$

This clearly indicates that, in a given material, the potential gradient (G) is directly proportional to the dielectric flux density (D), and if we can estimate the one, we can calculate the other. The chief problem in insulator design is so to proportion the parts that the flux density shall nowhere be so great as to cause breakdown of the air or puncture of the solid material of the insulator. An example illustrating the application of the formulas will be given in connection with the design of wall bushings, as this is a somewhat simpler problem than the design of high voltage line insulators.

73. Pin-type Insulators.—As previously mentioned, the sheds or petticoats with intervening air spaces which separate the wire at line potential from the pole or cross-arm which is usually at ground potential, may be thought of as a number of condensers

in series. If I_c is the charging current, and E is the potential difference causing this flow of current through a condenser of capacity C, we have the relation $I_c = 2\pi fEC$; and, when the frequency is constant, $E \propto \frac{I_c}{C}\cdot$ Thus, when a number of condensers are connected in series, the current I_c is the same through all the condensers, and the potential difference across any one condenser is *inversely proportional to its capacity;* whence the importance of care in design to avoid too great a stress across a given thickness of porcelain. With the pin type of insulator, a number of sheds or petticoats hanging close to the pin, with small air spaces between them, will not be effective, because, although the leakage path may be long, the capacity is high. The remedy consists in spreading the petticoats away from the pin, the outer shed in some designs being almost horizontal. This outer shed has, in some cases, been replaced by a metal shield. Apart from the advantage of lightness, which permits of a thin metal shed being made of larger diameter than would be permissible if the material were porcelain, the charging current will spread itself more uniformly over the surface of the outer shed, and so prevent the concentration of potential at the point where the conductor is tied to the insulator. An insulator of this type may flash over at a somewhat lower pressure than if the upper hood were of porcelain, but under wet conditions, the flash-over pressure may be higher.

Pin-type insulators are available for pressures up to 70,000 volts, but the suspension type, made up of two of more units, will generally be more satisfactory and economical for pressures above 50,000 volts. The pin type of insulator becomes too heavy and costly when designed for the higher voltages, and, owing to the great length of the supporting pin, the bending moment near the point of attachment to the cross-arm tends to become excessive.

Wooden supporting pins are not recommended for high-tension work; they are rarely used on lines working at pressures above 33,000 volts. Metal pins are generally preferable.

Fig. 46 shows the sparking distances as usually measured. The dotted line on the left side of the drawing shows the length of path on a dry flash-over. When the exposed surfaces of the insulator are wet with rain, these surfaces are looked upon as con-

ductors, and the flash-over distance is the sum of the separate distances A, B, and C; the lines A and B being usually drawn at an angle of 45 degrees to the vertical. In this particular case, the line C is also drawn at 45 degrees because the metal pin is surrounded by a porcelain base. The length of the pin should be such that the distance E to cross-arm is slightly greater than the sparking distance C from edge of inner petticoat to pin.

The flash-over distances as measured on actual insulators of the pin type are approximately as given below.[1]

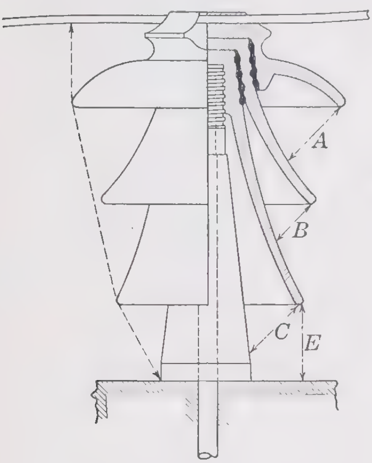


Fig. 46.—Sparking distances on pin type insulator.

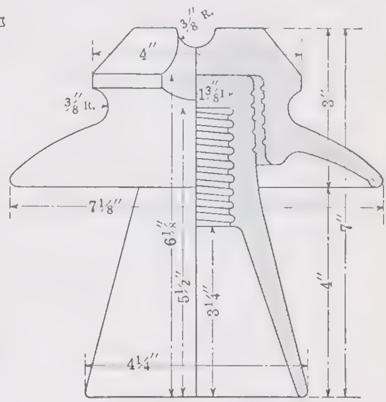


Fig. 47.—Pin type insulator.

Voltage, r.m.s. value	Flash-over distance, in.
40,000	3
60,000	4½
80,000	6⅜
100,000	8½
120,000	11
140,000	14
160,000	18¼

Figs. 47, 48, and 49 are illustrations of typical pin type insulators as made by R. Thomas & Sons of East Liverpool, Ohio.

[1] Average figures taken from curves given by Mr. J. Lustgarten in the *Jour.* of the Inst. E. E., vol. 49 (1912), p. 235.

The sections shown in Figs. 48 and 49 illustrate the trend in design of modern pin-type insulators toward smaller height in proportion to the head diameter. The same may be said of Figs. 50 and 51 which illustrate new standard types of "Victor" insulator manufactured by Locke Insulator Manufacturing Co. of Victor, N. Y. A tendency to increase the thickness of porcelain is noticeable in all recent designs of line insulators. The lead-

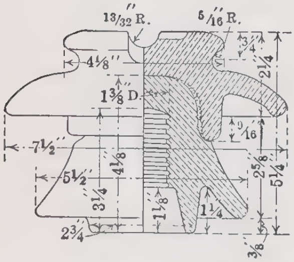


FIG. 48.

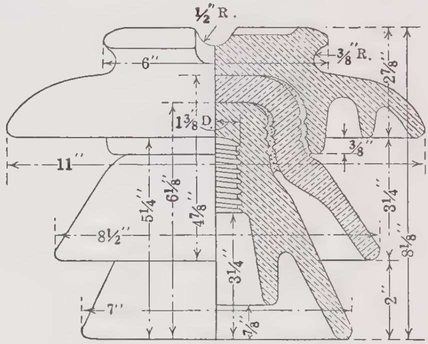


FIG. 49.

Pin type insulator.

ing particulars of these pin-type insulators, as furnished by the makers, are as follows:

	Fig. 47	Fig. 48	Fig. 49	Fig. 50	Fig. 51
Line pressure, volts........	25,000	25,000	45,000	45,000	50,000
Dry flash-over, volts........	80,000	89,000	147,000	150,000	175,000
Wet flash-over, volts.......	50,000	54,000	102,000	90,000	120,000
Leakage distance, inches....	12.5	12.0	28.00	20.0	23.5
Arcing distance (wet), inches	4.5	5.0	7.25		
Net weight, each, pounds...	4.5	6.5	20.00	13.5	24.5

74. Suspension-type Insulators.—With the *suspension type* of insulator, the conductor is hung below the point of support (which is usually grounded) at the end of a string of insulator units connected to one another by metal links. The potential difference which will cause a flash-over or breakdown on such a series of insulators will not be in direct proportion to the number of insulators in the string. This is due to the unequal distribution of the potential differences, which is again a question of relative capacities. The design of the individual units may ap-

pear to be good, and yet a string of such insulators, if these are not specially designed to fulfil certain requirements, may give surprisingly unsatisfactory results. A factor of importance is the ratio $\frac{\text{mutual capacity}}{\text{capacity to ground}}$ which determines the potential distribution; and this ratio will depend not only on the shape and size of the porcelain units, but also on the metal caps or means of attachment, and the spacing between units.

The distribution of the potential drop on a string of suspension insulators is easily calculated if certain assumptions are made with a view to simplifying the work. If symbols are used to denote the various voltages and currents, the formulas become somewhat complicated in appearance although actually simple to

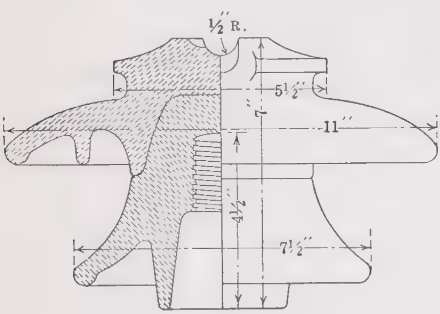


Fig. 50.—Pin type insulator.

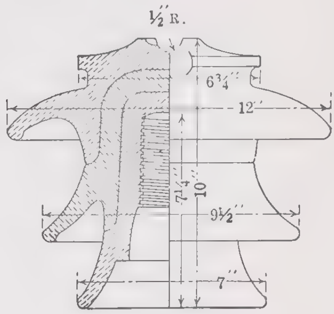


Fig. 51.—Pin type insulator.

apply. A very clear explanation of the voltage distribution on the string of suspension units has been given by Mr. F. W. Peek Jr.[1] The same treatment of the problem although not connected with Mr. Peek's name, will be found in the Handbook on Overhead Line Construction issued by the National Electric Light Association. A numerical example will best illustrate the manner in which the relative values of the several capacities affect the distribution of potential between the high-tension conductor and the point of attachment to the (grounded) cross-arm.

The metal-work (cap and link or bolt) on each side of the porcelain disc constitutes one terminal of a condenser which has a capacity C farads relatively to ground or grounded tower,

[1] "Electrical Characteristics of the Suspension Insulator," *Trans.*, A. I. E. E., p. 907, vol. xxxi, May, 1912.

and a capacity $C' = mC$ relatively to the metal-work (cap and link or bolt) on either side of the porcelain disc. This is shown diagrammatically in Fig. 52 where *a*, *b*, *c*, and *d*, are four imaginary condensers, each of capacity C farads, between the ground and the insulating metal caps, etc., while the condensers of capacity mC farads, formed by each individual unit, are numbered (1), (2), (3), and (4). We shall neglect the effects of *surface leakage* and *corona*. The former would tend to equalize the potential

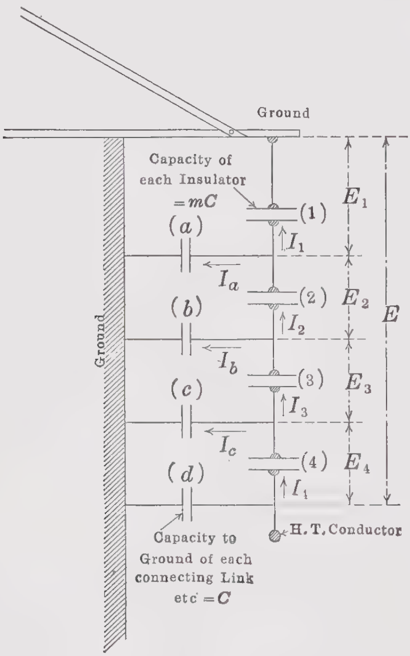


Fig. 52.—Diagrammatic representation of 4-unit suspension type insulator.

drop across the insulator units, while the latter—in addition to causing actual leakage losses through the air—might alter the capacity of the units subjected to the higher pressures. No error of appreciable magnitude is likely to be made by neglecting these items.

Assuming the voltage across the unit nearest to the grounded cross-arm to be $E_1 = 10{,}000$ volts, and the ratio $\dfrac{\text{mutual capacity}}{\text{capacity to ground}}$ to be $m = 10$, we have,

$$E_1 = 10{,}000 \text{ volts}$$
$$I_1 = 2\pi f E_1(mC) = \omega \times 10{,}000 \times 10 \times C$$
$$I_a = 2\pi f E_1 C = \omega \times 10{,}000 \times C$$

whence

$$I_2 = (I_1 + I_a) = \omega E_1(m + 1)C = \omega \times 10{,}000 \times 11 \times C$$

and

$$E_2 = E_1\left(\frac{I_2}{I_1}\right) = E_1 \times \frac{11}{10} = 11{,}000 \text{ volts}$$
$$I_b = \omega(E_1 + E_2)C$$

whence

$$I_3 = (I_2 + I_b) = \omega[E_1(m + 2) + E_2]C = \omega \times 131{,}000 \times C$$

and

$$E_3 = E_1\left(\frac{I_3}{I_1}\right) = 10{,}000 \times \frac{131{,}000}{100{,}000} = 13{,}100 \text{ volts}$$
$$I_c = \omega(E_1 + E_2 + E_3)C$$

whence

$$I_4 = (I_3 + I_c) = \omega[E_1(m + 3) + 2E_2 + E_3]C = \omega \times 165{,}100 \times C$$

and

$$E_4 = E_1\left(\frac{I_4}{I_1}\right) = 10{,}000 \times \frac{165{,}100}{100{,}000} = 16{,}510 \text{ volts.}$$

The total potential difference across the string of four insulators is therefore not $4E_1 = 40{,}000$, but $E = E_1 + E_2 + E_3 + E_4 = 50{,}610$ volts.

The stress across the insulator (No. 4) nearest to the high-tension conductor is $E_4 = 16{,}510$. Assume that it will flash over with twice this pressure, or $2E_4 = 33{,}020$ volts; then the pressure which will start a flash-over across the string of four insulators is $2E$ or 101,220 volts, which is less than four times $2E_4$ or 132,080 volts. The term "string efficiency" has been appliedby Mr. F. W. Peek, Jr.[1] to the ratio $\frac{\text{arc-over voltage of string of } n \text{ insulators}}{n \times \text{arc-over voltage of single insulator}}$ or $\frac{E}{nE_n}$ which, in this example, is

$$\frac{E}{4E_4} = \frac{50{,}610}{4 \times 16{,}510} = 0.765.$$

In this connection it is assumed that all the insulator units are of the same size and shape.

The fact that the ratio (m) of the mutual capacity to the capacity between the suspension link and ground is an important

[1] *Trans.* A. I. E. E., vol. xxxi, p. 907, May, 1912.

factor in determining the distribution of potential over the insulator string, suggests the importance not only of the shape and surface area of the metal fixtures on the individual insulators, but also the length of the connecting link, or spacing between insulators.

In actual tests made by Mr. Peek, the dry arc-over required 1¾ times the pressure of the wet arc-over on a single unit; but, on a string of nine such units, the wet and dry flash-overs occurred at the same voltage. The "string efficiencies" of these insulators on the dry test were as follows:

No. of units in series	Flash-over voltage	Efficiency, per cent.
1	74,000	100.0
2	137,000	93.0
3	186,000	84.0
4	238,000	80.5
5	281,000	76.0
6	318,000	72.0

Another series of tests, quoted by Mr. W. T. Taylor,[1] gives efficiencies as below:

No. of units in series	Flash-over dry test	String efficiency dry	Flash-over wet test	String efficiency wet
1	90,000	100.0	56,000	100.0
2	160,000	89.0	90,000	80.0
3	220,000	81.5	130,000	77.5
4	274,000	76.0	175,000	78.0
5	310,000	69.0	220,000	78.5
6	340,000	63.0	265,000	79.0

Arc-over voltages for strings of insulators are now usually given in manufacturers' catalogues.

When the mutual capacity is small relatively to the ground capacity, there soon comes a point beyond which it is useless to put more insulators in the string. Short strings, of well-designed and properly spaced units, will often be more effective and less costly than a longer series of insulators of which the single units may have excellent insulating properties, but may not have

[1] *Journal* Inst. E. E., vol. 46, p. 510 (1911).

been specially designed for the particular requirements. It is sometimes possible to increase the arc-over voltage of a string of insulators by *reducing* the distance between consecutive units; a fact that it is difficult to understand unless the importance of the proper capacity distribution has been realized. A well-arranged string of properly designed units not necessarily similar in shape and size, is less liable to damage by lightning and kindred phenomena than a series of insulators of low "string efficiency."

It is well to bear in mind that an insulator may, and does, behave differently when subjected to high-frequency charges as induced by lightning disturbances, than when the difference of potential is applied at the normal frequency of the transmission line. The distribution of potential among a number of condensers in series should be independent of frequency, but a high-tension insulator consists of many capacities in series with resistances such as the leakage paths through the body of the material and over the surfaces, and the distribution of the total potential drop across a condenser and resistance in series, even if the latter may be considered non-inductive, is not the same at high as at low frequencies. This is probably one of the chief reasons why insulators that will flash over rather than puncture on tests conducted at the ordinary line frequency, will sometimes fail through puncture during atmospheric electrical disturbances.[1]

Apart from the great advantages from the point of view of insulation, which are obtained by suspending the conductor from a string of insulators connected in series, this arrangement, as now generally adopted for pressures above 50,000 volts, has the further advantage that the conductor is less liable to be affected by lightning disturbances, since, at every point of support, the wire is hung *below* the attachment to the supporting tower which, in almost every instance is a well-grounded steel structure. Another advantage is the comparative flexibility of the attachment, which very considerably diminishes the possibility of crystallization of the conductor material, such as is liable to occur when the wire is rigidly attached to the pin type of insulator; this effect being more noticeable with aluminum than with copper.

In designing cross-arms for the attachment of the suspension type of insulator, it is not necessary to pay much attention to

[1] See "High Frequency Tests on Line Insulators," by L. E. Imlay and Percy H. Thomas in the *Trans.* A. I. E. E., vol. xxxi, p. 2121.

torsional stresses, which, however, must be carefully considered when pin-type insulators are used on heavy long-span lines; but, on the other hand, taller towers are necessary with the suspension type of insulator. Another possible disadvantage is the wider spacing between wires generally adopted when this type of insulator is used; and it is certain that rather more skill and experience are necessary in the stringing of the conductors than when pin-type insulators are used. It is usual to sling the wire

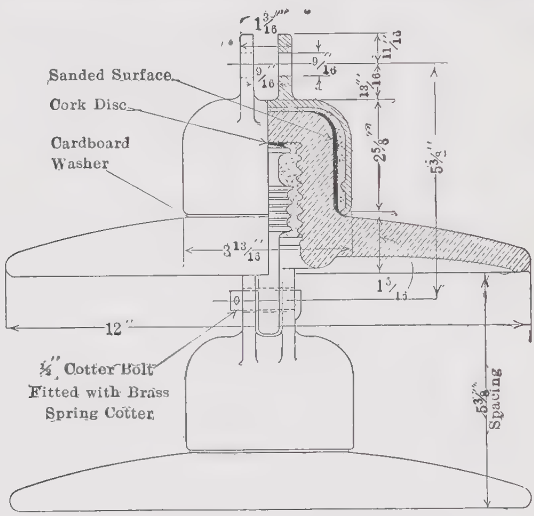


FIG. 53.—Suspension type insulator.

One unit			Number of units in string							Ultimate mechanical strength
Leakage distance	Wet arcing distance		1	2	3	4	5	6	7	
10″	4⅞″	Length of string.....	5⅜″	10¾″	16⅛″	21½″	26⅞″	32¼″	37⅝″	9000 lb.
		Dry flash-over, K. V.	62	124	184	240	292	340	383	
		Wet flash-over, K. V.	44	82	120	158	196	235	274	
		Net weight, lb.......	10½	21	31½	42	52½	63	73½	
		Gross weight, lb.....	...	....	42	56	69	83	97	

in the first instance from snatchblocks attached to the cross-arms of every tower over a distance of about a mile, or between anchoring or corner towers, and then draw up tight; the conductor being subsequently transferred at every point of attachment from the snatchblock to the permanent suspension clamp at the end of the lowest insulator in the series.

In the interlink type of suspension insulator, the porcelain

between the metal links is in compression, and in the event of the shattering of the porcelain parts, the conductor remains suspended; but although, from the mechanical point of view, this type is to be recommended, it is more liable to puncture owing to excessive electric stress in the thickness or porcelain between the metal links, than the more usual type of insulator with cap and bolt cemented to the porcelain parts as shown in the accompanying illustrations.

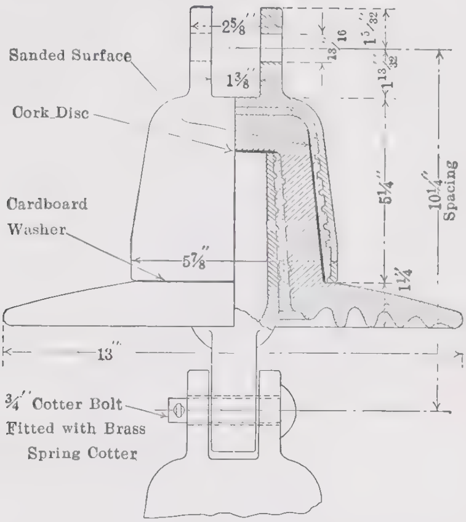


Fig. 54.—Suspension type insulator.

One unit			Number of units in string							Ultimate mechanical strength
Leakage distance	Wet arcing distance		1	2	3	4	5	6	7	
10½″	5¼″	Length of string.....	10¼″	20½″	30¾″	41″	51¼″	61½″	....	26,000 lb.
		Dry flash-over, K. V.	80	160	239	311	378	440		
		Wet flash-over, K. V.	49	103	157	211	265	319		
		Net weight, lb......	31	62	93	124	155	186		
		Gross weight, lb.....	...	78						

Figs. 53 and 54 are sectional views of suspension insulators manufactured by the R. Thomas and Sons Co. of East Liverpool, Ohio; and Fig. 55 is an insulator made by the Ohio Brass Co. of Mansfield, Ohio, as originally selected for the 140,000-volt transmission lines of the Au Sable Electric Co. (Au Sable to Battle Creek, Mich.), a string of 10 units being used on this voltage.

A great deal of attention has recently been devoted to the improvement of the suspension type of insulator. The parts of the early designs were cemented solidly together, with the result that the differential expansion of porcelain and metal caused cracking and consequent destruction of the procelain after being in use for only a few years. That experts in insulator design realize the effects of expansion and other causes leading to rapid deterioration of porcelain insulators is evidenced by the experimental work that has been carried on, and the number of papers published, relating to this subject, during the last two or three years.

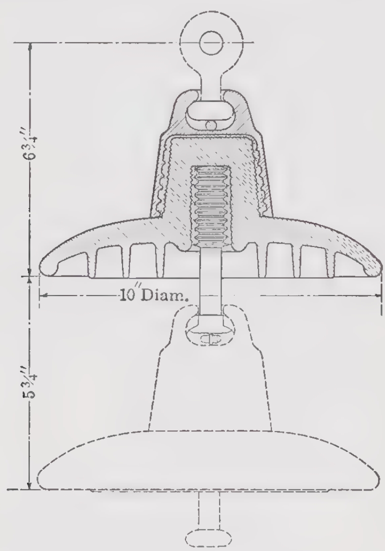


Fig. 55.—Suspension type insulator.

Fig. 56, reproduced from a drawing kindly supplied by the Locke Insulator Manufacturing Co., shows how allowance has been made for expansion. An insulator unit as shown in Fig. 56 may be used in both vertical and horizontal positions. When heavy strains have to be resisted, it is often preferable to use two or three strings of insulators in multiple. A good design of suspension unit can be made to have a breaking strength of about 6000 lb., but for greater mechanical strength some of the best features of design from the electrical viewpoint may have to be sacrificed.

An interesting discussion of the modern line insulator will be

found in the Paper by Mr. G. V. Twiss in Vol. 55, No. 267 (June, 1917) of the *Journal* of the Institution of Electrical Engineers. It is generally true that American engineers are the acknowledged experts in high-tension transmission work, and so far as overhead lines in England are concerned, these are so few and of such low voltage that they need not be considered. But British engineers have carried out important developments in India, Africa, and other Colonies abroad, and such work, although not invariably based on American practice, is always carefully planned and executed. The differences between British and American methods are worthy of some study, if only to put us on our guard against the tyranny of "established practice."

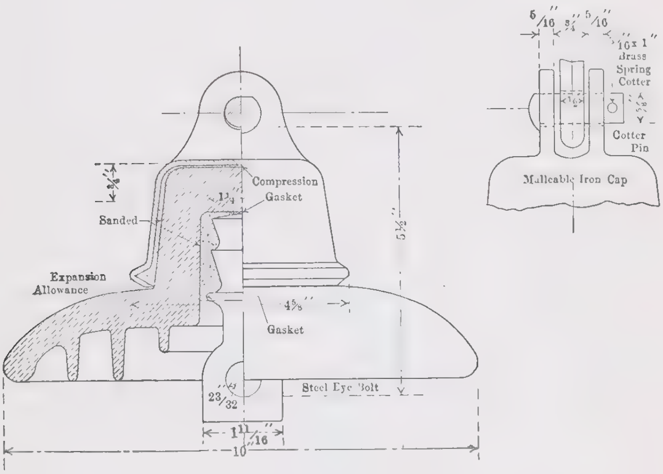


FIG. 56.—Suspension type insulator.
Line voltage = 16,000. Leakage distance = 12 in. Striking distance = $3\frac{1}{4}$ in.

75. Wall and Roof Outlets.—When overhead high-tension conductors have to be brought into buildings, the design of insulating bushings should receive careful consideration. The fact that every bushing acts as a condenser or series of condensers between wire and ground, must steadily be recognized just as in the case of the pole-line insulators, except that the effect is even more marked in connection with bushings entirely surrounding the conductor than with insulators that support the wire at one end only. When the climate and weather conditions are favorable, it is well to avoid bushings entirely. In such cases, the wires cannot be brought down through the roof of the building, but

they must enter at the side; a suitable protecting hood or roof being placed above the wires on the outside of the building.

The smallest dimension of the opening in brick, stone, or concrete wall should preferably not be less than as given below:

For line pressure, volts	Feet
22,000	1½
33,000	1¾
44,000	2¼
66,000	3½
88,000	4¾
110,000	6

On each side of the wall opening, the conductor is carried by line insulators, of the pin or suspension type, as the voltage may

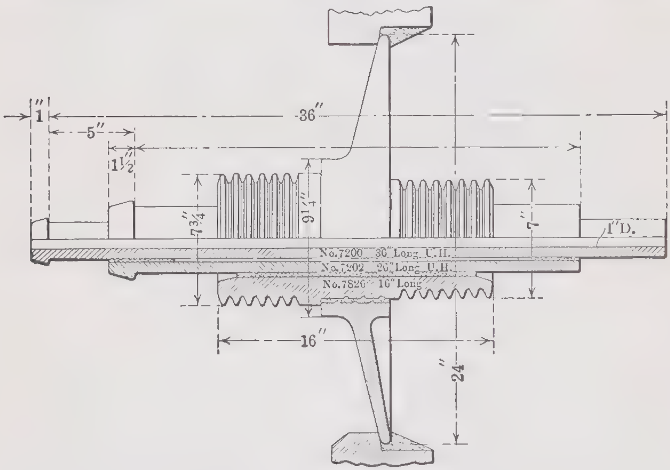


Fig. 57.—Porcelain entering bushing for 70,000 volts.

require, these being so arranged as to maintain the conductor in the center of the opening, with a slight downward incline toward the outside of the building to prevent rain-drops being carried to the inside.

When bushings are used, these must necessarily be thicker at or near the center, where the ground potential is brought up close to the conductor, than at either end. Fig. 57 shows a porcelain bushing built up of four parts, as supplied by the R. Thomas &

Sons Co., suitable for a working pressure of 70,000 volts. This bushing weighs about 100 lb. and will flash over with 170,000 volts (dry) and 138,000 volts (wet). Fig. 58 shows a floor bushing suitable for a working pressure of 44,000 volts. Its dry flash-over voltage is 112,000, and wet flash-over 75,000.

One method of bringing the high tension conductor through the wall of a building is illustrated by Fig. 59, which shows the overhead wire anchored to the wall by a string of suspension insulators, and passing through a standard porcelain wall bushing as manufactured by the Ohio Brass Co.

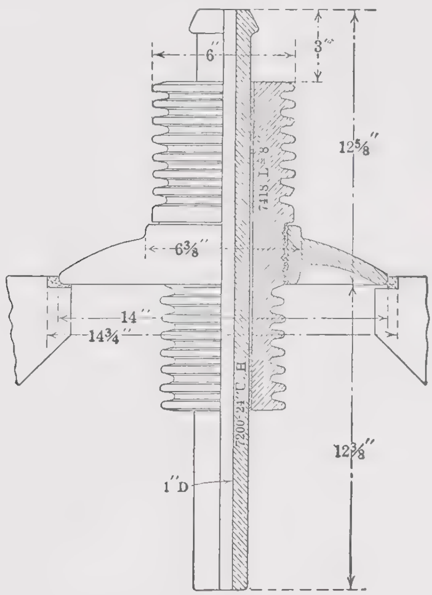


Fig. 58.—Floor bushing.

In the case of a roof bushing, permitting of the conductor passing vertically downward to the inside of the building, a hollow elongated barrel-shaped insulator (that is, of greater diameter at center, where it is supported on the outside, than at the two ends) filled with an insulating substance of higher specific inductive capacity and greater dielectric strength than air, makes a satisfactory arrangement provided there is no danger of the oil or other insulating filling leaking out of the containing shell. A high-grade insulating oil will frequently give good results, but it is liable to leak out at the joints, and, moreover, the use of oil calls for a

larger and heavier bushing than if the filling has a specific capacity more nearly equal to that of the porcelain shell (porcelain being the material most generally used). It will be understood

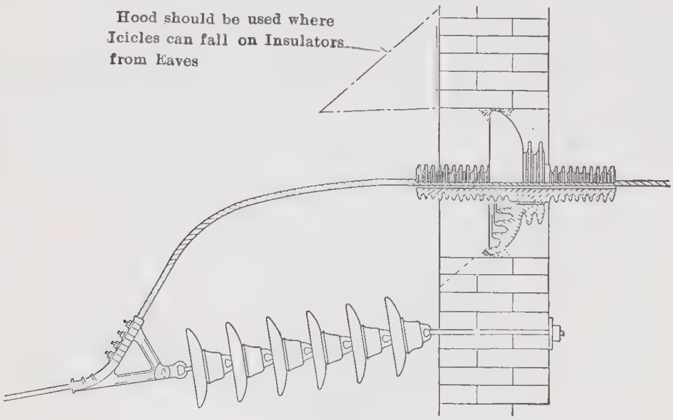


Fig. 59.

that if the substance filling the space between metal conductor and hollow bushing has the same specific capacity as the material of the bushing, there will be no change in the potential gradient at the inner surface of the bushing. The thickness of the porcelain shell and the distance between inner surface of shell and conductor surface should be proportioned in accordance with the insulating material intended for use as a filling.

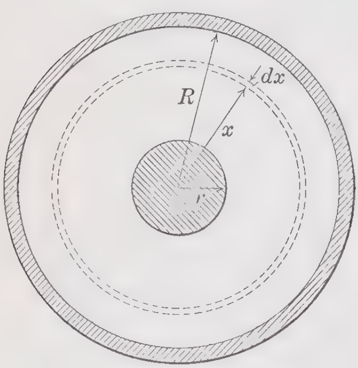


Fig. 60.—Section through insulating bushing.

76. Design of Insulating Bushings.—Without attempting to go into the details of design, the application of the principles and formulas of Article 72 may be illustrated by considering the stresses in the insulation which separates a cylindrical rod at high potential from a concentric cylindrical tube at ground potential. Fig. 60 is a section through a conductor of radius r separated by insulating material of specific

inductive capacity k from a concentric metal cylinder of radius R.

The equipotential surfaces will be cylinders, and the flux density over the surface of any cylinder of radius x and of unit length, say 1 cm., will be $D = \frac{\Psi}{2\pi x}$.

By formula (57) the potential gradient is,

$$G = \frac{D}{Kk} = \frac{\Psi}{2\pi x Kk} \tag{58}$$

In order to express this relation in terms of the total voltage E, it is necessary to substitute for the symbol Ψ its equivalent $E \times C$, and calculate the capacity C of the condenser formed by the rod and the concentric tube. Considering a number of concentric shells in series, the *elastance*[1] may be written as follows:

$$\frac{1}{C} = \int_r^R \frac{dx}{2\pi x Kk} = \frac{1}{2\pi Kk} \log_\epsilon \frac{R}{r} \tag{59}$$

Substituting in (58), we have

$$G = \frac{E}{x \log_\epsilon \frac{R}{r}} \text{ volts per centimeter} \tag{60}$$

the maximum value of which is at the surface of the inner conductor, where

$$G_{\text{max.}} = \frac{E}{r \log_\epsilon \frac{R}{r}} \tag{61}$$

This formula is of some value in determining the thickness of insulation necessary to avoid overstressing the dielectric; but it is not strictly applicable to wall bushings in which the outer metal surface is short as compared with the diameter of the opening. The advantage of having a fairly large value for r is indicated by formula (61), and a good arrangement is to use a hollow tube for the high-tension terminal.

Solid porcelain bushings with either smooth or corrugated surfaces may be used for any pressure up to about 40,000 volts. In designing plain porcelain bushings it is important to see that the potential gradient in the air space between the metal rod and

[1] The reciprocal of the *permittance* or capacity.

the insulator is not liable to cause brush discharge, as this would lead to chemical action, and a green deposit of copper nitrate upon the rod.

Fig. 61 shows a bushing that is liable to give trouble owing to brush discharge in the air space between the high-tension rod and the inner surface of the porcelain bushing. The reason for this will be best understood by working out a numerical example. Let us assume the following numerical values:

Radius of inner conductor, $r = 0.25$ in.

Inside radius of porcelain bushing, $r' = 0.375$ in.

Outside radius of porcelain bushing (in contact with grounded metal cylinder) $R = 1.5$ in.

Dielectric constants: for air $k = 1$; for porcelain $k = 4.5$.

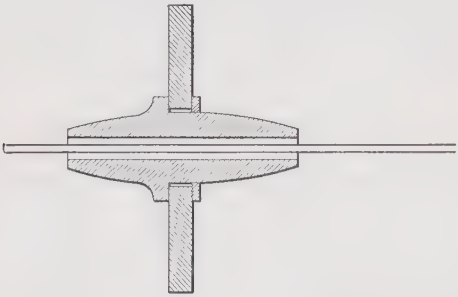


FIG. 61.—Type of bushing liable to cause brush discharge on surface of wire.

We have here the case of two capacities in series, the first between the high-tension rod and the inner surface of the porcelain, with air as the dielectric, and the second between the inner and outer cylindrical surfaces of the porcelain. The dielectric flux (and the charging current) being the same in both, and since $\Psi = E \times C$, it follows that the potential difference across each condenser will be inversely proportional to the capacity, or directly proportional to the *elastance* as given by formula (59).

Let E_a be the voltage across the air gap, and E_p the voltage across the porcelain sleeve: then,

$$\frac{E_a}{E_p} = \frac{\log\left(\frac{r'}{r}\right) \times 4.5}{\log\left(\frac{R}{r'}\right)} = 4.5\frac{\log\left(\frac{375}{250}\right)}{\log\left(\frac{1500}{375}\right)} = 1.315$$

which shows the voltage across the small air space to be greater than that across the greater thickness of the porcelain bushing.

The voltage gradient which will cause corona or brush discharge on the surface of a rod of radius r, according to Mr. Peek,[1] is $G_v = 31\left(1 + \frac{0.3}{\sqrt{r}}\right)$ where r is in centimeters. In this example, with $r = 0.25$ in. we find the value of G_v to be 42.7 k.v. per centimeter. Then, by formula (61) the maximum permissible value of the voltage across the air space is

$$E_a = 42.7 \times 0.25 \times 2.54 \times \log_\epsilon\left(\frac{0.375}{0.250}\right) = 11 \text{ k.v.}$$

The total permissible pressure across the bushing is therefore $11 \times \left(1 + \frac{1}{1.315}\right) = 19.37$ k.v. of which the virtual value is $\frac{19.36}{\sqrt{2}} = 13.67$ k.v.

Let us now consider the stress at the surface of the conductor if the porcelain sleeve is entirely removed, leaving air as the only dielectric between the rod and the outer metal cylinder. By formula (61),

$$\textit{max. value of } E = 42.7 \times 0.25 \times 2.54 \times \log_\epsilon\left(\frac{1.5}{0.25}\right) = 48.6 \text{ k.v.}$$

of which the virtual value is $48.6/\sqrt{2} = 34.4$ k.v.

Thus, if the porcelain bushing is entirely removed, and replaced by air, the voltage (in this example) may be increased from 13.67 to 34.4 k.v. before corona will form at the surface of the rod. Two remedies for the trouble resulting from the original design suggest themselves. We might (1) coat the inside of the porcelain bushing with tin-foil and connect this electrically with the high-tension rod, or (2) we might fill the intervening air space with oil or some solid insulating compound of higher specific inductive capacity than air.

(1) This method is equivalent to making the high-tension conductor of radius 0.375. Let us assume the total voltage to be 34.4 k.v. as calculated for the arrangement with air space only. The maximum potential gradient, by formula (61), will then be,

$$G_{\text{max.}} = \frac{34.4 \times 1.41}{0.375 \times 2.54 \times \log_\epsilon\left(\frac{1.5}{0.375}\right)} = 36.8 \text{ k.v. per centimeter}$$

[1] "Dielectric Phenomena," by F. W. Peek, Jr., McGraw-Hill Book Co.

which is less than with the smaller diameter of rod, notwithstanding the reduced thickness of the dielectric.

(2) If we assume, for simplicity in the calculation, that the filling compound has the same dielectric constant as the porcelain ($k = 4.5$), we can write,

$$G_{\text{max.}} = \frac{34.4 \times 1.41}{0.25 \times 2.54 \times \log_\epsilon \left(\frac{1.5}{0.25}\right)} = 42.7 \text{ k.v. per centimeter}$$

which is, of course, the same as when the entire space is filled with air; but there will be no corona formation on the surface of the high-tension conductor.

The disadvantage of small diameters for high-tension conductors is well brought out by this example. Without any increase in the size of the outer (grounded) cylinder, an increase in diameter of the conductor from ½ in. to ¾ in. has reduced the voltage gradient at the conductor surface (where its value is a maximum) from 42.7 k.v. to 36.8 k.v. per centimeter, and this notwithstanding the fact that the distance from metal to metal, is actually *less* in the case of the lower figure. There is obviously a limit—which is easily calculated—beyond which any increase in the diameter of the inner rod would lead to break-down on a lower voltage.

77. Condenser Type of Bushing.—By separating thin concentric layers of insulating material by tubes of tin-foil or other metal, and so proportioning the *lengths* of the tin-foil tubes that the *areas* remain the same, notwithstanding the variations in diameter, it is possible to design a bushing which virtually consists of a number of condensers of *equal* capacity all connected in series. In this manner the potential gradient can be made uniform throughout the thickness of the insulating bushing. Commercial entering bushings and transformer terminals have for some time past been built on this principle. The design of such terminals is not quite so simple a matter as this brief reference to the main principle involved might suggest, but the commercial insulators of this class have, on the whole, given good service.

The tendency to equalize the potential gradient throughout the concentric layers of insulating material, leads to a bushing of smaller overall diameter than when no attempt is made to equalize the stresses. Thus, if the figure of 36.8 k.v. per centimeter as calculated in the numerical example of the preceding article

is assumd to be the maximum working stress of a solid porcelain bushing of inside radius $\frac{3}{8}$ in. and outside radius $1\frac{1}{2}$ in., a bushing of the condenser type designed for the same maximum stress and suitable for the same total voltage, would have an outside radius of only

$$R = \frac{3}{8} + \frac{34.4 \times 1.41}{36.8 \times 2.54} = 0.893 \text{ in.}$$

This calculation does not take account of the thickness of the dividing layers of tin-foil, or of the fact that perfect equalization of the stress is not obtainable in a practical design of bushing.

Before considering such matters as factors of safety, spacing between wires, and probable limits of pressure on power transmission lines, it will be well to review briefly what is known about the brush discharges and corona formation, which are liable to become important factors when transmitting at the higher voltages.

78. Formation of Corona, and Accompanying Losses of Power.—When the pressure on an overhead transmision system exceeds a certain critical value depending upon the spacing and diameter of the wires, there will appear on the surface of the conductors a halo-like glow to which the name "corona" has been given. Apart from this luminous effect, the appearance of the corona is accompanied by a certain loss of power proportional to the frequency and the square of the amount by which the pressure between conductors exceeds a certain value known as the disruptive critical voltage. If the distance between outgoing and return conductors is comparatively small (less than fifteen times the diameter of the wire) there will be a spark-over when the disruptive critical voltage is reached; but with the greater separation such as occurs on practical high-tension transmission lines, the effect of the high potential at the conductor surface is to break down the resistance of the air in the immediate neighborhood of the conductor surface. A luminous cylindrical coating of air, acting as a conductor of electricity, is thus formed, the diameter of which will depend on the amount by which the actual value of the applied potential difference between wires exceeds the disruptive critical value of the potential difference. The result is equivalent to an increase of the diameter of the conductors, thus raising the value of the voltage necessary to break down new concentric layers of surrounding air, until it is approximately

equal to the voltage impressed on the wires. During the last few years much light has been thrown on the formation and effects of the corona. Among the earlier workers in this field were C. F. Scott, Harris J. Ryan, J. J. Thomson, H. B. Smith, Signor Jona, Lamar Lindon, E. A. Watson, and, among the later investigators, J. B. Whitehead[1] and F. W. Peek, Jr.[2]

Suppose a cylindrical wire of radius r is surrounded by a concentric metal cylinder of internal radius R and that visible corona starts when the observed voltage is E_v, then the maximum potential gradient at the surface of the wire, by the previously developed formula (61) is

$$G_v = \frac{E_v}{r \log_\epsilon \left(\frac{R}{r}\right)} \tag{62}$$

The formula for parallel wires is

$$G_v = \frac{E_v}{r \log_\epsilon \left(\frac{d}{r}\right)} \tag{63}$$

where d is the distance between centers of wires.

The value of G_v (the apparent strength of air) is found to be independent of the spacing between wires, but it is not found to be independent of their diameter. The apparent strength of air is greater at the surface of small than of large wires. Mr. Peek has found that, at a distance from the surface of any cylindrical wire equal to $0.301 \sqrt{r}$ cm., the breakdown gradient of all sizes of wire is the same, namely about 30 k.v. per centimeter. This leads to the formulas which will be found at the end of this article.

The losses which occur through corona are not different in kind from I^2R losses; but since they occur whenever a visual corona appears, and may reach high values if the line voltage appreciably exceeds the voltage at which corona is first formed, the importance of designing high tension lines so as to avoid excessive corona formation is evident.

The more important features and effects of the corona of interest to the practical engineer may be summarized as follows:

1. The loss due to leakage of current from the conductor into

[1] *Proc.* A. I. E. E., vol. xxix, p. 1059 (1910), and later contributions.

[2] *Proc.* A. I. E. E., vol. xxxi, p. 1085 (June, 1912). Also *Journal* Franklin Institute, Dec., 1913; and Book "Dielectric Phenomena," McGraw-Hill Book Co., 1915.

the surrounding air is practically negligible for pressure values below the disruptive critical voltage; no account need be taken of such leakage on alternating-current circuits operated at pressures below 44,000 volts, unless the wires are at high altitudes. On 80,000 volts, however, the loss may be appreciable, and a visible corona may even be formed if the wires are small in diameter.

2. The current passing from the wires into the air on an alternating system is an energy current in phase with the pressure.

3. On alternating-current systems, the critical break-down voltage will depend upon the maximum value of the e.m.f. wave, and therefore on the "form factor."

4. The break-down voltage—or, more properly, the disruptive critical voltage—is determined by the potential gradient at the conductor surface; it is therefore dependent upon the diameter and spacing of the wires; being higher with the larger diameters and spacings; it is also dependent upon the density of the air, and therefore on the temperature and barometric pressure.

5. The loss of power due to corona formation is approximately proportional to the frequency (within the usual commercial range), and to the square of the excess of line voltage over the disruptive critical voltage.

6. The disruptive critical voltage is the voltage at which the disruptive voltage gradient of the air is reached at the conductor surface. It is highest when the conductor surface is smooth and quite clean. It is lowered by roughness or dirt on the conductors, also by smoke and fog in the atmosphere, sleet on wires and falling sleet, rain and snow storms, and low barometric pressure. All these causes tend, therefore, to *increase* the corona losses.

7. The visual corona occurs only at a pressure above the disruptive critical voltage and is an indication that there is loss of power in the air.

When considering the effects of the corona formation on overhead wires, it is convenient, as in the case of the majority of electrical problems connected with transmission lines, to consider each wire separately in relation to the neutral plane or line. Since the formation of the corona depends upon the electric stress at the surface of the conductor, it is the potential gradient in the immediate neighborhood of the wire which, as previously mentioned, is the determining factor in corona formation. The dis-

ruptive critical voltage for any particular wire under specified atmospheric conditions will, as previously mentioned, depend upon the diameter of the wire and the distance of the wire or wires forming the return conductor; also upon the density of the air and, to some extent, upon the surface condition of the wire. Mr. Peek's formula is:

$$E_o = 21.1 m_o r \delta \log_\epsilon \frac{d}{r} \text{ k.v. to neutral (virtual value)} \quad (64)$$

in which r = radius of conductor in centimeters,

d = distance between centers of the outgoing and return (parallel) conductors, in centimeters,

m_o = a factor depending upon the surface condition of the conductor,

= 1 for polished wires,

= 0.98 to 0.93 for roughened or weathered wires,

= 0.87 to 0.83 for stranded cables (average = 0.85),

δ = a factor depending on the air density,

$$= \frac{3.92\,b}{273 + t}$$

in which b is the barometric pressure in centimeters of mercury, and t is the temperature in degrees Centigrade.

The luminosity, or visible halo of light surrounding the conductor, does not occur until a higher pressure has been reached, the increase over the critical disruptive voltage being dependent upon the diameter of the conductor. Mr. Peek's formula for the visual critical voltage (kilovolts to neutral) is:

$$E_v = 21.1 m_v r \delta \left(1 + \frac{0.301}{\sqrt{r\delta}}\right) \log_\epsilon \frac{d}{r} \quad (65)$$

where the surface factor m_v has the same value as m_o for wires, and may be taken at 0.82 for a decided visible corona on seven-strand cables. The notation is otherwise as above.

The formula for loss of power in fair weather, in kilowatts *per kilometer of single wire,* as given by Mr. Peek, is:

$$P = \frac{244}{\delta} \times (f + 25) \times \sqrt{\frac{r}{d}} \times (E_n - E_o)^2 \times 10^{-5} \quad (66)$$

where f is the frequency in cycles per second, and E_n is the actual (r.m.s.) pressure between wire and neutral, expressed in kilovolts. The approximate loss under storm conditions is obtained

by taking E_o as 80 per cent. of its (virtual) value as calculated by formula (64).

The transmission line engineer will usually prefer formulas with inch units and common logarithms; and the following may be used:

$$E_o = 123\, m_o r\delta \log_{10}\left(\frac{d}{r}\right) \text{ k.v. to neutral} \tag{67}$$

$$E_v = 123\, m_v r\delta \left(1 + \frac{0.189}{\sqrt{r\delta}}\right)\log_{10}\left(\frac{d}{r}\right) \text{ k.v. to neutral} \tag{68}$$

$$P = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}\,(E_n - E_o)^2\, 10^{-5} \text{ kw. per mile of single conductor} \tag{69}$$

The air density factor can, if desired, be calculated by the formula

$$\delta = \frac{17.9b}{459 + t}$$

where b = barometric pressure in inches of mercury, and t = temperature in degrees Fahrenheit. (Note that when $b = 29.9$ and $t = 77$ degrees, $\delta = 1$.)

As a guide in estimating the average pressure at high altitudes, the following figures may be used:

Elevation, sea level,	$b = 29.9$
2,000 ft.,	$b = 27.6$
4,000 ft.,	$b = 25.6$
6,000 ft.,	$b = 23.7$
8,000 ft.,	$b = 22.0$
10,000 ft.,	$b = 20.4$
12,000 ft.,	$b = 18.9$

As a practical *example* of corona losses, consider a 100 mile, three-phase, 110,000 volt, 60-cycle transmission, using No. 1 seven-strand conductors spaced 6 ft. apart. Calculate the approximate fine-weather corona loss if the air density factor is unity ($\delta = 1$).

By formula (67)

$$E_o = 123 \times 0.85 \times 0.165 \times \log\left(\frac{72}{0.165}\right) = 45.6 \text{ k.v.}$$

By formula (69)

$$P = 390 \times 85 \times \sqrt{\frac{0.165}{72}} \times \left(\frac{110}{\sqrt{3}} - 45.6\right)^2 \times 10^{-5}$$

$$= 5.1 \text{ kw. per mile of single conductor.}$$

The total loss to be expected under fair weather conditions is therefore $5.1 \times 3 \times 100 = 1530$ kw.

The assumption is here made that the line voltage (and therefore E_n) remains constant over the entire length of 100 miles.

79. Corona Considered as "Safety Valve" for Relief of High-frequency Surges or Over-voltage Due to Any Cause.—The loss, as calculated in the above example, is not small; and since it is proportional to the *square* of the excess of pressure over the disruptive critical voltage, a small increase of pressure will lead to an enormously increased dissipation of energy in the air. Thus, if the pressure of 110 kv. in the above example be supposed to increase only 10 per cent. the total dissipation of power, instead of being 1530 kw., would be 2800 kw. It has indeed been stated that on the 110,000-volt system of the Grand Rapids-Muskegon Power Co., the line loss due to corona discharge actually increases 100 per cent. for a 10 per cent. rise in pressure.

This property of the corona suggests the possibility of working high-voltage transmission lines at a normal pressure in the neighborhood of the critical disruptive voltage where the loss would be inappreciable. An extra-high-voltage discharge, due either to atmospheric lightning, or to internal causes, would then be largely dissipated in the corona itself. This may, to some extent, account for the fact that fewer lightning troubles are experienced on the very high voltage transmissions than on the lower voltage lines. The insulation of the conductors being such as to withstand, without breakdown, pressures considerably in excess of the disruptive critical voltage of the corona, a large amount of oscillating energy can be dissipated in the air before the voltage rises to such a value as to pierce or shatter insulators or damage apparatus connected to the line. On the other hand, too much reliance should not be placed on the corona as a means of dissipating large amounts of suddenly impressed energy; because lightning and similar disturbances, being to a great extent local, must discharge their power locally, and the corona losses over a *short* section of the transmission line cannot under any circumstances be very great.

80. Spacing of Overhead Conductors.—It is difficult to lay down rules for the proper spacing of overhead conductors. The question has been settled in the past by the individual engineer who has usually striven to be "on the safe side" in the matter of possible discharges between wires under abnormal conditions

such as strong and variable winds. The result is that great differences are to be found in the wire spacings in different countries or on different transmission systems in the same country.

The spacing of the conductors should be determined by considerations partly electrical and partly mechanical. With the longer spans, the spacing should be greater than with short spans,

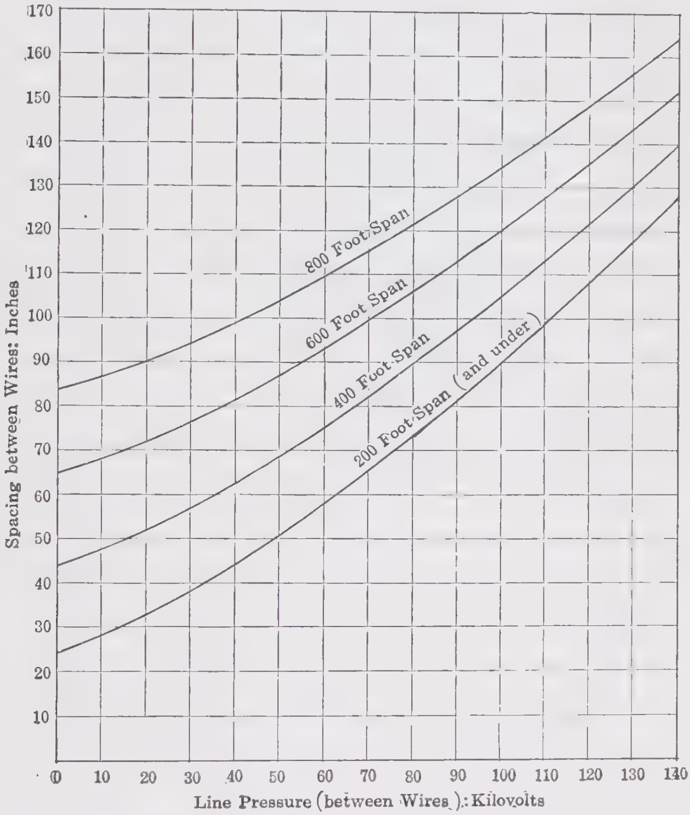


FIG. 62.—Approximate separation of overhead conductors.

apart from voltage considerations. The material and diameter of the conductors should also be taken into account when deciding upon the spacing, because a small wire—especially if of aluminum—having a small weight relatively to the area presented to a cross wind, will swing out of the vertical plane farther than a conductor of large cross-section. Usually wires will swing synchronously in a wind; but with long spans and small wires,

there is always the possibility of the wires swinging non-synchronously, and the size of wire, together with the maximum sag at center of span, are factors which should be taken into account in determining the distance apart at which they shall be strung. A horizontal separation equal to something between one and one and a half times the sag at the temperature corresponding to the season of highest wind velocities should be sufficient to prevent wires swinging within sparking distance of each other; the closer spacing being used with copper conductors of large diameter.

The curves of Fig. 62 will be found to give spacings generally in accordance with present-day practice. These figures can be used as a guide in arriving at a suitable value for the *horizontal* spacings. The *vertical* spacing may be less; but it is usually undesirable to suspend wires in the same vertical plane, especially in locations where sleet and ice deposits are likely to occur.

Distance Between Conductor and Pole or Tower.—The following clearances are recommended:

Line pressure, k.v.	Clearance, in.
10 (and under)	9
15	10
22	11
35	14
44	17
66	24
88	32
110	36

In the case of suspension type insulators it is well to arrange for the clearance, even under conditions of greatest deflection caused by high winds, to be not less than the sparking distance over the string of insulator units.

81. Practical Limitations of Overhead Transmission-line Voltages.—From the foregoing review of the insulation problems to be met with on long-distance overhead transmissions, it will be clear that manufacturers are now in a position to provide insulation amply sufficient for present requirements. Power is actually being transmitted at 150,000 volts. The lines of the Au Sable Electric Co. in Michigan, transmitting power at

140,000 volts, consist of stranded copper conductors about $\frac{3}{8}$-in. diameter on 500-ft. spans, with a sag allowance of approximately 12 ft. The shortest distance between conductors is 12 ft., this being the vertical height between the two conductors on one side of the steel supporting towers. There is practically no visible corona, but a buzz or hum, due no doubt to brush discharge, can be heard in the neighborhood of the transmission lines.

Although there are no insurmountable difficulties in providing ample insulation for these high voltages, it is the engineer's business to provide such insulation as will be justified by economic considerations. It is also his business to determine the voltage of transmission on the same basis, and resist the temptation to experiment in voltages higher than may be justified by commercial considerations.

It is well to bear in mind that the economical transmission voltage depends not only on the length of the line, but also on the amount of power to be transmitted; and although a 200,000-volt transmission offers no serious engineering difficulties, the conditions under which a transmission at so high a voltage would be a commercial success, are very seldom found.

82. Factors of Safety: Rating and Testing of Line Insulators.—When selecting insulators and deciding upon the spacing and arrangement of conductors suitable for a given voltage, the factor of safety to cover abnormal pressure-rises is a matter of great importance, since it is obviously bad engineering to provide insulation in excess of what experience has shown to be a reasonable safeguard against interruption of service. Generally speaking, the insulators should, when dry, withstand a pressure test of $2\frac{1}{2}$ to 3 times the working pressure to ground, applied for five minutes, and a wet test of not less than twice the working pressure. This would sometimes be considered too small a margin of safety; but the ratio between test pressure and working pressure will depend upon whether the line voltage is high or low. The following safety factors, representing ratio between wet-test pressure and working pressure are generally in accordance with usual practice; but the engineer should use his judgment in a matter of this sort. It is clear that, on the coast, where gales and salt sea mists are prevalent, the factor of safety should be rather higher than in a district where the climatic conditions are more favorable. The effects of high altitude—to be referred to later—must also be taken into account.

Working pressure (voltage between line wires)	Safety factor (wet test)
20000	2.5
40000	2.2
80000	2.0
Above 80000 volts	1.8 to 2

As the wet or "rain" test will give different results, depending on the method of conducting the tests, there should be a clear understanding between the purchaser and manufacturer on this point. A very common specification is that the spray shall be directed at an angle of 45 degrees, under a pressure of 40 lb. per square inch at the nozzles; the flow being regulated to give a precipitation of 1 in. in 5 minutes. The method of attaching test wire and ground connection to the insulator should also be clearly defined. The test pressure is usually measured by means of a spark gap; and the alternating e.m.f. used should conform as nearly as possible to the sine wave.

The striking distances in air between No. 3 *sharp* needles, as given by the Locke Insulator Mfg. Co., are as follows:

Kilovolts	Inches	Kilovolts	Inches
20	1.00	200	20.50
40	2.45	225	23.05
60	4.65	250	25.60
80	7.10	275	28.30
100	9.60	300	31.00
125	12.25	350	36.10
150	15.00	400	41.20
175	17.80		

It will be observed that for pressures above 100,000 volts the gap between needle points is approximately 1 in. per 10,000 volts. The pressures referred to in the table are the virtual or root-mean-square values of the test pressure, on the sine wave assumption.

Effect of Altitude.—The insulators on a transmission line erected at high altitudes will flash over with a lower voltage than if the line were erected at sea level. The reason for this is the reduced air pressure at the higher elevation. The flash-over voltage will not be exactly proportional to the barometric pres-

sure because the electrostatic field is not uniform, but depends upon the type and design of the insulator. Mr. Peek gives some results of experimental work on various standard designs of insulator in his book on Dielectric Phenomena previously referred to; but since the departures from the theoretical relation for uniform dielectric fields is very small, the correction for altitude can safely be made by assuming the flash-over voltage to be directly proportional to the barometric pressure.

As an example: suppose the flash-over voltage of a pin type insulator is found to be 100 kv. on test at sea level; then, if used on a transmission line erected at an elevation of 8000 feet, it would be liable to flash over with a pressure of only $100 \times \frac{22}{29.9} = 73.5$ kv. (The proportional figures for air density are taken from the table on page 155.) It is thus easy to decide upon a factor of safety which shall make proper allowance for the elevation at which the insulator may have to operate.

Rating of Insulators.—The fact that conditions of tests and factors of safety may have to be modified in accordance with the conditions under which the insulator will be expected to operate, suggests the subject of insulator rating. At the present time this question is far from being settled in a satisfactory manner, and it would seem that manufacturers and users should get together with a view to deciding upon an acceptable basis which would permit of any design of insulator being placed in a particular class, and so facilitate the comparison between tenders submitted by competing firms for insulators to operate under given conditions. At first sight the most reasonable method appears to be to adopt some arbitrary rule depending upon the dry and wet flash-over voltage of the insulator: for instance, to specify that for the higher line pressures the dry flash-over should be three times the line voltage, and the wet flash-over twice the line voltage. This would, however, probably lead to confusion because of the number of variable factors involved in determining the dry and wet flash-over. It is possible—as suggested to the writer by an engineer connected with one of the best known firms manufacturing porcelain insulators—that a method of rating depending not upon *tests*, but upon design *dimensions*, such as the overall height and the minimum arcing distances (both wet and dry), might prove a useful basis of classification.

Detecting Faulty Insulators while Line is in Operation.—Another question which is now receiving attention, but is not settled or standardized, is the best means of detecting faults in insulators while in use. It is only of recent years that accurate data on the "life" of high tension insulators is becoming available; and the continued action of alternate heat and cold, dryness and dampness, on the porcelain—or rather on the complete assembly of porcelain, metal, and cement—is found to necessitate a very large percentage of replacements after a line has been in operation many years.

The causes of rapid deterioration—especially after several years of service—are being investigated, and eliminated as far as possible in the later designs;[1] but in the meanwhile, lines that have been in use for a considerable time are giving trouble in the matter of insulation, involving increasing vigilance and activity on the part of the operating staff. There is room for improvement in the methods now available for detecting incipient faults in line insulators without interrupting the supply or disconnecting insulators from the live wires. One method, which involves the use of a telephone, and is said to give reliable information as to the condition of the insulators, is described on page 821 of Vol. 64 of the *Electrical World* (Oct. 24, 1914). The reader who is interested in this matter of insulation troubles should also refer to an article by Prof. H. J. Ryan in the *Journal of Electricity* (San Francisco), Feb. 27, 1915; and to the article "Testing Insulators to Assure Continuous Service" by Professor R. W. Sorensen in the *Electrical World* of Sept. 1, 1917 (Vol. 70, p. 426).

[1] Refer *Trans.* A. I. E. E., vol. xxxiii (1914), pp. 111 and 119 (J. A. Brundige), and p. 1731 (A. O. Austin). Vol. xxxiv (1915), p. 465 (E. E. F. Creighton). Vol. xxxvi (1917), p. 527 (W. D. Peaslee), p. 535 (J. A. Brundige), and p. 545 (A. O. Austin).

CHAPTER VI

PROTECTION AGAINST LIGHTNING—TRANSIENT PHENOMENA

83. Theoretical Considerations.—Before taking up the matter of lightning disturbances and the means adopted for minimizing their destructive effects on line insulation and station apparatus, the causes leading to surges, oscillations, and travelling or standing waves, will be very briefly discussed. No attempt will be made to deal thoroughly with this somewhat difficult subject which has lately come into prominence because of the increasing distances to which energy is being transmitted electrically. Nothing will be included here beyond the elementary considerations with which the engineer engaged on the design of high-tension long-distance transmission lines should be familiar. For a complete study of the principles underlying transient phenomena—energy surges and oscillations, and the peculiarities of hypothetical "quarter wave length" transmission lines—the reader is referred to authorities such as Dr. Steinmetz[1] and the writings by various authors, that have appeared recently in the technical Journals and the publications of the Scientific and Engineering Societies.

The relation existing between the voltage and the current of any transient electrical disturbance occurring in a circuit depends upon the relation between the magnetic flux-linkages per unit current—or the *inductance*—and the *permittance* or electrostatic *capacity*.

Consider a circuit in which there is alternating or oscillating energy which is not utilized by any form of receiving apparatus, and is not dissipated in the form of heat through the ohmic resistance of the conductors, or through "dielectric hysteresis" or corona: it is obvious that, at the instant when the current wave passes through zero value, the whole of the energy must be stored in the electrostatic field, and similarly, at the instant when the pressure wave passes through zero value, the whole of the oscillating energy must be stored in the electromagnetic field.

[1] "Transient Electric Phenomena and Oscillations."

Moreover, so long as the interchange of energy from one form to the other continues without diminution of amount, *these two quantities must be exactly equal.* This conception of the oscillations of energy in a circuit having negligible resistance, but appreciable inductance and capacity, is fundamental, and we shall examine it in further detail with a view to arriving at a definite relation between the amplitudes of the voltage and current waves.

Energy Stored in Magnetic Field.—Since the engineer usually prefers to think of volts and amperes, the product of which represents power (watts) or the rate at which work is being done, we can say that the energy stored in a magnetic field during a short interval of time dt seconds is $ei \times dt$ watt-seconds or *joules.* In this connection the voltage e is the e.m.f. developed in a conductor carrying i amperes when a change in current di causes a change of flux $d\Phi$ in the short interval of time dt. Thus, since we are considering the flux linking with a circuit of one turn (a transmission line conductor) in a medium (air) of constant permeability, we can write,

$$e = \frac{d\Phi}{dt} = \mathbf{L}\frac{di}{dt}$$

whence

$$\left.\begin{array}{l}\text{Energy stored in magnetic field during the}\\ \quad\text{interval of time } dt\end{array}\right\} = \mathbf{L}\frac{di}{dt} \times i \times dt$$
$$= \mathbf{L}i \times di$$

and since the current grows from zero to its maximum value in a quarter of a period, we have,

$$\left.\begin{array}{r}\text{Energy stored in magnetic field}\\ \text{during one quarter period}\end{array}\right\} = \mathbf{L}\int_0^{I_{max.}} i \times di$$
$$= \tfrac{1}{2}\,\mathbf{L}I^2_{max.}\text{ joules} \qquad (70)$$

It is easy to show in a similar manner that the energy stored in the dielectric circuit in one quarter period while the value of e grows from zero to $E_{max.}$ is $\frac{1}{2}\,CE^2_{max.}$ where C is the electrostatic capacity of the circuit, or portion of circuit considered.

Thus, in the case of a pure, undamped, oscillation, when no energy is supplied from the outside to the circuit, or by the circuit to the outside, it follows that

$$\mathbf{L}I^2 = CE^2$$

or

$$\frac{E}{I} = \sqrt{\frac{\mathbf{L}}{C}} \tag{71}$$

where $\mathbf{L}$ is expressed in henrys, and C in farads.

The quantity $\sqrt{\frac{\mathbf{L}}{C}}$ is thus seen to be of the nature of a resistance or impedance, and it may be expressed in ohms. It is generally called the *natural impedance* of the circuit; but the expressions *wave impedance* (or resistance) and *surge impedance* (or resistance) are also used to denote the ratio $\frac{\text{volts}}{\text{amperes}}$ of the oscillating energy.

In the case of an overhead transmission line, the approximate value, per mile, of the inductance betwen one conductor and neutral, as given in Article 48, Chapter IV (page 88) is

$$\textit{External Inductance, } \mathbf{L} = 0.000741 \log \frac{d}{r}$$

and the approximate formula for capacity, as given in Article 10 of Chapter II, is

$$C = \frac{0.0388}{\log \frac{d}{r}} \times 10^{-6} \text{ farads}$$

whence the *surge impedance* of an overhead transmission line is approximately,

$$\sqrt{\frac{\mathbf{L}}{C}} = 138 \log \frac{d}{r} \text{ ohms} \tag{72}$$

In practical overhead work, the limiting values for the ratio $\frac{d}{r}$ will probably be 800 and 50; which, when inserted in formula (71), show that the "natural impedance" of an overhead transmission line must lie between 400 and 230, or, to be well on the safe side, between (say) 500 and 200 ohms.

A knowledge of this quantity renders it possible to determine the maximum value of any surge pressures that can possibly occur on the line due to the sudden interruption of the current. Thus, if the "natural impedance" is 300 ohms, and the instantaneous value of the current at the crest of the wave is 200, the surge pressure, however suddenly the current is interrupted, cannot possibly exceed $200 \times 300 = 60{,}000$ volts; because this is the maximum value of the pressure wave necessary to store in the electric field the whole of the energy stored in the magnetic field at

the moment when the current was interrupted. It is safe to say that, on a practical transmission line, the surge pressure is never likely to exceed 200 times the current in amperes; but, with heavy currents, this may well be sufficient to break down insulation and cause considerable damage to power plant. It must not be overlooked that it is often more difficult to handle heavy currents at comparatively low pressures than small currents at the very highest pressures yet attempted. When the current is large, the opening of switch or fuse on full-load, or an accident causing a break in the circuit, with or without the formation of an arc across the gap, may lead to insulation troubles on many widely separated parts of the system; but on a high-pressure system, even if the current were as large, the insulation is frequently so good that it will withstand without injury the stress imposed on it by the highest possible value of the surge pressure.

In underground cables, the capacity is much larger relatively to the inductance than in overhead systems, and the surge impedance, $\sqrt{\frac{L}{C}}$, has then a smaller value, which may be about one-tenth of the value for overhead lines; but the transformers connected to transmission systems will always have a surge impedance very considerably higher than that of the line itself.

The effect of the ohmic *resistance* in series with the inductive and condensive *reactances* of a circuit, is to damp out the oscillations by dissipating the energy in the form of I^2R losses. With a sufficiently high value of resistance in the circuit, surges or oscillations of energy cannot take place.

For the condition of massed resistance, capacity, and inductance, the critical resistance is

$$R = 2\sqrt{\frac{L}{C}}$$

and if R has a higher value than this, oscillations of energy cannot occur. The case of a transmission line with distributed capacity and inductance is much more complicated, and the mathematical analysis is very difficult; but if the receiving end of a long transmission line of negligible resistance is closed through a resistance (line to neutral) of value

$$R = \sqrt{\frac{L}{C}} \tag{73}$$

there will be no oscillations of energy resulting from a sudden change of potential. In other words, a surge travelling along the line will be completely absorbed, and there will be no "reflected" waves. (The proof of this statement will be given in Article 86).

If R has a value smaller than that given by formula (73) there will be "reflection" of some part of the impressed energy, and oscillations will occur; but these will gradually decrease in amplitude according to the logarithmic law.

84. Frequency of Oscillations.—The rate at which the oscillating energy will pass back and forth between the magnetic and the dielectric fields is entirely independent of the frequency of the power current in a transmission line. By formula (71) we have

$$E = I\sqrt{\frac{\mathbf{L}}{C}} \tag{74}$$

but since I may be considered as the charging current of a condenser of capacity C with a voltage E across the terminals, we can also write

$$E = \frac{I}{2\pi f C} \tag{75}$$

which is obtained from formula (36) of Article 54, Chapter IV. The value of f obtained by equating (74) and (75) is

$$f = \frac{1}{2\pi\sqrt{\mathbf{L}C}} \text{ periods per second} \tag{76}$$

which is the periodicity of the oscillations when the inductance and capacity are supposed to be concentrated at a given point. When capacity and inductance are *distributed* as on a long-distance power-transmission line, the formula for the frequency of oscillation as developed by Dr. Steinmetz is

$$f = \frac{1}{4\sqrt{\mathbf{L}C}} \tag{77}$$

which is the frequency of *resonance* and is called the *natural frequency* of the line. It should be noted that $\mathbf{L}$ and C, as in the previous formulas, are expressed in henrys and farads; but since we are dealing with a *product*, not a *ratio*, of the two quantities, it is the inductance per mile $\times$ length in miles, and the capacity per mile $\times$ length in miles, that these symbols now stand for.

By formula (37) of Article 54, Chapter IV, the approximate value of the product CL for an overhead transmission one mile long is seen to be

$$CL = \frac{1}{34700 \times 10^6}$$

which, when substituted in formula (77), gives for the *natural frequency* of an overhead line,

$$f = \frac{1}{4} \times \frac{186000}{L} \qquad (78)$$

where L is the distance of transmission, or length of a single conductor, in *miles*.

85. Wave Length.—The rate of travel of an electric impulse along an overhead wire is approximately the same as the velocity of light, or (say) 186,000 miles per second. Thus, if we imagine

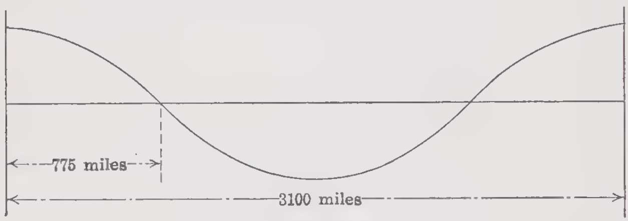


FIG. 63.—Diagram showing instantaneous value of current at different points on a long transmission line.

an alternating e.m.f. of frequency $f = 60$ applied to the ends of a circuit of length $\frac{186000}{60} = 3100$ miles, the maximum value of the current wave would occur at the receiving end of the line simultaneously with the maximum value at the sending end, but it would be the crest of a current wave which had left the sending end exactly $\frac{1}{60}$ second earlier. Fig. 63 is an attempt to indicate the travel of the electrical impulse over a wire of great length. The ordinates of the curve show the value of the current at any point along the line at the instant when the impressed sine wave of current has attained its maximum value at the sending end of the line. The *wave length*, or distance covered by one

complete wave, is $\frac{186000}{f}$ miles, which in this example is 3100 miles.

A *quarter wave length line* with a frequency of 60 would be $\frac{186,000}{4 \times 60} = 775$ miles long; it would have the peculiarity that the pressure (or current) wave would have zero value at one end of the line at the same instant of time as its value was a maximum at the other end. The characteristics of such a line are quite different from those of an ordinary transmission line, and although at ordinary frequencies trouble from this cause is not likely to result, it is possible to get the resonance effect of a quarter wave line with the higher harmonics of a distorted wave, even on practical transmission lines.

The *natural period* of an overhead line, as given by formula (78) is

$$f = \frac{1}{4} \times \frac{186000}{L}$$

in which L is now seen to be the length in miles of a *quarter wave* transmission line, although it was not previously pointed out that the constant 186,000 in formula (78) actually represents the velocity of light (or of an electric impulse) in miles per second.

Without attempting to explain or analyze the properties peculiar to a quarter wave length transmission line, it may be said that these are largely due to the fact of the quarter wave displacement providing the charging current for the line, and so leaving the generators to supply the load and losses. The inductive pressure drop and the charging current are, in effect, wiped out by the peculiar overlapping of the travelling waves of energy. The power factor of a line specially designed to make use of this peculiarity would, therefore, be very nearly 100 per cent. at all loads, and the regulation, even if the load were inductive, might be surprisingly good. With distorted waves, and complications due to limited length of line, branch circuits, and other causes, it is usually desirable to avoid the conditions of resonance in practice.

As an example, consider a line 200 miles long: what frequency will cause the *quarter wave* effect? This is the frequency which causes the conditions at the sending end to be repeated at the

receiving end exactly one quarter of a period later, and by formula (78) we have

$$f = \frac{186000}{4 \times 200} = 233 \text{ cycles per second}$$

which is the lowest frequency at which free oscillations can occur on a transmission line of this length. It corresponds to the third harmonic of a wave of fundamental frequency 77.7, which should therefore be avoided; but either 60 or 25 cycles will be satisfactory on a line of this length.

86. Reflection of Travelling Waves.—Imagine a three-phase transmission line arranged as shown in Fig. 64. Here G is a three-phase generator from which a voltage e as measured between line (1) and ground (or neutral point of star connection) is suddenly impressed for a very short space of time, by closing and then immediately opening the three-pole switch S. We have here the case of a "wave pulse" travelling along the transmission line with the velocity of light. Let us consider what happens when it reaches the end of the line, for the three conditions,

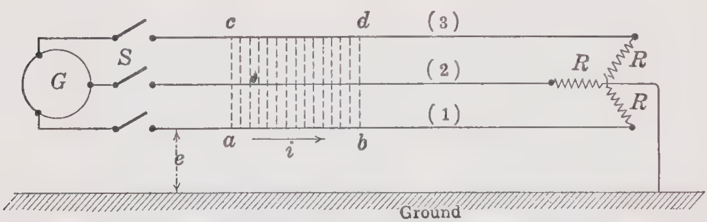


FIG. 64.—Diagram illustrating "wave pulse" travelling along a three-phase transmission line.

1. Line open ($R = \infty$)
2. Line short-circuited ($R = 0$)
3. Line closed through a non-inductive resistance of value R (line to neutral).

Suppose that the operations of closing and opening the switch have been performed so quickly that we have a rectangular wave pulse of which the length ab depends upon the time during which the switch was closed. The current i flows in the wire between the points a and b, but neither forward of b nor backward of a. It carries with it the system of magnetic and dielectric flux lines in the space comprised between the planes ac and bd; the complete energy pulsation being supposed to move away from G with the velocity of light.

Assuming the resistance of the conductors to be negligible, the relation between i and e is given by formula (71).[1]

To satisfy Case (1), the current arriving at the open end of the line must, at every instant, be equal to the current leaving this point; and since $R = \infty$ (*i.e.*, since the circuit is open) $i + i' = 0$; the symbol i' standing for the "reflected" current.

In order to reverse the current and start the wave pulse back toward the generator end of the line, we must conceive of a "piling up" of the e.m.f. at the open end of the line sufficient to force the reflected current i' against the incoming current i. But that is exactly what must necessarily occur because when half of the wave pulse of current has been reflected, the resultant current in the line will be zero, and since no energy has been lost, it must all be in the electrostatic field.

Apart from the relation $i + i' = 0$, at the open end of the line, the equations

$$\frac{e}{i} = \sqrt{\frac{L}{C}}$$

for the outgoing wave, and

$$-\frac{e'}{i'} = \sqrt{\frac{L}{C}}$$

for the reflected wave, must be satisfied; whence it follows that $e = e'$ when $i = -i'$.

In other words, during the period of reversal of the current wave, at all portions of the line where $i = -i'$, the voltage must be $2e$. On arrival at the point where the reversal of the wave pulse is complete, the voltage will again have its original value of e volts.

Case 2.—With a short-circuit at the end of the line, complete reflection of the wave pulse will also occur, but it will now be the *voltage* instead of the current that is reversed. Since there

[1] This relation is not obviously true of a travelling wave or impulse; but in order to avoid devoting a disproportionate amount of space to this subject, it is necessary to refer the reader to other sources of information, should he find this brief treatment inadequate to his needs. The most practical and the clearest explanation of the peculiarities of travelling waves, known to the writer, will be found in Franklin and MacNutt's "Advanced Electricity and Magnetism." These authors present the subject in a manner that will satisfy the requirements of most electrical engineers far better than the many highly mathematical writings on the same subject by other authors.

is a short-circuit at the end of the line, the volts must be zero when the wave pulse is in process of reversal, and in order to maintain the total energy constant (a necessary condition) the current where the overlapping occurs must be $2i$. When reversal is complete, the current will be i as before—that is to say, of the same amount and direction; but the voltage e will be of opposite sign because the wave pulse of energy is now travelling along the line in the reverse direction.

Case 3.—Comparing the two preceding extreme cases, it is evident that since $e' = e$ and $i' = -i$ when $R = \infty$, while $e' = -e$ and $i' = i$ when $R = 0$, there must be a particular intermediate value of R which will *absorb* the wave pulse and prevent reflection.

When R is made equal to $\sqrt{\frac{L}{C}}$, reflection cannot occur because both voltage and current waves will enter the resistance unchanged.

Using symbols, we can say that for any value of R, if i is the outgoing current and i' is the reflected current,[1] the balance which enters the resistance is $i_r = (i + i')$. The volts at the end of the line must therefore be $(e + e') = R\,(i + i')$ and since there is to be no reflection, both e' and i' must be zero; whence

$$R = \frac{e}{i} = \sqrt{\frac{L}{C}}$$

Although the cases considered have been chosen merely to explain general principles, and do not exactly represent conditions likely to arise in practice, they should nevertheless be helpful in giving some indication of when troublesome surges or oscillations are likely to occur. Instead of a detached "wave pulse" travelling along a line, we must usually think of a "wave train" of harmonic functions of gradually decreasing amplitude travelling along the line in both directions from the point where the disturbance occurs. Arriving at the ends of the line, or at points where branch circuits or transformers are connected, these travelling waves of energy may be totally or partially reflected. The reflected waves meeting the outgoing waves may lead to considerable magnification of the original trouble.

[1] Both i' and e' (the "reflected" current and voltage) may be positive or negative quantities depending upon their *direction* relatively to the direction of these components of the original energy wave pulse.

Standing Waves.—If the conductor resistance is so small as to be negligible, the oscillations of energy may be thought of as a wave train of harmonic functions of constant amplitude travelling to the end of the line which we shall suppose to be unloaded (*i.e.*, open). The waves are therefore reflected in the manner previously explained under Case (1), and the reflected waves, meeting the outgoing waves, produce *nodes*, or points of zero potential, and *antinodes* where the voltage is exactly double the maximum voltage of the original disturbance. At these points of double voltage the current must be zero—since the total energy remains constant—and the nodes of the current waves therefore occur at the antinodes of the voltage waves. The combination of the outgoing and reflected waves is thus seen to produce *standing waves* which remain stationary in position although varying in amplitude. The resistance of the conductors prevents this *simple mode of oscillation* being exactly realized on a practical transmission line.

87. Line Disturbances Caused by Switching Operations.—It is hardly necessary to add anything to what has already been said, in order to emphasize the possible danger of suddenly switching a source of electrical energy on or off a long transmission line. Unfortunately the calculations of the probable surges or oscillations are not easily made, and moreover accurate data concerning the characteristics of the various circuits and apparatus connected to the system are rarely available. It follows that the engineer cannot predetermine accurately what will happen under the different probable or possible conditions of operation; but a general understanding of the principles underlying the creation of energy surges in a system of electric conductors will enable him to avoid obvious mistakes in the design and operation of a particular transmission scheme.

There is frequently danger of abnormally high voltages due to surges at the points where there is a change in the constants of the circuit. Thus, if a transformer is connected across the ends of a long overhead transmission line, there will be a rise of pressure when a travelling wave arrives at this point because the *surge impedance* $\left(\sqrt{\frac{L}{C}}\right)$ of the transformer winding may be between 2000 and 4000 ohms, which is very much higher than that of the line itself (about 400 ohms as previously explained). For this

reason the end turns of the transformer primaries should be specially insulated to withstand much higher voltages between turns than the remainder of the winding.

In the case of a change from underground to overhead transmission, a surge originating in the cable will produce a rise in pressure at the junction with the overhead line, while the contrary will occur (*i.e.*, the voltage will be reduced) if the surge is originated in the overhead line and passes into the cable system of which the surge impedance will always be *smaller* than that of the overhead transmission.

With the good insulation provided on modern high voltage systems, it is doubtful if the interruption of the current by opening switches under load is likely to cause serious voltage disturbances, except in the case of air-break switches where a long arc may be formed and suddenly interrupted—as for instance by a draught of air—when the current is of considerable value. Oil-break switches almost invariably open the circuit at the instant when the current is passing through zero value.

88. Lightning.—The foregoing considerations do not take into account the effects of lightning, either by direct stroke or by induction, because in such cases a pressure from an outside source is impressed upon the circuit, and the potential of these atmospheric charges may be tens of times greater than any surge voltage due to a redistribution of the energy stored in the circuit itself.

Although our knowledge of lightning phenomena is still far from complete, it is generally agreed that a single stroke of lightning is of short duration, frequently not exceeding the one-thousandth part of a second. If an overhead conductor receives a direct stroke of lightning, the potential value of the lightning charge is generally so enormously in excess of the working pressure on the conductors that the lightning leaps over the insulators down the pole to ground. Any charge on the line, which is not sufficiently high in potential above ground to jump over the insulators, will travel along the line in both directions until it is grounded through a lightning arrestor or dissipated as I^2R losses in the conductors. The frequency of such travelling waves will depend upon the natural frequency of the line, and may be of the order of 1000 to 5000 cycles per second. If the resistance of an arrestor or the path through which a discharge occurs, were zero, the current passing would be a maxi-

mum. If C is the capacity in farads, and $\mathbf{L}$ the inductance, in henrys, of unit length of line, then $\sqrt{\frac{\mathbf{L}}{C}}$ is the surge impedance of the circuit; and the maximum possible value of the current will be $I_{\text{max.}} = E \div \sqrt{\frac{\mathbf{L}}{C}}$, where E is the impressed voltage, which may be considered as something less than the pressure which will cause a flash-over at the insulators.

The intense concentration of lightning disturbances is the cause of the difficulties experienced in protecting transmission lines by means of lightning arrestors; experience tends to show that an arrestor does not adequately protect apparatus at a greater distance than 500 ft., yet it is unusual to find arrestors on a transmission line at closer intervals than 2000 ft.

Disturbances are most likely to occur on exposed heights, and on open wet lowlands; special attention should therefore be paid to lightning protection at such places.

Although the quantity of electricty in a lightning flash may not be very great, the short duration of the flash accounts for currents which are probably of the order of 20,000 to 50,000 amperes.

Apart from the effects of atmospheric electricity, it is necessary to guard against the abnormal pressure rises that will occur on long transmission lines through any cause, such as switching operations, or an intermittent "ground." Over-voltages up to 40 per cent. in excess of the normal line voltage can be produced by switching in a long line. High-frequency impulses or surges are set up, which, in the special case of an arcing ground, may give rise to a destructive series of surges, a state of things which will continue until the fault is removed. An arrestor which may be suitable for dealing with transitory lightning effects may be quite inadequate to dissipate the charges built up by such continual surges.

89. Protection of Overhead Systems against Direct Lightning Strokes and Sudden Accumulations of High Potential Static Charges.—Under this heading the ordinary lightning rod and grounded guard wire will be briefly dealt with. If no guard wire is used, lightning rods should be provided at intervals along the line. They may be fixed to every pole or tower, but, in any case, they should not be spaced farther apart than 300 to 400 ft. unless the spacing of the supporting poles or towers has to be greater than this, for economic reasons. It is especially important to

provided them on the poles or towers in exposed positions such as hill tops. They should project from 3 to 6 ft. or more above the topmost wire. A convenient form of lightning rod is a length of galvanized angle iron bolted to pole top or forming an extension to the structure of a steel tower. Long lines have been worked satisfactorily for extended periods without lightning rods or guard wires, but these are extra high pressure transmissions which, on account of the better insulation throughout, are always less liable to trouble from lightning than the lower voltage systems.

Although engineers are still divided in opinion as to the value of the protection afforded by overhead grounded guard wires, carried the whole length of the line above the conductors, it is now generally recognized that this method of protection is efficient. The objections to the guard wire are the additional cost and the possibility of the grounded wire breaking, and falling across the conductors below, thus causing an interruption to continuous working. Trouble due to this cause is, however, exceedingly rare.

It has been suggested that the guard wire or wires should be of the same material as the conductors, in order that the "life" of all the wires may be the same; but there are other considerations in favor of using a galvanized stranded steel cable for the guard wire. This may be the ordinary cable, $\frac{5}{16}$ to $\frac{7}{16}$ in. in diameter, as used for guying poles; but, where great strength is required, the Siemens-Martin steel cable, with or without hemp core, is preferable. Bessemer steel wire has not been found satisfactory for this purpose. In the case of the "flexible" steel tower type of line, a strong steel guard wire joining the tops of the towers, adds greatly to the strength and stability of the line, and may even, on long lines, save its cost, by allowing the use of lighter structures and fewer intermediate (dead-ending) towers.

In regard to the position of overhead guard wires relatively to the conductors, it is obvious that a number of grounded wires surrounding the conductors will afford better protection than a single wire above the conductors; and two guard wires are sometimes provided; but the additional cost is rarely justified. Perfect protection cannot be obtained even with two guard wires, and cases have been reported of lightning missing the grounded wire and striking a conductor situated immediately below.

The best position for a single guard wire placed above the con-

ductors is, according to Dr. Steinmetz[1] such that all the current carrying wires are included within an angle of 60 degrees below the guard wire. Additional wires can be installed in exposed positions, such as the summit, or very near the summit, of a range of hills, or by the shores of lakes or seas where the prevailing winds come over the water. In such positions, an additional guard wire on the side of the conductors may be useful. The guard wire should preferably be grounded at every pole, or at least every 500 ft.

90. Protection of Insulators from Power Arcs.—As a special means of protecting insulators from the flash-over caused by

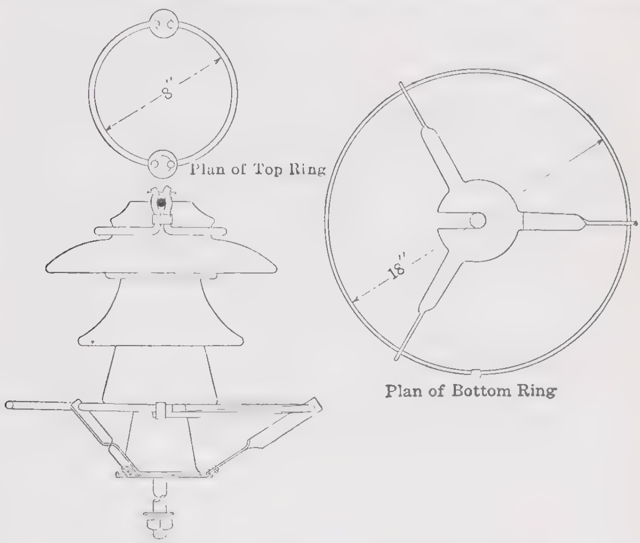


FIG. 65.—Arcing ring on pin type insulator.

lightning, or the power arc following a high potential discharge, the "arcing rings" first introduced by Mr. L. C. Nicholson, may be mentioned. These rings, which are grounded, are placed in such a position as to take the arc, and hold it at a sufficient distance from the porcelain of the insulator to prevent cracking or breakage by heat. The illustrations, Figs. 65 and 66, show the arrangement of the grounded arcing rings attached to standard types of insulator made by the Locke Insulator Manufacturing Company. It is not claimed that these rings will protect an

[1] Discussion of the Committee on "Lightning Protection," of the National Electric Light Association, May, 1908.

insulator against a direct lighting stroke; but their utility on high-pressure lines transmitting large amounts of power has been proved without doubt.

Although a pair of metal rings, one near the top and one near the bottom of the insulator will probably afford the best protection, the arrangement of arcing rods shown in Fig. 67 will prove almost as effective; the object being not only to provide a path for the high-voltage flash-over which shall keep the arc away from the porcelain, but also to prevent puncture of the insulator due to concentration of potential at the points of

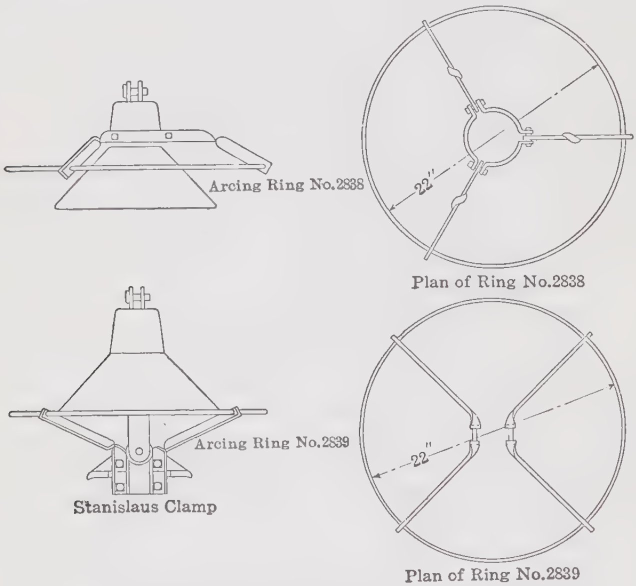


FIG. 66.—Arcing rings on suspension type insulator.

attachment to the metal fittings. The arcing horns as shown in Fig. 67 are used on the insulators of the 66,000 volt lines of the Peninsular Power Company, of Iron Mountain, Mich., described by Mr. Max. H. Collbohm in the *Electrical World* of April 18, 1914.

91. Methods of Grounding.—The ground wire from lightning rod, guard wire, or arrestor, on high-tension transmission circuits, should be as short and as straight as possible; it need not be of very low resistance;[1] but small reactance is of first importance.

[1] The question of resistances in series with spark gap arrestors will be taken up in Article 93.

The ground plate should have a large surface but the material is of little importance, except that it is not wise to bury aluminum wires in the ground, because of possible electrolytic action. Galvanized iron is a good material. If the ground contact is made with one or more iron pipes buried or driven into the ground, these pipes may be from 1 to 1½ in. in diameter, and a good connection should be made to the *top* of the pipe, as the inductive effect of an iron tube surrounding the ground wire might be considerable if a connection were made only at the bottom of the pipe. One or more pipes 8 to 10 ft. long, driven into the ground with 6-in. to 12-in. projecting above, will generally be found more effective than buried plates. A very low resistance ground is not essential on a high-tension system, and, generally speaking, the special forms of ground plate made of perforated copper, designed to hold, or to be in contact with, crushed charcoal, are unnecessary. If a plate is used, this should be not less than 12 in. square, but need not be larger than 18 in. square; it may be of galvanized iron 3/16 in. or ¼ in. thick, buried as deep as possible in the ground, and, in all cases, an effort should be made to secure permanently damp soil for ground plates or pipes.

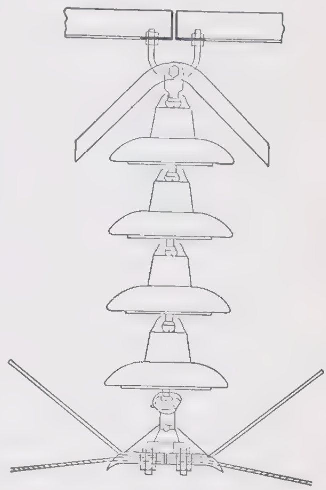


Fig. 67.—Arrangement of arcing rods above and below string of insulators.

92. Relieving Conductors of High Potential "Static." Water Jet Arresters.—By directing a stream of water from the nozzle of a grounded metal pipe on to the high-tension conductors, a high-resistance non-inductive path to ground is provided for the extra-high potential charges on the line; but there will be very little leakage of power current. It is claimed that arresters constructed on this principle have been found useful in practice; but the employment of jets of water has its objections. It is usual to put the jets in action only at times when electric storms are pending; and the reliance on the "human element" renders the apparatus less valuable than an equally effective

device which is always ready to act. Patents have been granted for various forms of water-jet arresters, but they are not extensively used at the present time. The chief function of the water jet is to prevent the building up of static pressures on the line caused by the contact of dust, snow, or rain drops, blown against or falling upon an insulated line of considerable length, or by variations in the potential of the atmosphere, at different parts of a line traversing hilly country.

Since other means, such as highly inductive resistances, or the grounding of the neutral point of transformers connected to the line, are available for preventing the accumulation of static charges on an overhead line, the advantages claimed for water jets are not obvious. Many engineers of high standing in Europe use water-jet dischargers, and recommend them for pressures up to 40,000 and even 50,000 volts. For higher line pressures, iron-cored choke coils are preferable for the purpose of draining the system of static charges. These should always be of the single-phase type connected between the line and ground through damping resistances, the purpose of which is to prevent energy oscillations. The connection to the line should be made at a point between the lightning arrester and the apparatus to be protected. It has been stated that water jets are effective in preventing damage by direct lightning strokes in their immediate neighborhood; but no very definite evidence to this effect is forthcoming. If the column of water is of so high a resistance that it causes no appreciable leakage of the power current, it is not easy to believe that it can prove equal in an emergency to shunting the hundreds or thousands of amperes which would otherwise pass to ground through the apparatus connected to the line.

93. Horn Gap.—Nearly all lightning arresters are designed on the principle of one or more spark gaps between the conductors and ground, the air space being so adjusted that the normal difference of potential between the line and ground is insufficient to jump the gap; but abnormally high pressures will break down the insulation of the gap, and so find a path to ground before the pressure is sufficiently high to damage the insulation of the line or the apparatus connected thereto.

The ordinary horn gap arrester of the type shown in Fig. 68, is so well known that it requires no detailed description. When the potential rises to such a value that it can jump the gap at the base of the curved wires, the power arc will follow the dis-

charge, but, owing partly to the upward tendency of the heated air, and mainly to the magnetic field produced by the current itself, the arc is driven upward toward the ends of the "horns" where, after being sufficiently drawn out in length, it is finally ruptured. The horn gap is not effective when set to discharge at pressures below 13,000 volts, because, with a small gap (less than 1 in.), the arc may not rise and break properly. The usual settings for horn gaps are as follows: the voltages in the table are the r.m.s. values on the sine-wave assumption, and they must not

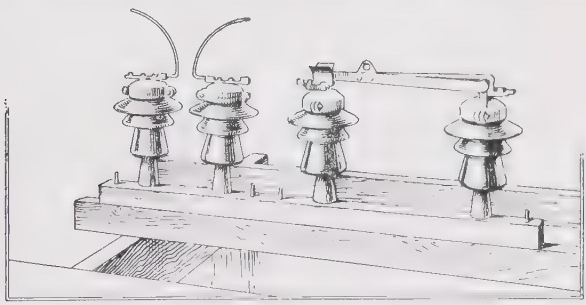


FIG. 68.—Horn arrester with disconnecting switch.

be used except as a rough indication of the probable gap between horns:

Gap, in.	Working voltage across gap	Spark-over voltage, approximate
1	21,000	30,000
1½	29,000	41,000
2	34,000	48,000
3	44,000	63,000
4	52,000	74,000
6	65,000	93,000
8	77,000	110,000
10	88,000	126,000
12	98,000	140,000

A non-inductive resistance should be connected in the ground wire from the horn arrester. An ordinary wooden barrel filled with water, with a connecting plate at the bottom, and the upper terminal carried about 6 in. below the surface of the water, makes an effective resistance. If no resistance is provided in the ground connection, the momentary discharge of the power

current may be excessive, dangerous surges may be set up in the line, and there is the possibility of synchronous machines being thrown out of step.

As an example of how a suitable value for the resistance in the ground connecton may be estimated, consider a 60,000 volt three-phase line of surge impedance $\sqrt{\frac{L}{C}} = 300$ ohms. The maximum value of the working pressure between wire and ground under normal conditions, on the sine wave assumption, is

$$E_n(\text{max.}) = \frac{60000}{\sqrt{3}} \times \sqrt{2} \text{ volts.}$$

Let the safety factor of the line insulators be 3, that is to say, the insulators will flash over with $\frac{60000}{\sqrt{3}} \times 3$ volts (r.m.s.) to ground. Any voltage exceeding this value will cause a "spill-over" and it therefore follows that the maximum possible transient voltage which can be added to the crest of the normal e.m.f. wave and travel along the line, is

$$e = 2\left(\frac{60000}{\sqrt{3}} \times \sqrt{2}\right) = 98000 \text{ volts.}$$

The maximum transient current that the arrester will have to take care of is therefore

$$i = \frac{e}{\sqrt{\frac{L}{C}}} = \frac{98000}{300} = 327 \text{ amperes.}$$

If the arrester is to prevent a spill-over and afford reasonable protection to the insulation of the station apparatus, it must be set for a lower voltage than $3E_n$, the usual setting being about $1.5E_n$. If R_a is the *non-inductive* resistance in the grounded connection of the arrester, the potential drop across it, if the maximum possible value of the surge current passes through it to ground, will be iR_a, whence

$$1.5\, E_n = iR_a + E_n$$

and $$R_a = \frac{0.5E_n}{i} = \frac{17000}{327} = 52 \text{ ohms.}$$

In this calculation, the resistance of the air-gap is supposed to be zero at the time of the discharge; but in any case in does not follow that 52 ohms will be the best possible value for the

resistance: the problem is not quite so simple as these considerations would appear to imply. Experience is the best guide in arriving at the most effective value of the resistance in the ground connection. It is evident that an appreciable amount of resistance is desirable, but that too high a resistance will allow insufficient current to pass to ground, and moreover may stand in the way of the rapid extinction of the arc across the horns. When deciding upon a suitable value, the effect of the power current following the discharge should be considered. Thus, in the above example the power current on one phase (conductor to ground) would be $\frac{60000}{\sqrt{3} \times 52} = 666$ amperes, representing an output of 23,000 kw. on one phase, if the arrester is near the generating station. For this reason alone, it might be desirable to use a grounding resistance somewhat larger than the calculated 52 ohms.

One serious disadvantage to the ordinary horn-gap arrester is the liability of an intermittent arc setting up surges and high potential disturbances which may lead to more trouble than the original cause of the spark-over. Fairly satisfactory results have been obtained by providing a number of horn gaps on a high-tension transmission and "grading" these, by adjusting some of them to discharge with a very small rise of pressure through a high resistance; while other sets would have larger gaps and lower resistances in series; the very largest gap being such as to break down only rarely, under exceptionally high pressures, and this should have a very low resistance in series but may with advantage be protected by a fuse.

Horn arresters, *if intelligently placed and properly connected and adjusted,* are capable of affording good protection on circuits (both A.C. and D.C.) up to 20,000 volts; but the multi-gap and "low equivalent" arresters, as originally suggested by Mr. P. H. Thomas, have some special features which have led to their frequent adoption on alternating current circuits for pressures up to about 35,000 volts.

94. Multiple-gap Low Equivalent Arrester.—In this type of arrester there are many air gaps in series between the line and ground. No single gap is greater than $\frac{1}{32}$ in. or $\frac{1}{16}$ in. and it occurs between the adjacent surfaces of small cylinders made of a so-called "non-arcing" metal as used in the earlier types of Wurtz arrester. The number of gaps in series depends upon

the working voltage of the line, and the last of the metal cylinders is connected to ground (or to one of the return conductors, as the case may be) though a non-inductive resistance, which may, with advantage, be shunted by a fuse in series with a spark gap. Sometimes a portion of this resistance is bridged by a number of

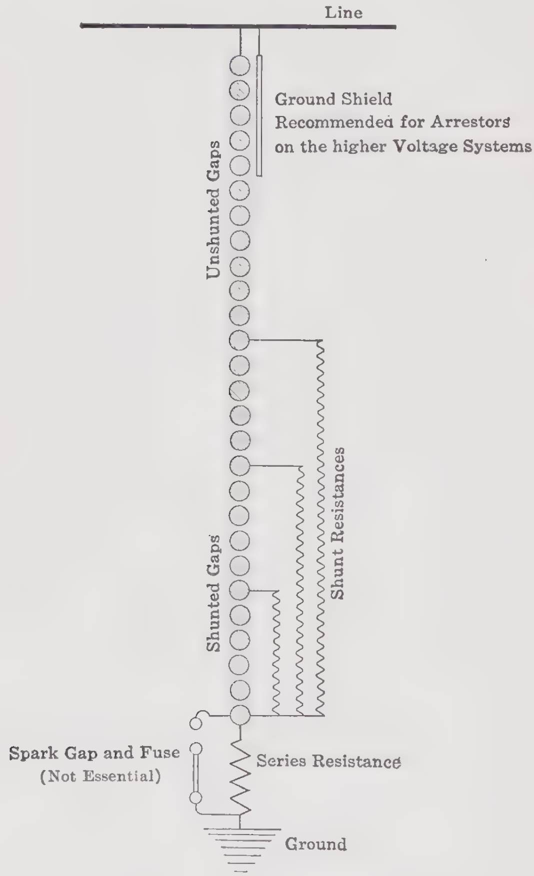


FIG. 69.—Diagram of multiple-gap low equivalent arrester.

spark gaps, all as shown in the diagram Fig. 69. These shunted gaps act as a sort of by-pass for heavy discharges, the amount of the series resistance, through which all discharges have to pass, being comparatively small. The theory of the low equiva-

lent arrester has been ably discussed by other writers.[1] Its action, is, briefly, as follows. There is a certain electrostatic capacity between consecutive cylinders, and between each one of these cylinders and ground; and the potential gradient is considerably greater at the high voltage end of the arrester, with the result that, when the total voltage across the arrester reaches a certain critical value, the breakdown occurs between the first and second cylinders. The second cylinder is then connected to the first by an arc, so that its potential rises accordingly, until a breakdown occurs between the second and third cylinders; and so on. The line current then follows the discharge, and in so doing, tends to produce a uniform fall of potential along the line of cylinders, with the result that the maximum potential difference between cylinders is considerably less than that required for the initial breakdown, and the power arc is ruptured as the current passes through zero value. When a breakdown occurs between two cylinders, the potential of the lower cylinder of the series will depend upon the quantity of electricity which passes to it from the more highly charged cylinder. The initial current is really a capacity current, and it will therefore be greater at the higher frequencies; but, by a scientific proportioning of the shunted resistance, a very satisfactory arrester of this type can be made, for use on circuits up to about 13,000 volts; it is less effective on higher voltages, but is actually used on 20,000-volt, and even 35,000-volt transmission lines.

One reason why the multiple-gap low equivalent arrester is not satisfactory on very high voltage systems is that the necessary increase in the number of gaps to prevent arcing over by the line voltage alone, is out of all proportion to the increase in voltage. There is also much uncertainty as to the number of gaps required, which will depend on the position of the arrester relatively to surrounding grounded objects. With the ground potential brought very near to the arrester, the potential gradient at the end near the line frequently becomes high enough to ionize the air between the cylinders, thus carrying the line potential to lower cylinders, until the remaining gaps are so few that a discharge occurs. In order to obtain the more equal division of

[1] See Dr. Steinmetz on the theory of this type of arrester in vol. xxv (1906), of the *Proceedings* of the A. I. E. E. Also the excellent book "Les surtensions dans les distributions d'energie electrique," by I. Van Dam (Van Mantgem & De Does, Amsterdam) whose theoretical treatment of this and kindred problems always has the practical end in view.

the total potential difference, and so allow of a reduction in the total number of gaps, such as would be obtained by removing the whole arrester to a considerable distance from grounded objects, a metal guard plate or shield is sometimes placed near the gaps at the high potential end of the arrester, and connected to the line wire as indicated in Fig. 69. The theory of the potential distribution over the string of insulated metal cylinders in the multiple-gap arrester need not be discussed here because the problem has already been considered in connection with the suspension type of insulator (refer Article 74, Chapter V). It is evident that, if the electrostatic capacity between the consecutive elements of the arrester can be made large relatively to the capacity to ground, a more uniform drop of potential over the series of elements will result, thus rendering this type of arrester more suitable for the higher pressures. The introduction of condensers to increase the capacity from element to element is the basis of the Moligniani system which has been patented and is in use in Italy. It seems probable, however, that the high cost of the condensers will stand in the way of this method proving superior to other possible alternatives.

95. Spark-gap Arresters with Circuit Breakers or Re-setting Fuses.—If the resistance in series with a gap arrester is very small a good path is provided to ground for taking a very heavy discharge; but there will be a large flow of power current in the arc following the discharge. This current may be interrupted by connecting some self-acting device such as a fuse or automatic circuit breaker in the ground connection; and arresters, whether of the horn type or with any other kind of spark gap, are sometimes provided with fuses so arranged that when one fuse blows, the dropping of a lever or an equivalent device, automatically inserts another fuse, so that the system is not left unprotected. Even without automatic replacement, if a number of gaps with fuses are connected in parallel, it will generally be found that one discharge will not blow all the fuses, and that during the passage of a single storm, the line will be adequately protected.

In the Garton-Daniels arrester, for use on alternating-current circuits up to 20,000 volts, the principle of the multiple gap is combined with a simple type of automatic circuit breaker connected as a shunt to some of the spark gaps, in order that the discharge path for the lightning shall remain unaltered even during the operation of the arrester. The arrester is built up

of several unit parts connected in series; each unit being rated for 3300 volts. The illustration, Fig. 70, shows a complete single-phase arrester for 10,000 volts. On a 20,000 volt circuit, there would be eight units in series, the total air gap distance being 1⅛ in., with a series resistance averaging 3800 ohms. The diagram, Fig. 71, refers to a single unit of the Garton-Daniels arrester. The discharge follows the straight path through the two sets of air gaps and the resistance rod, as indicated by the round dots. The power current following the discharge will, after passing through the two upper gaps and the resistance rod, be shunted by the low resistance winding of the circuit breaker;

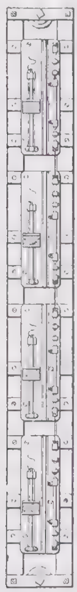


Fig. 70.—Garton-Daniels arrester for 10,000-volt circuit.

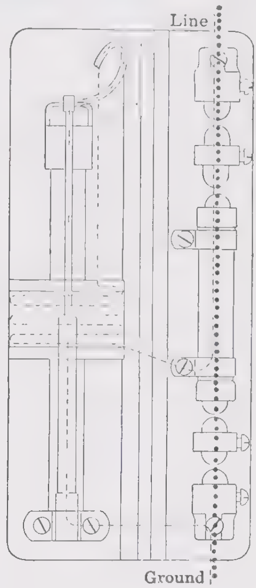


Fig. 71.—Diagram of Garton-Daniels arrester.

and if this following current is too heavy to be ruptured by the combined action of these two gaps and the resistance rod, the iron armature of the circuit breaker will be lifted by the action of the solenoid, thus throwing the two lower spark gaps in series, and extinguishing the arc.

96. Aluminum Cell Arrester.—When two aluminum electrodes are immersed in a suitable electrolyte, an insulating film of hydroxide of aluminum is formed on the surface of the metal; this effectually prevents the passage of any appreciable amount of

current until a certain critical voltage is reached, when the film breaks down and the current is limited only by the resistance of the electrolyte. On lowering the voltage, the film is reformed and the flow of current again limited to a very small amount.

With alternating currents, the critical potential difference per pair of plates is about 350 volts, and the practical construction of lightning arresters on this principle consists in stacking a large number of cone-shaped aluminum plates one within the other, with suitable separating washers of insulating material between them. In this manner a column is formed of a large number of cells in series, capable of withstanding high voltages. The whole is enclosed in a case containing oil, which improves the insulation and prevents the evaporation of the electrolyte which fills the spaces between adjacent trays within a short distance of the edge.

If cells built up in this manner are connected directly between line and ground, there will be an appreciable current passing through them, which is partly a leakage current, but chiefly a capacity current. It is therefore customary to insert a spark gap, usually of the horn type, in series with the aluminum cell arrester; the gap being set to break down with a pressure slightly in excess of the normal working voltage.

Although the film of hydroxide is formed on the plates at the factory before the arresters are installed, it is necessary to maintain it by periodic "charging" of the cells; this being done by closing, or nearly closing, the spark gap in series, so as to put the full line pressure across the arrester. It is generally recommended that this be done once every day.

In principle the aluminum cell arrester would appear to offer an ideal solution of the problem of lightning protection; because, once the critical voltage is exceeded, and the film broken down, a very large current—depending on the amount of separation and the area of the plates, and also the nature of the electrolyte—is allowed to pass to ground; and the device is capable of dealing with continual surges, such as will occur with an intermittent ground, for a period of about half an hour without excessive heating. In practice it has proved fairly satisfactory, especially on the higher voltages; but, apart from its large initial cost, it has frequently been found to be somewhat costly in upkeep, as the aluminum cells are liable to become damaged through frequent and heavy discharges, and have to be periodically reformed or replaced. Then again, the necessity of charging with the line

current is an objection where there is not an operator constantly in attendance; and lastly, it must not be overlooked that the device suffers from the disadvantage common to all spark-gap devices, namely that high-frequency surges are liable to be set up in the system when the spark gap discharges. In this particular case the trouble is liable to occur, not only when the horn gap breaks down while fulfilling its function of discharging an excess of pressure through the cells, but also when the spark is deliberately formed for the purpose of charging the cells. On very high pressure systems it is possible that the surges set up by spark gaps in series with the resistance of the cells are not likely to cause trouble; but this suggests the possibility of simpler and less costly devices such as the graded horn gaps previously referred to, being equally effective. On the other hand, when used on low voltage systems operating at about 11,000 volts (especially if the generators are directly connected to the transmission line, without the intervention of step-up transformers) the operation of charging the aluminum cell arresters in the generating station has been known to break down the insulation of the generators.

The makers of this type of arrester appear to have recognized the danger of damage to apparatus arising from the operation of charging the cells, and they now recommend that the charging current be passed through a resistance in series with the arresters. Suitable resistances are provided in connection with the horn gaps so that the line pressure is not put directly across the aluminum arresters at the time of charging.

The latest development in this direction consists of a somewhat complex arrangement combining sphere gaps, horn gaps, and resistances; the object being to obtain a quick and free discharge of disturbances of steep wave front across sphere gaps *without resistance in series.* If the arc persists across this gap it rises to another sphere gap which has resistance in series to dampen the oscillations of energy. Other lightning discharges are dealt with by a horn gap of the ordinary type mounted immediately above the second sphere gap.

97. Condensers.—Although much has been done, and more good work will probably be done in the future by the intelligent "grading" of a number of spark gaps, to afford a path to ground and yet avoid the setting up of dangerous high-frequency surges, the objections to all spark gaps are (1) the necessity of an appre-

ciable increase in pressure above normal line pressure to break down the resistance of the gap, and (2) the danger of high-frequency oscillations being set up in the network of conductors.

A non-inductive low resistance direct connection to ground can obviously not be made on a high-tension alternating-current overhead transmission line; but a path to ground may be provided either through a highly inductive choke coil, or through a condenser, or both, without the necessity of providing a spark gap in series. The inductive resistance may easily be designed to pass only an inappreciable current of normal or higher frequency, and it will therefore be useless for affording relief in the case of high-frequency surges; but it is capable of relieving the line of slowly accumulated static charges. The condenser, however, acts as an almost perfect insulator so far as direct currents are concerned; but it is pervious to high-frequency currents, and a suitably designed condenser, or rather battery of condensers, connected between line and ground without the intervention of any spark gap, is certainly an ideal device for dealing with the very high-frequency oscillations that accompany lightning phenomena. This is the chief function of the Mosciki condensers which, although not largely used on this continent, have found favor in Europe—where they have been in use for many years in almost every country—and also in South Africa and China.

The travelling waves induced by a lightning discharge on a transmission line may have a frequency of the order of 1000 to 5000 cycles per second, and a set of condensers which will pass but a very small current at 25 to 60 cycles, will deal with much larger currents on these higher frequencies. As a matter of fact, an electric discharge between cloud and ground or between cloud and cloud may induce in the line travelling waves having frequencies considerably in excess of 100,000 cycles per second. This is proved by the fact that wireless telegraphic apparatus which responds only to frequencies ranging between about 100,000 and 1,000,000 cycles per second is interfered with by atmospheric electric storms. A 40-amp. fuse in series with a condenser has been blown during atmospheric discharges, although the condenser could not possibly pass more than a hundredth part of this current on frequencies of 3000 or 4000.

It is perhaps not generally understood that high-frequency travelling waves may break down the insulation of generators or transformers even when the voltage of the induced charges is

small as compared with the normal operating voltage. The trouble is that the wave is short, and the point of zero potential may be only a few hundred feet behind the point of maximum potential. If, therefore, a travelling wave of this nature enters a piece of electrical machinery such as a generator or transformer, the full difference of potential, which may amount to only a few thousand volts, may be applied across adjacent layers of the coil winding, thus causing a puncture and ultimate breakdown of the insulation, even if the apparatus as a whole is insulated to withstand pressures of 100,000 to 200,000 volts to ground. As a protection against trouble of this sort from high-frequency induced charges, the condenser appears to offer a good solution.

It must not be understood from these notes on the uses of condensers as lightning arresters that the discharge is diverted to ground through the condenser and so dissipated, much as energy would be dissipated in a resistance; because the condenser cannot absorb or dissipate any but the smallest percentage of the energy passing through it. The energy is necessarily re-delivered to the line from which it originally came, and is ultimately dissipated through the ohmic resistance of the conductors. The function of the condenser is, in fact, somewhat analogous to that of an air chamber on a water pipe in which the rate of flow is subject to sudden variations.

In view of the fact that the condenser merely *shunts* the high-frequency oscillations, and so prevents damage to the apparatus to be protected, but returns nearly all this energy to the line where it ultimately dies out owing to the conductor resistance, it would seem advisable to provide some small amount of resistance in series with the condensers, even at the risk of slightly higher surge pressures across the apparatus to be protected. The intelligent combination of condensers, reactance coils, and resistances, may be expected to afford good protection; but there is always a danger of resonance effects at certain critical wave frequencies.

Since a condenser of appreciable capacity is obtainable from a comparatively short length of insulated underground cable, it is certainly advantageous to lead the current from an overhead transmission into generating- and sub-stations through a length of (say) 200 to 300 feet of underground cable. Of course this may not be possible in the case of very high pressures because

of the high cost of the cable, but as a matter of fact any form of condenser becomes very costly when designed for use on high voltage systems. The effect of an underground cable on a surge travelling along an overhead line has already been referred to in Article 87. The greater capacity (per unit length) of the cable is such that its surge impedance $\sqrt{\frac{L}{C}}$ would be of the order of 40 ohms, compared with about 400 ohms for the overhead line.

98. Spacing of Lightning Arresters.—A reasonable distance must be allowed between the live metal parts of arresters placed side by side; the following limiting distances are suggested for guidance in installing lightning arresters, such as those of the horn gap type where large arcs may be formed and blown or drawn from one element to another. These distances may be reduced if suitable partitions are provided between the arresters.

Potential difference, volts	Separation, in.
11,000	24
22,000	32
33,000	42
44,000	50
66,000	66
88,000	84
110,000	108

99. Choke Coils.—When a lightning arrester is connected between line and ground in or near generating or substations for the purpose of providing a path to ground for high-frequency surges, an inductive reactance is placed in series with the apparatus to be protected. This reactance must not be so great as to cause a serious drop in pressure when carrying the normal line current, neither must it be so small as to allow the induced charges travelling along the line to pass through it rather than jump the air gap of the lightning arrester. This reactance usually takes the form of an air-insulated coil of copper wire or rod, supported at each end on a suitable insulator. The "hour glass" form of coil, in which the diameter of the turns increases from the center toward both ends, is mechanically stiffer than a cylindrical coil, and any arc that might be started between adjacent turns has a greater tendency to clear itself. The air space between turns is usually from $\frac{1}{4}$ in. to $\frac{3}{8}$ in. Too little attention has been given

in the past to the proper design and proportioning of choke coils for use in conjunction with lightning arresters. It has sometimes been argued that, except for the drop of pressure under working conditions, and the higher cost, there is no objection to installing very large choke coils having a high inductance. This argument is, however, incorrect, except for the special case in which some protection against surges is provided on the machine side of the reactance in addition to the lightning arresters on the line side. A high reactance may be quite satisfactory if it is merely intended to hold back high-frequency currents travelling along the line; but surges may originate near the generators or transformers due to switching operations or other causes, and a very high reactance between the electrical plant and the line will tend to aggravate the effect of comparatively low-frequency surges which might otherwise be dissipated in the line, or even through the lightning arrester. In fact, choke coils should be designed with due regard to the apparatus they are intended to protect, with a view to avoiding the building up of high voltages at the terminals of the generating plant in the event of surges being set up in or near the plant itself. When the lightning arrester discharges, it does not follow that high-frequency waves do not find their way through the choke coil to the machines; but the inductance of the choke coil will lower the frequency of such waves; or, in other words, will reduce the steepness of the wave front to such an extent that the insulation of the machines will not be injured. The first few turns of a transformer or generator winding will act as a choke coil and usually prevent damage to the turns farther removed from the terminals; but they are liable themselves to suffer injury, as the charge will leap across the insulation and so get to ground. If it is assumed that the reactance of the first six turns of a transformer winding is sufficient to afford protection to the seventh and subsequent turns of the winding, then a choke coil having a reactance equal to that of the six turns of transformer winding will afford the necessary protection to the transformer. A higher reactance in series is unnecessary and may be dangerous.

The tendency among engineers is apparently toward the use of choke coils of too great reactance. As an example of what appears to be generally sufficient to afford reasonable protection to modern machinery, about 25 turns of copper rod wound into a coil 10 in. in diameter may be used on voltages from 10,000 to

25,000, while for pressures of the order of 100,000 volts, two such coils would be connected in series. The diameter of the copper rod would depend upon the current to be carried; but it is best to have it large enough in all cases to be self-supporting, although coils wound on insulating frames, with separating pieces between turns, are not necessarily objectionable.

It is possible that copper is generally used for choke coils because the calculation of the reactance at various frequencies is more easily made than in the case of a "magnetic" material, such as iron; but the cost can be reduced by using iron bar or strip in place of copper, and a peculiarity of the iron choke coil is its property of passing currents of normal frequency with comparatively small loss of pressure, while the choking effect with high-frequency currents is very much greater.

100. Arcing Ground Suppressor.—If any one conductor of a transmission system is connected to ground through an arc such as might occur over an insulator in the event of a rise of pressure due to any cause, there is the possibility of the arc continuing during an appreciable length of time, sufficient to do serious damage to the insulator, even if it should not totally destroy it. Apart from this danger, every intermittent arc is liable to set up dangerous high-frequency surges in the line, especially at the moment when it is finally interrupted. To protect a line against troubles due to this cause, a device known as the arcing ground suppressor has been introduced. This is an automatic device for momentarily short-circuiting the arc through a switch. By providing a metallic connection between the conductor and ground, the arc is suppressed, and it will usually not re-form when the switch is again opened, because the air in the path of the arc has had time to cool, and the line pressure, which was sufficiently high to maintain the arc once started, is not able to break down the insulation of the new layers of cooler air. The arcing ground suppressor was fully described in the *Proceedings* A. I. E. E. of March, 1911,[1] but the diagram, Fig. 72 will explain the principle of its action. Automatic switches are provided which will connect any one conductor to ground during the very short time necessary to allow the arc to clear itself. The principal feature of the device is the selective relay which

[1] "Protection of Electrical Transmission Lines," by E. E. F. Creighton, *Trans.* A. I. E. E., vol. xxx p. 257. Refer also to article by R. A. Marvin in the *General Electric Review*, March, 1913.

will energize the solenoid operating the switch on the faulty line. On high-pressure systems, this relay may be of the electrostatic type, generally on the principle of the electrostatic ground indicator. On comparatively low-pressure transmissions, the forces would be too low to operate such a device satisfactorily, and recourse is then had to an electromagnetic relay worked through transformers. There are no difficulties or new principles involved in the design of such a relay. When the relay operates, the switch between line and ground is momentarily closed. On re-opening, a suitable resistance is inserted before the final break, to prevent the creation of oscillating currents in the line.

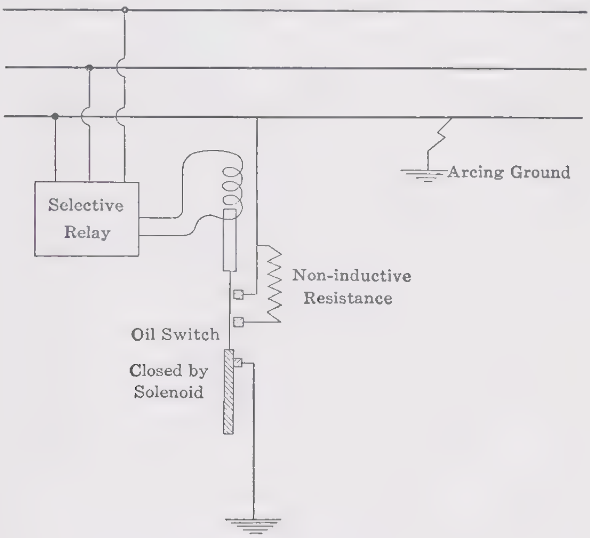


FIG. 72.—Diagram of arcing ground suppressor.

101. General Remarks on Lightning Protection.—The best means to adopt for the protection of any particular line or portion of a line against lightning disturbances is still largely a matter of conjecture, but by the exercise of sound judgment, an experienced engineer should be able to provide reasonable protection against discontinuity of service during atmospheric disturbances. There are many devices to choose from, each of which has a particular field of usefulness. It is probable that, in a few years' time, the additional information on this subject which is continually being accumulated, will lead to uniformity in the protective arrangements adopted under the various conditions arising in practice.

In the meanwhile, however, a careful record of all accidents due to lightning or abnormal pressure rises should be kept in connection with each power system with overhead transmission, as this will generally lead, after careful investigation, to certain amplifications or modifications of the existing protective arrangements such as to prevent the repetition of similar accidents. In this manner, very fair protection can be afforded at the present day to almost any overhead transmission system; but it is doubtful if it will ever be possible to protect apparatus against a direct lightning stroke. Damage to machinery, due to this cause, is, however, very rare.

In regard to the protection of the line itself, it is obvious that protective devices, however complete or perfect they may be, provided at the two ends of a long transmission, afford no protection to the insulators along the line. The frequently grounded guard wire would appear to be a good protection to a line; but here again the engineer must use his judgment, because certain portions of a line may require far more protection than other portions, and even if the cost of guard-wire protection be considered excessive for the entire length of a long-distance transmission, it may yet be a decided advantage to provide guard-wire protection near the generating and transforming stations and on those parts of the line most likely to be affected by atmospheric disturbances.

In some cases, it may be wise to improve the insulation and to raise the voltage at which a "spill over" will occur; while under other circumstances it might be better to provide an easy path for a discharge over insulators, by means of suitably disposed arcing rings or equivalent arrangement. Mr. P. H. Thomas once explained the matter of line insulation by making use of a very simple analogy. Where a discharge strikes the line, a wave starts and the potential of this wave will be such as can be allowed by the line itself; the energy of the discharge is limited by the static capacity of the line and the voltage at which a "spill over" will occur at the insulators. The energy of the travelling waves "grows less and less as they proceed. This action may be likened to the formation of a wave in a long, narrow trough with high sides containing water and normally less than half full, by sudden flooding of the trough by a large quantity of water at some particular point; the excess water spills over and escapes from the trough at the point of the flooding, but there is still a wave started in each direction as high as the sides of the trough

will permit; this passes along until the end is reached or the energy of the wave is gradually dissipated. It makes no difference how much water is thrown into the trough, there can be a wave only as high as the sides will permit."

One point that is sometimes overlooked is the effect of the line *current* on pressure disturbances. The disturbances that are set up by switching operations or by power arcs following a lightning discharge, will be far more serious when the current is large than when it is small. This is one reason why extra high-tension transmissions suffer less from lightning disturbances than moderate voltage systems on which the current is often larger. It is hardly an exaggeration to say that the handling of heavy currents on long distance transmissions presents more engineering difficulties than insulation problems on the high-voltage schemes.

Very high-voltage transmission lines may, indeed, operate satisfactorily without lightning protection, especially when working at pressures near the critical voltage of the corona formation, and some relief to high-pressure energy is afforded by the corona itself. Low-pressure lines, working at about 10,000 volts are usually far less exposed than the high-pressure lines, and the low-pressure lightning arresters are rather more effective than those for the higher pressures. Such lines do not give so much trouble as those working at voltages between 30,000 and 80,000.

It will be gathered from the foregoing remarks that not only the pressure of transmission, but the amount of power transmitted, is an important factor in the problem of lightning protection.

Without attempting a detailed analysis of the troubles due to switching operations, it may be stated that, as a general rule, it will be best to energize a dead line of considerable length by first connecting to the line the step-down transformers at the distant or receiving end, and then switching the step-up transformers at the generating end on to the low-tension bus-bars.

It is possible that the near future may see some developments in the matter of facilitating the dissipation of high-frequency energy in the line itself, with the object of rapidly limiting the amplitude of the travelling waves and the distance from the center of disturbance at which their effects can be of practical account. It is obvious that what is required is a line that will transmit, without undue loss, the power currents at normal frequency, and yet afford means for the rapid dissipation of high-frequency energy. Apart from the property peculiar to the

corona, which leads to the dissipation of energy on over-voltages, there is a property common to all metallic circuits which leads to the more rapid dissipation of high-frequency than of low-frequency energy. The so-called "skin effect" which apparently increases the resistance of a conductor carrying alternating or fluctuating currents, owing to the forcing of the current toward the outside portions of the wire at the higher frequencies, is clearly of value in limiting the distance over which high-frequency disturbances are propagated. By covering the conductor with a thin layer of high-resistance metal, astonishing results can be obtained. Experiments made with wires having a coating of nickel only 0.07 mm. thick, showed that the resistance offered to currents of 300,000 cycles per second was four times the resistance offered by the same wires without the coating of high resistance metal. This was referred to by Mr. Gino Campos at the Turin International Electrical Congress of 1911.

In conclusion, it would seem that much may be accomplished by a careful study of local conditions, and by the intelligent selection of such protective apparatus as may be available. On the other hand, it is not improbable that the coming years will see fewer rather than more protective devices than are now used. Careful design of the line as a whole, with the provision of suitable choke coils at the ends, or the special insulations of the end turns of the transformer windings to withstand very much higher pressures between turns than the remainder of the high-tension winding, will frequently be all that is necessary. Even if the installation of costly apparatus may slightly decrease the risk of damage by lightning, the economic aspects of the problem should be carefully considered before deciding to provide the additional protection. On low and medium voltage systems, the extra expenditure on a few hundred feet of underground cable between the terminals of the overhead line and the transformers or h.t. bus-bars, will generally be justified; while an improved system of grounded guard wires near the terminals of extra high-pressure lines will tend to keep the original disturbance at a safe distance from the station apparatus.

For comparatively low voltages the use of condensers appears to be justified; but they are always costly, and it is questionable whether the money invested in them might not with advantage be spent on improved insulation, especially in the immediate vicinity of the terminal apparatus.

CHAPTER VII

TRANSMISSION OF ENERGY BY UNDERGROUND CABLES

102. Introductory.—The principal use of underground electric cables is in connection with distributing systems in cities, at comparatively low pressures; but they are also used, to a limited extent, for the transmission of energy at fairly high voltages. In this chapter, underground cables will be considered chiefly in relation to straight long-distance transmission of energy, and what follows will therefore treat mainly of power cables for the transmission of energy at high voltages. No attempt will be made to deal with the historical aspect of the subject, or with the practical considerations connected with the handling, laying, and jointing of underground cables; but the construction engineer and the student desiring more information than can be contained in a single chapter, are referred to the many other writings on this subject, including the excellent book on "Underground Transmission" by Mr. E. B. Meyer.[1]

Notwithstanding the higher cost of underground transmission, it replaces overhead transmission—for comparatively short distances—in many cases where the latter system is not suitable. High-tension underground cables are used in populous districts where overhead construction is not permissible or advisable. The underground cable is not subject to damage by wind, ice, or thunderstorms, and the danger to life is obviously reduced by placing the high-voltage conductors underground. The unsightly appearance of poles and overhead conductors in the neighborhood of cities is another reason for putting wires under ground, notwithstanding the increase of cost. This consideration has more weight in Europe than in America and partly accounts for the fact that Europe is, and always has been, somewhat in advance of America in the design and manufacture of underground electric cables. The shorter distance of transmission,

[1] "Underground Transmission and Distribution," by E. B. Meyer. McGraw-Hill Book Co. (1916).

resulting in lower economical pressures on the Old World systems, is another reason why underground power cables are more extensively used in Europe than on the American continent. The writer has therefore no hesitation in using data and other information referring mainly to modern British practice, especially since he has been able to secure the collaboration of Mr. C. J. Beaver, chief engineer to Messrs. W. T. Glover and Co. of Manchester, England. Mr. Beaver has not only furnished much useful advice and material for this chapter, but he has also kindly consented to read and correct the manuscript.

Even when it is inadvisable to transmit by underground cable the whole distance of transmission, sections of the line passing through populous districts may be put underground, while overhead conductors are used in the open country. Also, as mentioned in the preceding chapter, a short length of underground cable is desirable as a protection against high-pressure surges, at each end of a long overhead transmission line, provided the voltage is not so high as to render the cost prohibitive.

Another use for insulated cables is as feeders on electric railway systems. In the trunk-line electrification of the New York Central Railroad, there are no less than 1,600,000 duct-feet of conduit for insulated feeder cables, some of which are of tile, the balance consisting of iron pipe.

The Thury system of transmission by continuous currents, which is explained in the succeeding chapter, lends itself to the extensive use of underground cables. Smaller and cheaper cables may be used for the same voltage and energy transmitted on a D.C. than on an A.C. transmission. Mr. J. S. Highfield has shown[1] that if the overall diameter of a 100,000 volt single-core lead-covered cable for alternating currents is 3.27 in., the equivalent cable for 100,000 volts continuous current would have an outside diameter of only 1.75 in. It is true that the comparison is between the r.m.s. values and not the maximum values of the voltage; but it serves to show that a considerable reduction of size—and therefore of cost—is effected when cables are used on D.C. instead of A.C. systems.

103. Submarine Power Cables.—When transmitting electric energy across water which cannot be spanned by overhead conductors, the insulated cable becomes a necessity. Two

[1] Discussion of Mr. Beaver's paper on "Cables," *Jour.* Inst. E. E., vol. 53, Dec. 15, 1914 and Mar. 1, 1915.

examples of submarine power transmission occur in San Francisco, where a three-phase 12,000-volt cable 18,800 feet long has been laid across the bay, and two 11,000-volt cables have been laid across the Golden Gate. The carrying out of the latter project is described in the *Electrical World* of March 4, 1916 (Vol. 67, No. 10, p. 532). The greatest depth of water at this point is 210 feet. The cables are 3-core, each 13,500 feet long, the cores being insulated with rubber (inside) and varnished cambric (outside). Impregnated jute is used as a filler. The lead sheath is $\frac{5}{32}$ in. thick, and the armoring consists of 42 No. 4 B.W.G. galvanized steel wires. The deep water portion of the cable has cores of 250,000 circ. mil section, with an outside diameter of 4 in., and a weight of 19 lb. per foot. At the shore ends, the cores are of 350,000 circ. mil section, with an overall diameter of $4\frac{1}{2}$ in. and a weight per foot of 22 lb.

In Germany there is the Stralsund channel project transmitting three-phase power at 15,000 volts from the generating station at Stralsund to the island of Rügen, the total length of submarine cable being 5600 feet. Another example occurs in the straits between the mainland and the island of Fehmern, where two 3300-foot lengths of lead-covered, iron wire armored, paper-insulated 11,000-volt cables are laid under water.

The most important existing submarine power transmission is the international cable between Sweden and Denmark, which was put into operation in December, 1915. It is 3.4 miles long and connects a point near Palsjö in Sweden with Marienlyst in Denmark. The cable is three-core with impregnated paper insulation suitable for a working pressure of 35,000 volts (test pressure 87,500 volts). The cross-section of each core is 0.108 sq. in. and steel wire armoring is applied over the lead sheath. The greatest depth of the Oresund at the point traversed by the cable does not exceed 125 feet. The cable was laid in nine lengths of about 2000 feet.

104. Voltage Limitations of Underground Cables.—Transmission and distribution by underground cables with alternating currents at 20,000 and 22,000 volts is not at all uncommon; but there are not many instances of higher voltage cable systems at the present day. Various schemes are in progress or in contemplation in England for linking up separate systems or parts of systems at pressures from 30,000 to 60,000 volts. A few years ago an extensive system of 30,000 volt three-phase cables was

laid in Berlin, following the experience gained in England with 20,000 volts. There are a few short lengths of 40,000-volt three-core three-phase cables in France, and this is probably the highest working voltage applied to three-core cables at the present time. The difficulties—apart from high cost—in the way of increasing the pressure limit, include heavy capacity currents and large dielectric losses, the latter being "all day" losses like the iron losses of distributing transformers. The size, or diameter over the lead sheathing, must not be overlooked; a high-tension cable with an outside diameter much in excess of 4 to 4½ in. would not only be difficult to handle, but would be liable to injury while being laid in position unless precautions were taken to avoid sharp bends and mechanical injury during the process of laying.

It seems nevertheless probable that the conclusion of the war will see important developments in high-voltage cable transmission, notably in connection with electric railways in Switzerland and elsewhere on the continent of Europe, where power cables may be used for pressures up to 80,000 and even 100,000 volts, and of sufficient size to transmit from 10,000 to 20,000 k.v.a. per cable. In this connection it should perhaps be pointed out that the much advertised 60,000-volt underground transmission for supplying single-phase power to the Dessau-Bitterfeld railroad in Germany from the power station at Muldenstein, employs 30,000-volt single-core cables, the middle point of the 60,000-volt system being grounded through a resistance at the generating station. The use of single-core cables in this manner entails the use of costly methods of laying, because it precludes armoring (owing to the excessive losses that would occur), and necessitates insulation between the lead sheaths and ground. The feature of chief interest from the cable maker's point of view is the employment in these cables of aluminum in place of copper conductors, the object being to reduce the voltage gradient at the surface of the conductor. This point will be referred to again later.

105. Types and Construction of Power Cables.—For the transmission of three-phase alternating currents, three-core cables may be used for pressures up to about 60,000 volts. When the pressure exceeds this figure, a separate single-core cable should be used for each phase. These separate cables of an extra high-tension three-phase transmission should be symmetrically ar-

ranged with a small amount of insulation between the lead sheath and ground. Steel armored single-core cables are inadmissible for alternating currents because of the inductive effects.

The lead sheathing referred to is a necessity because impregnated paper is practically the only material at present available for the insulation of e.h.t. underground cables. It is somewhat hygroscopic in character, and the protection of the lead covering is therefore provided. Fig. 73 shows a section through a three-core 30,000-volt paper-insulated underground cable, while Fig. 74 shows one of three separate cables designed for a working pressure of 173,000 volts between phases.[1] This cable is provided

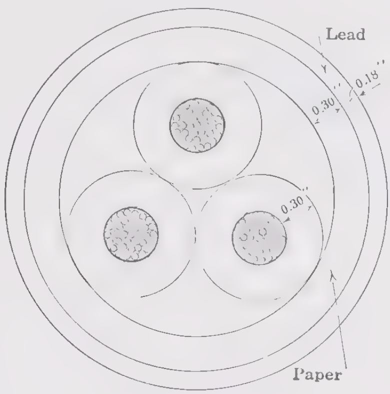


FIG. 73.—Three-core 30 kv. lead covered cable.

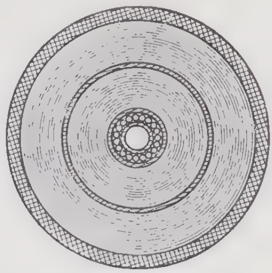
with a so-called "inter-sheath" of lead, the purpose of which is to improve the design by reducing the potential gradient at the surface of the conductor. The theory of this method will be discussed later.

The "clover leaf" or sector type of three-core cable is illustrated in Fig. 75. This design permits of a smaller overall diameter for a given voltage and so leads to a saving in cost; but it is rarely used for pressures exceeding 20,000 volts. The sector shape of cross-section is obtained by specially stranding wires of suitably varied diameters, or by rolling or hammering a circular strand to the desired shape. This construction was introduced in Europe many years before it was adopted in America; but British practice does not favor the use of sector-

[1] From Mr. C. J. Beaver's paper on "Cables" in the *Jour.* Inst. E. E., vol. 53, Dec. 15, 1914.

shaped cores in three-phase cables for working pressures exceeding 11,000 or 12,000 volts.

Concentric cables are used for single-phase transmissions and



		Radial thickness.	Radius.
Conductor.—	Inner lead tube—bore 0.27 in.	0.060 in.	0.195 in.
	Copper wires 19/15 S.W.G.	0.072 "	0.267 "
	Outer lead tube	0.050 "	0.317 "
Dielectric.—	First paper layer	0.545 "	0.862 "
	Lead inter-sheath	0.050 "	0.912 "
	Second paper layer	0.565 "	1.477 "
Outer sheath.—	Lead covering	0.160 "	1.637 "
	Complete diameter	3.27 in.	

Fig. 74.—Section through single-phase 173 kv. paper-insulated cable, with lead "inter-sheath."

are therefore practically confined to railway work. Fig. 76 is a section through such a cable, the return conductor being in the form of a layer of segmental strips laid over the insulation surrounding the inner core.

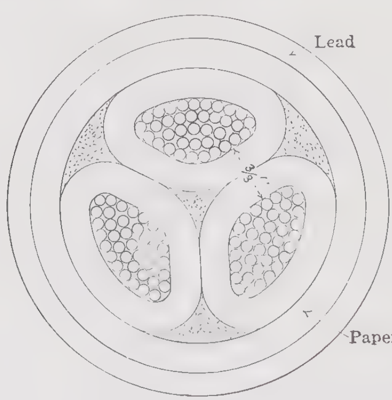


Fig. 75.—Three-core cable with shaped cores.

rounding the inner core. When this outer conductor is grounded —as it usually is—the arrangement is very satisfactory and efficient. Concentric cables are simple to construct, and even

when used with alternating currents, they can be armored and therefore buried directly in the ground, thus facilitating the dissipation of heat which is easily carried from the outer conductor to the lead sheath and armor.

There are two methods of manufacturing paper-insulated cables. In both methods a roll of paper is cut into narrow ribbons which are lapped on the copper conductor in successive layers until the required thickness is attained. In one method the paper is put on dry and the whole cable is immersed in the insulating oil. In the other, the impregnation is effected by passing the paper, *before* it is cut into ribbons, through a bath of the insulating oil. This last method, which was evolved and

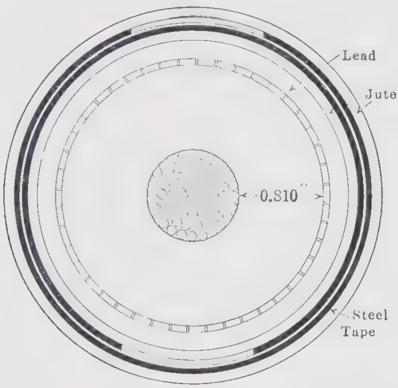


FIG. 76.—Section through concentric cable for single-phase transmission.

has been used for the last twenty years by Messrs. W. T. Glover and Co. of Manchester, England, appears to have the advantage of more certain and thorough impregnation of the insulation.

With paper insulation the lead sheath must be very carefully applied to ensure the absence of flaws and "pin holes" through which moisture might be admitted. The lead sheathing is applied by passing the insulated cable through dies in a hydraulic press which forces hot lead in a semi-fluid condition around the slowly moving cable, thus forming a closely fitting lead cylinder on the outside of the insulation. It is the practice of one English factory to apply a hydraulic test to the finished cables in order to detect the presence of imperfections in the lead covering, by forcing water into the dielectric so that its presence may be detected before the cable leaves the factory. It is partly in order

to facilitate such testing that the same firm employs a metallic test sheath[1] enclosed in the cable, and lightly insulated from the lead covering. This lead sheath is also useful for maintenance tests, fault localization, the detection of incipient faults on live cables, and for use in connection with special protective devices (Br. Pat. 13681/17 Beaver, Richards, and Claremont.)[2]

Insulating Materials Other Than Insulated Paper.—The reason why reference has not previously been made to vulcanized rubber and vulcanized bitumen as materials for cable insulation is not because these are not in general use, but because they are not so suitable as paper for e.h.t. power cables. Vulcanized bitumen as an insulator appears to have met with greater success in Europe than in America. It is a substance of which the physical properties are somewhat similar to those of vulcanized rubber. Cables insulated with vulcanized bitumen (without lead sheathing) have a special field of utility in mining work, where they are used not only as feeders to carry the electric energy down the shaft, but also as distributors under ground. The results of a very complete investigation of the properties of this material will be found in Mr. Beaver's Institution Paper on "Cables" previously referred to.

A type of insulation which is largely used in America is varnished cambric, or varnished cloth. The prepared cloth is applied to the conductor in the form of tape, a thin layer of a non-hardening viscous filler being applied between the layers to ensure flexibility by permitting relative movement of the layers. Since it is customary to provide a lead sheath over power cables insulated with varnished cambric, there is no economic advantage in using this material as a substitute for paper.

106. Methods of Laying Underground Cables.—The present-day methods of laying underground cables may be classified under three headings:

1. Laid direct in the ground (armored cables).
2. Drawn into weldless steel tubes, stoneware ducts, or fiber ducts in concrete.
3. Laid solid in wood, cast iron, or special asphalt troughing (this last is a patented system known as the Howard asphalt troughing).

[1] British patent No. 22355/12, Beaver and Claremont.

[2] Particulars of the test sheath cable and its potentialities will be found in *The Electrician* (London) of July 5, 1918.

The first-mentioned system is usually adopted for cross-country runs; the second, in congested areas where streets cannot be disturbed and where facilities for adding to the number of cables are desired. The third system may be adopted where occasional disturbance of the street is not a matter of great importance, or where chemical protection for the cable is desirable on account of soil conditions or other causes.

When lead-covered and armored cables are laid direct in the ground, it is customary to place a wooden board over the cable to act as a warning to workmen and so protect the cable in the event ot the ground surface being disturbed after the cable has been laid.

The drawing-in system is the one most commonly adopted at the present time. The ducts for the cables may be tile or fiber set in concrete, or they may consist of wrought-iron pipes.

Tile or stoneware conduits can be supplied single-way or multiple-way. The opening is usually $3\frac{1}{2}$ in. square when used for distributing purposes in or near cities. The inside corners are well rounded, and the outside dimensions are approximately as follows:

Single-way duct	5 in. by 5 in. by 18 in. long.
Two-way duct	5 in. by 9 in. by 24 in. long.
Three-way duct	5 in. by 13 in. by 24 in. long.
Four-way duct	9 in. by 9 in. by 36 in. long.
Six-way duct	9 in. by 13 in. by 36 in. long.

These ducts are set in concrete forming a wall about 3 in. thick on all four sides.

Fiber conduits are light in weight and easy to handle. They are cylindrical pipes made of wood pulp saturated with an asphalt or bituminous compound containing a small amount of creosote, and are usually supplied in 5-ft. lengths.

Iron pipes are useful when slight bends to clear obstructions are of frequent occurrence. These can be supplied in 20-ft. lengths with threaded ends and couplings.

In all cases, the internal width of the duct should allow not less than $\frac{1}{2}$-in. clearance for the largest cable to be drawn into it.

Manholes must be provided at intervals of 300 to 500 feet, the latter distance being rarely exceeded, as it represents the practical length of cable of large size which can be drawn into an underground duct without injury. The manholes, if required

merely for drawing-in purposes and for joints (apart from the installation of transformers or other apparatus), should measure about 7 ft. × 6 ft. × 6 ft. high. If the construction permits of an arched roof, the walls might be about 5 ft. high with a total head room at center of about 7 ft. The floor should be above the sewer level, and proper arrangements provided for drainage and ventilation. If transformers are placed in manholes, a space of about 4.5 cubic feet per k.v.a. should be provided.

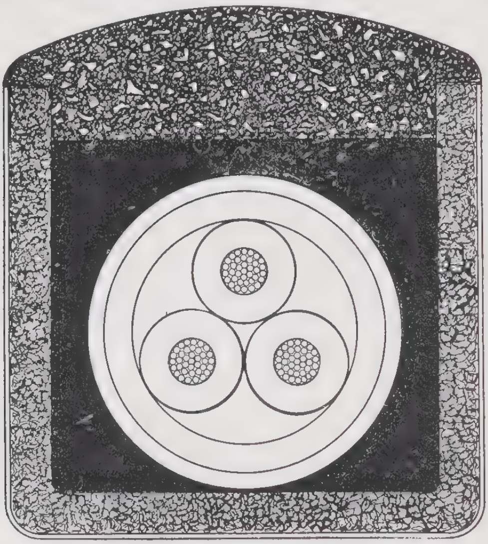


Fig. 77.—Section through three-core cable laid in Howard asphalt troughing.

Fig. 77 is a section showing cables in the Howard asphalt troughing.[1] The chief advantages claimed for this system are imperviousness, chemical inertness, ductility, ease of repair, electric insulation, and excellent dissipation of heat by conduction. In the latter respect it has the advantage over all other troughing systems, and compares favorably with armored cables laid directly in the ground.[2]

107. Costs of Underground Transmission Lines.—The cost of underground systems of transmission varies greatly with the

[1] British patents 5945, 5946, 5947/06, Stratton, Claremont, Beaver, Tanner.

[2] See Beaver, *Jour.* I. E. E., vol. 47, p. 747 (1911).

type of construction, the nature of the ground, the road surface, conditions of transport, labor facilities, etc.; but the following figures should be useful as a rough indication of probable cost, and they may be used for preliminary estimates when reliable figures, based on the specific conditions and requirements, are not available.

	Cost per mile per cable
1. *Laid direct.*	
Cable laid direct in the ground and covered with a tarred warning board	$2200.00
2. *Drawn in.*	
Cable drawn into weldless steel tubes	$6000.00
Cable drawn into stoneware ducts	$3200.00
Cable drawn into fiber tubes laid in concrete	$4500.00
3. *Laid solid.*	
Cable laid solid in Howard Asphalt trough, complete with bitumen and asphaltic concrete	$4200.00
Cable laid solid in wood troughing with wood covers, complete with bitumen	$4500.00
Cable laid solid in cast iron troughing with tile covers, complete with bitumen	$6200.00

The troughing and ducts above referred to allow for suitable clearances for cables of about 4 in. diameter. These costs are based on the assumption that not less than six cables are laid at a time in the case of systems (1) and (3), and not less than twelve duct-ways in the case of (2).

In order to arrive at the total cost of a given system of underground transmission, the cost of cables must be added to that of the conduit. It is obviously impossible to give close prices, and quotations for cables should always, if possible, be obtained from the manufacturers; but the prices here given will serve as a rough indication which will often determine whether or not it will be worth while proceeding further in the matter of a proposed undertaking.

The ducts and troughs for which costs have been given would be capable of accommodating cables of the approximate carrying capacities as listed in the following table.

The expression "voltage graded" as here used refers to the system of providing metallic layers between sections of the insulation with a view to economy of size and cost, in the manner which will be explained later.

System	Type of cable	Working pressure, k.v.	Kilowatts	Approximate cost per mile per cable	Approximate cost per mile per kw.
3-phase	3-core	33	15,000	$12,000	$0.80
3-phase	3-core	66	20,000	13,500	0.675
3-phase	Single core	175 per phase; 100 to ground (voltage-graded)	40,000 per core	15,000	0.375
Single phase	Concentric (grounded outer)	100 (voltage-graded)	30,000	12,000	0.40

NOTE.—Single wire armoring and jute serving for laying direct in the ground would add approximately 30 per cent. to the above figures of cost.

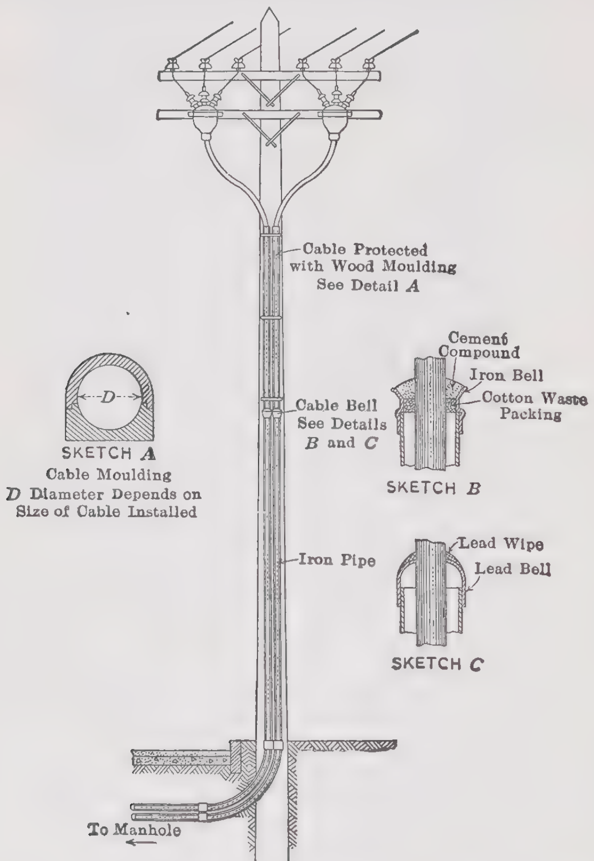


FIG. 78.—Junction between underground and overhead conductors.

108. Cable Terminals. Junction with Overhead Lines.—The connection between overhead wires and underground cables must be made with great care to avoid trouble at this point. The ends to be attained and the precautions to be taken are fairly obvious, and the engineering difficulties may readily be overcome. Each particular case should be considered as a special problem,

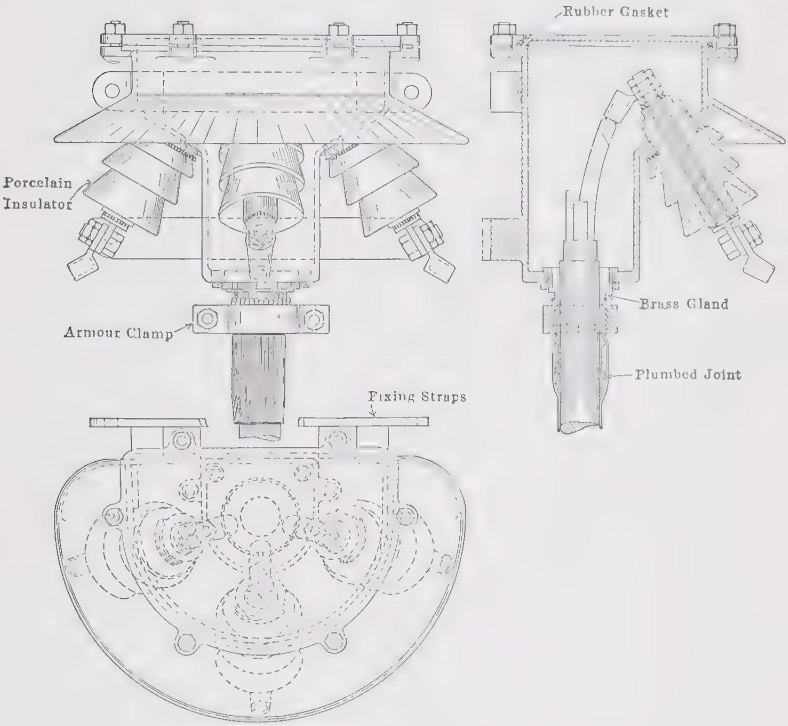


FIG. 79.—Detail of cable ends at junction with overhead line.

and the proper steps taken to provide adequate insulation and prevent deterioration or damage by water owing to improperly sealed joints.

Fig. 78 shows the terminal pole of an overhead line and the methods employed for the protection of the cable. It is reproduced by kind permission of Mr. E. B. Meyer and his publishers from "Underground Transmission and Distribution." The design of the actual terminals or "pot-heads" as they are sometimes named, varies considerably. One form—suitable for a working pressure of 11,000 volts, three phase—is shown in Fig. 79

which has been reproduced from a drawing kindly supplied by Messrs. W. T. Glover & Co. Ltd., Manchester, England.

109. Design of Cables.—The theory of the *Potential Gradient* at the surface of a wire of radius r surrounded by a concentric metallic cylinder of internal radius R has already been discussed in Article 76 of Chapter V, in connection with insulating bushings. With a dielectric of constant specific inductive capacity throughout the total thickness, the maximum potential gradient—which occurs at the surface of the inner conductor—is given by formula (61):

$$G = \frac{E}{r \log_\epsilon \frac{R}{r}} \text{ volts per centimeter} \tag{61}$$

where E stands for the potential difference between the conductor and the outside metallic sheath, in volts, and the dimensions are expressed in centimeters. This formula is therefore applicable to single-core cables with lead sheath, and to the concentric type of two-conductor cable.

The breakdown gradient of paper-insulated cables is about 200 k.v. per centimeter, and it is advisable not to exceed a figure of 120 to 150 k.v. for the guaranteed test pressure. Since the test pressure may be as high as $2\frac{1}{2}$ times the working pressure, the maximum working stress will be of the order of 50 k.v. per centimeter. It is quite possible to design practical paper-insulated cables for working stresses of 60 to 80 k.v. per cm.; but owing to the fact that dielectric heating is likely to be excessive with these high gradients, from 40 to 50 k.v. per cm. is a reasonable maximum. All the voltages here referred to are r.m.s. values, and not peak values.

The *Capacity* of a single-core cable per centimeter of length is the reciprocal of the elastance as given by formula (59), whence

$$\text{Capacity per centimeter} = \frac{2\pi K k}{\log_\epsilon \frac{R}{r}} \text{ farads} \tag{79}$$

where k is the relative inductive capacity or *dielectric constant* of the insulating material, the value of k for air being unity. The numerical value of K, as given in Article 72 of Chapter V, is 8.84×10^{-14}, and since there are 161,000 centimeters in a mile, the capacity per mile of single-core cable, in microfarads, is

$$C_m = \frac{161000 \times 2\pi \times 8.84 \times 10^6 \times k}{10^{14} \times \log_\epsilon \left(\frac{R}{r}\right)}$$

$$= 0.0896 \frac{k}{\log_\epsilon \left(\frac{R}{r}\right)}$$

$$= 0.0388 \frac{k}{\log_{10} \left(\frac{R}{r}\right)} \tag{80}$$

Approximate values of k which apply to materials used for cable insulation are:

$k = 3.3$ for impregnated paper.
$k = 4.2$ for varnished cambric.
$k = 4.5$ for vulcanized bitumen.
$k = 3$ to 6 for vulcanized india-rubber.

The *permittivity*, or relative specific inductive capacity (k) will depend somewhat upon the temperature of the dielectric. The effect of the higher temperatures in *increasing* the value of k being greatest for varnished cambric insulation. The above values are averages for normal working temperatures.

The formula for the *Insulation Resistance* of a single-core cable is obtained as follows:

Let ρ stand for the resistivity, or specific resistance, of the dielectric, in megohms. It is the electrical resistance of one centimeter cube of the cable insulation. Considering the insulation resistance of a length of cable l centimeters long to be the sum of the resistances of successive concentric layers of insulation, we may write,

$$dR_i = \rho \frac{dx}{2\pi x l}$$

where dx is the length, and $(2\pi xl)$ the area, of a cylindrical element of radius x and length l. Whence

$$R_i = \frac{\rho}{2\pi l} \int_r^R \frac{dx}{x}$$

$$= \frac{\rho}{2\pi l} \log_\epsilon \left(\frac{R}{r}\right) \tag{81}$$

Let R_i stand for the insulation resistance, in megohms, of one mile of cable; then $l = 161{,}000$ cm.; and if we convert the natural into common logarithms, we get:

$$R_i \text{ (of one mile)} = 2.28 \times 10^{-6} \times \rho \log_{10} \left(\frac{R}{r}\right) \text{megohms} \quad (82)$$

Some average values of ρ are:
For impregnated paper, $\rho = 8 \times 10^8$
For varnished cambric, $\rho = 3 \times 10^8$ to 5×10^8
For vulcanized bitumen, $\rho = 2.4 \times 10^8$

For vulcanized india-rubber, $\rho = \begin{cases} 4.8 \times 10^8 \\ \quad \text{to} \\ 1.2 \times 10^{10} \end{cases}$

Temperature has a marked effect on the specific resistance of cable insulation, which decreases rapidly with increasing temperatures. This effect is particularly noticeable with paper insulation. Vulcanized india-rubber is the material with the best temperature characteristics.

The *Reactance* of a single-conductor cable without steel armoring would be calculated as for bare wires (refer to Articles 43, 44, and 45 of Chapter IV) and will depend upon the position of the return conductor. Single cables carrying alternating currents should not be armored or drawn into iron pipes.

The reactance of two-conductor concentric cables is usually negligible. Formulas for three-core cables are referred to in Article 112.

110. Economical Core Diameter of High-pressure Cables.—For a given voltage and constant diameter over the insulation of a single-core cable, there is a definite diameter of the core which will cause the potential gradient at the conductor surface to be a minimum. A small core will allow of a greater thickness of insulation; but on the other hand the smaller radius of curvature will tend to increase the stress; while the effect of too large a diameter of core is also to cause an increase in the stress through reduction of the total thickness of insulation. Formula (61) may be written,

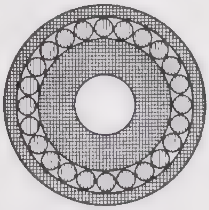
$$r \log_\epsilon \left(\frac{R}{r}\right) = \frac{E}{G}$$

and if we assume both R and G to be constant, the greatest maximum permissible voltage (E) will be obtained when the quantity $r \log_\epsilon \left(\frac{R}{r}\right)$ is a maximum, a condition which is satisfied when $\log_\epsilon \left(\frac{R}{r}\right) = 1$, or $R = 2.718\,r$. Assuming this ideal requirement to

be fulfilled, even if the inner core has to be made hollow in order to economize conductor material, we obtain the relation:

$$r = \frac{E}{G} \tag{83}$$

from which an approximate value of economic conductor radius can be obtained. This radius will generally (in high voltage cables) be found to be greater than that required to provide the necessary cross-section for current carrying purposes, and this accounts for the fact that aluminum may prove to be more economical than copper as a conductor in high-tension underground cables. The 30,000-volt cables for the electrification of the Dessau-Bitterfeld railway, previously referred to, have cores of stranded aluminum 0.512 in. diameter (0.157 sq. in. cross-section) covered with impregnated paper of a radial thickness of 0.512 in. The lead sheath is 0.138 in. thick and it is covered with a layer of jute, the overall diameter of the cable being 2.05 in. By formula (61) the maximum gradient is



Inner lead tube.	0.256 in. bore, or 0.140 in. radial.
Conductor.	24/0.082 in.
Outer lead sheath.	0.05 in. radial.
Complete diameter.	0.80 in.

FIG. 80.—Specially constructed core for e.h.t. underground cables.

$$G = \frac{30000}{2.54 \times 0.256 \log_\epsilon \left(\frac{0.512 + 0.256}{0.256}\right)}$$

$$= 42{,}000 \text{ volts per centimeter.}$$

If stranded copper had been used (without a hollow, or non-metallic core) the diameter of the core of equivalent current-carrying capacity would have been about $\frac{3}{8}$ in. which, with the same thickness of insulation, would cause the maximum gradient to be about 48,000 volts per centimeter.

The application of formula (61) to the calculation of the maximum potential gradient in cables with stranded wire cores,

gives only approximate results. The comparatively small radius of the individual wires will bring about a concentration of stress appreciably greater than what would be indicated by formula (61). One method of providing a smooth cylindrical surface to the cable core is to sheath over the stranded copper core with a thin wall of lead. Fig. 80 shows the complete core (without insulation) of a high-pressure cable as designed by a British cable factory.[1] In this case the calculated "economic" radius for a maximum gradient of 50,000 volts (r.m.s.) is 0.8 in. and since the conductor is of sufficient carrying capacity when made up of 24 copper wires of 0.082 in. diameter, these wires are disposed around a hollow lead core of the necessary diameter, as shown in the illustration.

111. Grading of Cables.—The potential gradient at any distance x from the center of a single-core cylindrical cable is given by formula (58) of Article 76, Chapter V, as

$$G = \frac{\Psi}{2\pi x K k}$$

For a given voltage and total capacity of the cable—which determine the total dielectric flux Ψ—it follows that if k is also constant, the gradient will have a value inversely proportional to the distance (x) from the center of the core. With a view to reducing the outside diameter of the cable, the value of k should change continually in successive layers of insulation in order to maintain G as nearly as possible at the maximum permissible value throughout the thickness of the insulation. In other words, if $k \propto \frac{1}{x}$ the potential gradient will be constant at all parts of the dielectric. By varying the nature of insulation in adjacent layers, a practical approximation to this condition can be obtained —especially with rubber, which can be made in varying qualities having markedly different permittivities. It cannot be said, however, that the grading of cables by this method—*i.e*, by controlling the *capacities* of successive thicknesses of insulation—has met with much success in practice, although developments along these lines may be looked for when the higher voltages for underground transmission become more common.

Another method of grading cables, which might be described as the conducting layer method, gives promise of coming to the front as soon as very high voltage cables are in greater demand than

[1] British patent No. 20549/14.

they are at present. The principle is somewhat similar to that of the condenser type of bushing (see Article 77, Chapter V) except that the intermediate metallic cylinders are now all of the same length, and the potential gradient is controlled by "anchoring" the voltage of these metallic intersheaths. The term "voltage grading" might be used to distinguish this method from "capacity grading." The insulation is the same throughout the total thickness, but being divided into two or more sections by means of metallic cylinders, each section can be made to take its proper share of the total potential difference by applying a definite voltage from an outside source to the intermediate metallic sheaths. Various methods of accomplishing the necessary distribution and "anchoring" of potential for both D. C. and A. C. cables have been patented by Tanner and Claremont.[1] The most obvious way of obtaining the required voltage control on an A. C. system is to take tappings from the high voltage side of the power transformers for connection to the intermediate sheaths.

112. Three-core Cables.—The formulas for use with three-core cables are not so easily developed as those for single-core cables, and they are largely empirical, especially when the shape of the cores departs from the true circular section. Tables giving the approximate inductance and capacity of three-core cables may be found in the electrical engineering Handbooks and in makers' catalogues.

The reactive voltage drop per mile of single conductor of a three-core cable may be calculated by means of formula (29) of Article 45, Chapter IV, which is correct for the usual frequencies, even when the distance (d) between centers of cores is as small relatively to the radius (r) of the conductor as in three-core underground cables.

113. Capacity and Charging Current of Three-core Cables.—Mathematical formulas for calculating the electrostatic capacities of three-core cables are complicated and not very reliable owing to certain assumptions having to be made which are not always satisfied in a practical cable. The figures relating to capacities of cables, as furnished by cable makers, are therefore based on test data obtained from the finished cable.

The capacity between two parallel overhead wires, in microfarads per mile, is

[1] British patents No. 27858/08 and No. 27859/08.

$$C_m \text{ (between wires)} = \frac{0.0194}{\log \frac{d}{r}} \tag{84}$$

and the charging current (r.m.s. value), on the sine-wave assumption, is

$$I_c = 2\pi f C_m E \times 10^{-6} \times L \text{ amperes} \tag{85}$$

where E is the (r.m.s.) voltage between wires, and L the distance of transmission in miles, while d and r in formula (84) stand respectively for the distance between centers of the conductors and the radius of the conductor cross-section.

If three wires occupy the corners of an equilateral triangle the capacity as measured between wires is still given by formula (84), and the capacity between wire and neutral will be just twice this value, or

$$C_m \text{ (to neutral)} = \frac{0.0388}{\log \frac{d}{r}} \tag{12}$$

as given by formula (12) Article 10, Chapter II. The voltage across this imaginary condenser is now $\frac{E}{\sqrt{3}}$ instead of $\frac{E}{2}$ as it would be in the case of a single-phase transmission, and the charging current per wire of the three-phase system is therefore

$$I_c = 2\pi f \frac{E}{\sqrt{3}} C_m \times 10^{-6} \times L \qquad \text{amperes} \tag{13}$$

as given by formula (13) of Chapter II. In this formula, the capacity C_m to neutral is just twice as great as the capacity between wires as given by formula (84), whence it follows that the charging current per wire of the three-phase system is $\frac{2}{\sqrt{3}}$ times greater than that of the single-phase system with the same spacing and line voltage.

In overhead systems, the capacity to ground is generally negligible; but in a cable transmission with or without lead sheath, the condenser formed by the comparatively small thickness of insulation between conductor and ground must be reckoned with. Fig. 81 is a section through a three-core h.t. cable with lead sheath. The capacity of each of the imaginary condensers shown in the diagram cannot be measured directly; but certain measurements can be made on the finished cable, from which the necessary data may be obtained.

Let C_e stand for the effective equivalent capacity per conductor to neutral for one mile of cable. Then, since the voltage to neutral is $\frac{E}{\sqrt{3}}$, the charging current per conductor, on the sine-wave assumption, is

$$I_c = 2\pi f \frac{E}{\sqrt{3}} C_e \times 10^{-6} \times L \qquad \text{amperes.} \tag{86}$$

Two measurements of capacity can readily be made on the finished cable:

(*a*) between one core (1) and the two remaining cores and the lead sheath, all connected together (2, 3, *S*).

(*b*) between any two cores, as (1) and (2).

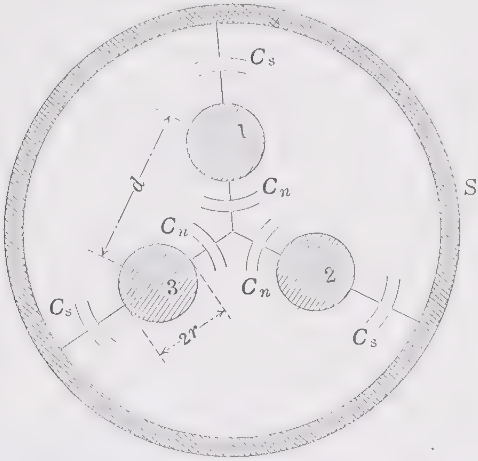


FIG. 81.—Diagrammatic representation of electrostatic capacities in three-core cable.

In terms of the imaginary capacities C_s and C_n of Fig. 81, we have:

$$\text{Capacity } (a) = C_s + \frac{1}{\frac{1}{C_n} + \frac{1}{2C_n}} = C_s + \frac{2}{3} C_n \tag{87}$$

and

$$\text{Capacity } (b) = \frac{1}{2}(C_s + C_n) \tag{88}$$

There is no constant ratio between wire to wire capacity and the wire to sheath capacity; but, generally speaking, the former (*b*) is from 57 to 68 per cent. of the latter (*a*). The most usual

value is 60 per cent., whence, by equating (88) with 0.6 times (87), it follows that $C_s = C_n$.

The standard test for electrostatic capacity is between one core and the two remaining cores grounded to the sheath, namely (*a*) as expressed by formula (87), and since the total capacity (C_e) per core to neutral is obviously

$$C_e = C_s + C_n \tag{89}$$

it follows that the approximate ratio of these two capacities is,

$$\frac{\text{Effective equivalent capacity, core to neutral } (C_e)}{\text{Measured capacity, one core to the remaining cores and sheath } (a)}$$

$$= \frac{(1 + 1) \times 3}{3 + 2} = 1.2 \tag{90}$$

The measured capacity (*a*) for 3-core paper-insulated cables designed for a working pressure of 10,000 volts will be about 0.4 microfarad per mile for a $^{37}/_{0\cdot083\text{ in.}}$ cable, and 0.26 for a $^{19}/_{0\cdot058\text{ in.}}$ cable. With shaped instead of circular cores, the capacity is slightly greater, being from 1.08 to 1.1 times the capacity with circular cores of the same cross-section.

Calculation of Capacity of Three-core Cables.—Although measured values of cable capacities are usually obtainable from manufacturers, it may be necessary to determine approximately the capacity of a cable for a special purpose before it has been made. It would seem at first sight from Fig. 81 and the fact that C_s is found to be approximately equal to C_n, that C_e would be obtained by merely *doubling* the capacity to neutral as calculated by the usual transmission line formulas (see Article 52, Chapter IV). It should be observed, however, that the proximity of the sheath will modify the distribution of the electrostatic flux between core and core, and the imaginary capacity C_n of Fig. 81 is not what it would be if the lead sheath were removed and replaced by a considerable extra thickness of insulation.

It is found in practice that the capacity of a three-core cable with shaped cores, having the same thickness of insulation between core and core as between core and sheath, is about the same as the capacity of a single-core cable having the same conductor cross-section and the same thickness of insulation between core and sheath. On this assumption, we can use formula (80) of Article 109 for predetermining the probable capacity of a three-core cable.

The value of R in formula (80) is taken as the radius of the

conductor plus the thickness of insulation between cores, or between core and sheath. If these thicknesses are not exactly the same, the mean of the two thicknesses is taken. If the conductor is not circular in cross-section, the dimension r in the formula is the radius of a circular core of the same cross-section as the actual conductor.

As an example, let the diameter of each core of a three-core paper-insulated cable be 0.58 in., with insulation between cores 0.38 in. thick, and between core and sheath, 0.30 in. thick. Assuming the specific inductive capacity of the insulation to be 3.3, we have,

$$k = 3.3$$
$$r = 0.29$$

and

$$R = 0.29 + \left(\frac{0.38 + 0.3}{2}\right) = 0.63$$

whence,

$$\textit{Capacity } (a) = \frac{0.0388 \times 3.3}{\log\left(\frac{0.63}{0.29}\right)} = 0.38 \text{ microfarad per mile.}$$

The approximate value of the capacity to neutral, for shaped cores, will therefore be,

$$C_e = 0.38 \times 1.2 = 0.456$$

and for cores of circular cross-section,

$$C_e = \frac{0.456}{1.08} = 0.422 \text{ microfarad per mile.}$$

114. Example of Design of Single-phase Concentric E.H.T. Power Cable.—Let the working pressure be 100 k.v. (alternating) between the inner and outer conductors. The further assumption will be made that the maximum stress must not exceed 40 k.v. (r.m.s.) per centimeter. This is a low value, and 50 k.v. would probably be permissible; but the lower figure has been chosen in order to provide a large factor of safety and keep the dielectric loss (and heating) in the neighborhood of its lowest practical limit.

Consider first the case of a cable without either "voltage" or "capacity" grading. By formula (83)

$$r = \frac{E}{G} = \frac{100}{40} = 2.5 \text{ in.}$$

If a solid core of stranded cable were used, this would correspond

to a cross-section of about 2¼ sq. in. which would certainly be in excess of the requirements, and a hollow core, constructed as shown in Fig. 80, should be adopted.

Solving for R in formula (61), we have,

$$\log_\epsilon \left(\frac{R}{2.5}\right) = \frac{100}{40 \times 2.5} = 1$$

whence

$$\frac{R}{2.5} = 2.718, \text{ and } R = 6.8 \text{ cm.}$$

The dimensions of the cable, in inches, would be approximately:

Diameter over outer conductor = 5.56 in.
Diameter over lead sheath = 5.93 in.
Diameter over armor and jute = 6.6 in.

Consider now the alternative of intersheath- or voltage-grading. If a pressure of 50 k.v. is maintained on each side of a single metallic intersheath, the radius r will now be,

$$r = \frac{50}{40} = 1.25 \text{ cm.}$$

To calculate the radius over the insulation between the intersheath and the inner conductor, we have

$$\log_\epsilon \left(\frac{R}{1.25}\right) = \frac{50}{40 \times 1.25} = 1$$

whence

$$R = 2.718 \times 1.25 = 3.4 \text{ cm.}$$

The radial thickness of the lead intersheath would be about 0.05 in., or 0.127 cm., whence the outside radius of the intersheath is $r' = 3.4 + 0.127 = 3.527$ cm. Considering this as the core of a cable with 50 k.v. across the total thickness of insulation, and the same maximum voltage gradient as before, we have for the radius over the insulation,

$$\log_\epsilon \left(\frac{R'}{3.527}\right) = \frac{50}{40 \times 3.527} = 0.354$$

whence

$$\frac{R'}{3.524} = 1.425 \text{ and } R' = 5.02 \text{ cm.}$$

The approximate dimensions of this cable, in inches, would be as follows:

Diameter over outer conductor = 4.08 in.
Diameter over lead sheath = 4.45 in.
Diameter over armor and jute = 5.12 in.

In order to realize the advantage of this method of grading large high-voltage cables, these figures should be compared with those previously calculated for the cable without voltage grading.

It will be interesting to calculate the charging currents per mile of this cable. Assuming the specific inductive capacity of the paper insulation to be $k = 3.3$ as given on page 213, the capacity per mile between core and intersheath, by formula (80), is $C_m = \dfrac{0.0388 \times 3.3}{\log_{10} 2.718} = 0.295$ microfarad. If the frequency is 25 cycles per second, the charging current, on the sine-wave assumption, will be

$$I_c = 2\pi \times 25 \times 0.295 \times 50000 \times 10^{-6} = 2.32 \text{ amp.}$$

Similarly, between the intersheath and the outer conductor, we have

$$C_m = \frac{0.0388 \times 3.3}{\log 1.425} = 0.832$$

and

$$I_c = 2.32 \times \frac{832}{295} = 6.55 \text{ amp.}$$

The difference between these two values of capacity current must obviously be carried by the intersheath, and for every mile of cable the values of the charging current would be as follows:

In the core	2.32 amp.
In the intersheath . . .	4.23 amp.
In the outer conductor .	6.55 amp.

Thus, if the cable were 10 miles long, the current in the intersheath—if fed from one end only—would be 42.3 amperes, which might be excessive. This is a point which must not be overlooked in the design and installation of intersheath cables.

115. Losses in Underground Cables.—In addition to the I^2R losses in the conductors, which can easily be calculated, some loss occurs in the dielectric of an underground cable. The ohmic resistance of the insulation being very high, the losses, when a high-tension cable is used on a continuous current circuit, are very small; but with alternating currents there is a further loss due to dielectric hysteresis, which is proportional to the frequency of alternation of the electrostatic field.

The total charging current in a cable may be considered as made up of two components, one being the true capacity current of which the phase is exactly 90 degrees in advance of the impressed e.m.f., the other being the "energy" component in

phase with the e.m.f. This last component is relatively small, being due to what is known as dielectric hysteresis and not to the ohmic resistance of the dielectric, the effect of which is usually negligible. The dielectric loss is equal to the product of the voltage and the "energy" or in-phase component of the charging current. Thus

$$\text{Watts lost} = \text{e.m.f.} \times \text{charging current} \times \cos\varphi \quad (91)$$

where $\cos\varphi$ stands for the power factor of the cable. This will usually be about 0.03 for paper-insulated power cables, at ordinary working temperatures; but, with high temperatures, it is very much greater. As a rough indication of the manner in which the power factor (and therefore the dielectric loss) increases with temperature, the following figures may be useful; they refer to paper-insulated cables.

Temperature of insulation, degrees C.	Power factor, per cent.
50	3 to 5
60	5.5 to 10
70	9 to 15
80	13 to 20
90	19 to 30

As an example of how the losses in a cable may be calculated, we shall use the data of the numerical example in Article 113. The calculated capacity per core to neutral was $C_e = 0.422$ microfarad per mile. Assuming the voltage between wires to be 11,000, the frequency 50, and the distance of transmission 10 miles, the charging current per core, by formula (86), will be,

$$I_c = 2\pi \times 50 \times \frac{11000 \times 0.422}{\sqrt{3}} \times 10^{-6} \times 10$$
$$= 8.4 \text{ amperes,}$$

whence the total dielectric loss is seen to be

$$W = 3 \times \left(\frac{11000}{\sqrt{3}}\right) \times 8.4 \times 0.03 \times 10^{-3}$$
$$= 4.8 \text{ kilowatts.}$$

The *apparent* power required is, however, $\frac{4.8}{0.03} = 160$ k.v.a., which is an indication of the size of generator required to keep the full voltage on the line when the receiving end is disconnected from the load.

116. Temperature Rise of Insulated Cables.—The layers of insulating material close to the conductors will be hotter than those near the surface of the cable; but the difference in temperature between the conductor and the external surface is not easily predetermined. It will depend upon the nature and thickness of the insulation, and also upon the type of cable, *i.e.*, whether two- or three-core, or concentric. The difference of temperature between the core and the external sheath of a fully loaded power cable will usually be between 10° and 25° C.; but the actual temperature of the insulation at the hottest parts will be determined not only by the rate at which the heat can be conducted from the cores to the surface through the insulation, but also by the rate at which it can be radiated or conducted from the outside surface of the cable.

The best conditions for cooling will usually occur with submarine cables; but armored cables buried direct in certain kinds of soil are also capable of dissipating large amounts of energy. When many cables are laid close together in multipleway ducts, it is not possible to consider each cable independently of the others, since the temperature of the duct will depend upon the total cooling surface and the total amount of energy lost in all the cables. The problem of calculating temperature rise is thus seen to be a difficult one, and indeed more data must be accumulated before reliable empirical formulas will be available. Very little can be said here that will be of service to the engineer in determining exactly what will be the safe current for a given cable laid in a particular manner; but it is important to keep the temperature of the insulation within certain limits which cannot be exceeded without injuring the cable or leading to a greatly increased dielectric loss which aggravates the trouble and leads to rapid deterioration of the insulation. This limit may be set at about 85° C. for paper-insulated cables.

For pressures up to 20,000 volts, the dielectric loss in three-phase high-grade paper cables is so small as to be negligible; and the permissible I^2R loss in the conductors will depend (1) upon the rate at which heat can be conducted through the insulation from the conductor to the outside surface, and (2) upon the facilities afforded for the cooling of the outside surface of the cable.

It is possible to run the current densities in three-core 20,000-volt power cables up to 1000 amp. per sq. in. in cables of

0.25-sq. in. core section, and even 1500 amp. per sq. in. if the core is not more than 0.1 sq. in. cross-section. It is rarely safe to allow the lead sheath to reach a temperature greater than 40° C.; but no hard and fast rule can be laid down in this connection. When the price of copper is high, it is important to load cables up to the safe limit, and research work is being carried on with a view to furnishing additional information on the heating of underground cables of different types and under different conditions of laying.

A simple case, which admits of calculation without a large amount of empirical data, is that of the single-core or concentric cable. Thus, if it is desired to calculate the difference in temperature between the core and external sheath of such a cable, the procedure would be similar to the method followed in calculating the ohmic resistance, except that the "heat conductivity" of the dielectric would take the place of the electrical conductivity.

As an *example*, and following the method outlined in Article 109 when developing an expression for the insulation resistance, consider a lead-covered single-core paper-insulated cable of core diameter $2r = 0.9$ in. and diameter over the insulation of $2R = 1.9$ in. The heat conductivity of the insulation must be determined experimentally, but we shall assume that it is $k = 0.002$. This coefficient may be defined as the number of watts that will be conducted through each square centimeter of a slab of insulating material 1 cm. thick when the difference of temperature is 1° C.

The cross-section of this conductor will be about 0.5 sq. in., the resistance being 0.1 ohm per mile, and if we assume a current density of 1300 amp. per sq. in., the power loss per mile will be $0.1 \times (650)^2 = 42{,}250$ watts.

Considering a cylinder of the dielectric of thickness dx at a radius x from the center of the core, the difference of temperature between the two sides of this layer of insulation will be

$$dT = \frac{W dx}{k\ (2\pi x l)}$$

where l is the length in which the loss of W watts occurs (in this example $l = 161{,}000$ cm.). Thus,

$$T = \frac{W}{2\pi l k} \int_r^R \frac{dx}{x}$$

$$= \frac{W}{2\pi lk} \log_\epsilon \left(\frac{R}{r}\right)$$

$$= \frac{42250}{2\pi \times 161000 \times 0.002} \log_\epsilon \left(\frac{1.9}{0.9}\right)$$

$$= 15.5° \text{ C.}$$

If the cable were suspended in air, the difference of temperature between the lead sheath and the surrounding air could be calculated by assuming about 0.0012 watt to be radiated from each square centimeter of surface per degree Centigrade difference of temperature. The outside diameter of this cable—if there is no jute or steel armoring over the lead—will be about 2.15 in., and the rise in temperature of the lead sheath will therefore be

$$\frac{42250}{\pi \times 2.15 \times 2.54 \times 161000 \times 0.0012} = 12.7 \text{ degrees.}$$

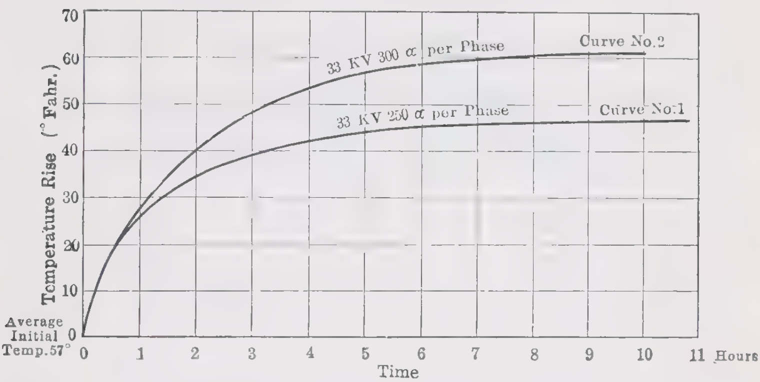


FIG. 82.—Temperature rise of 33 kv. 0.25 sq. in. three-phase, paper insulated, lead-covered cables in iron pipes laid 12 in. apart 3 ft. below ground surface.

The total temperature rise of the copper, on this basis, is therefore $15.5 + 12.7 = 28.2°$ C.; but since the cable is not likely to be suspended in air, correction coefficients derived experimentally would have to be applied in order to determine the probable temperature rise under practical conditions. A further correction would have to be made if the cable is provided with an outer covering of jute.

Since the final temperature rise of a cable under given conditions will be very nearly proportional to the square of the current, we may write

$$I^2 = KT$$

and if the temperature rise T_1 is known for a given current I_1, the value of the constant K can be determined, and the temperature rise with any other current I_2 will be approximately

$$T_2 = \frac{I_2^2}{K}$$

The curves of Fig. 82 show not only the final temperature rise attained by three-core paper-insulated power cables drawn in iron pipes underground, but also the time required to bring about any particular rise in temperature. The vertical scale indicates degrees Fahrenheit above an initial temperature of 57 degrees, while the horizontal scale gives the hours during which the current has been on the cable. The cross-section of each core of this cable is 0.25 sq. in., and the tests were made with current densities of 1000 and 1200 amperes per square inch; the frequency of the supply being 50. The value of the constant K in the expression $I^2 = KT$ as calculated for $I = 250$ amperes, is 1330, whence the calculated temperature rise for 300 amperes is 67.6° F. which is somewhat higher than the observed rise of 62 degrees, as one would expect it to be.

No appreciable rise of temperature was observed with the full working voltage on the cable without load. In other words, the dielectric losses were not of such magnitude as to add appreciably to the heating caused by the I^2R losses in the conductors.

117. Reliability of Cable Systems. Joints; Electrolysis.—Apart from mechanical injury, which must be guarded against by giving attention to the method of laying and to the handling of the cables during their installation, trouble can usually be traced (1) to poor joints, (2) to general or local overheating, and (3) to electrolysis.

Joints.—Even when the trouble is caused by overheating rather than initial weakness of insulation, this frequently occurs in the neighborhood of poorly made joints. A considerable amount of skill is necessary in making satisfactory joints on h.t. underground cables, and the difficulty experienced in obtaining skilled and reliable workmen has led to the development of designs employing special insulating spacers and accurately made metal sleeves or bridge pieces, which depend less upon the skill and experience of the jointer than the older methods involving paper or tape wrappings.

Careful bonding of the lead sheath is also a matter of consider-

able importance if electrolytic troubles are to be avoided, and this is provided for in the designs illustrated by Figs. 83 and 84.[1] The former shows a straight joint on a three-phase cable suitable for about 20,000 volts; paper sleeves are slipped over the joints, and rubber rings keep these the proper distance apart. A high grade of insulating compound is poured inside the lead

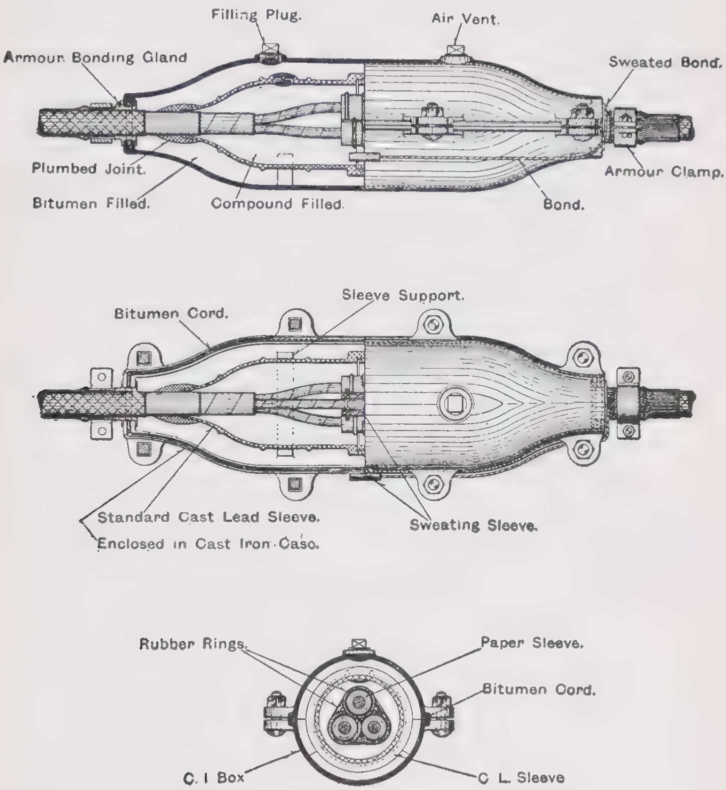


FIG. 83.—Joint in high tension (20 kv.) cables laid direct in ground.

sleeve forming the bond between the lead sheathings, while bitumen may be used in the space between the lead and the outer case of cast iron. It will be noted that all sharp corners are avoided; the castings being designed with curves of large radius to obtain the proper distribution of the dielectric flux.

[1] Designs of Messrs. W. T. Glover and Co.

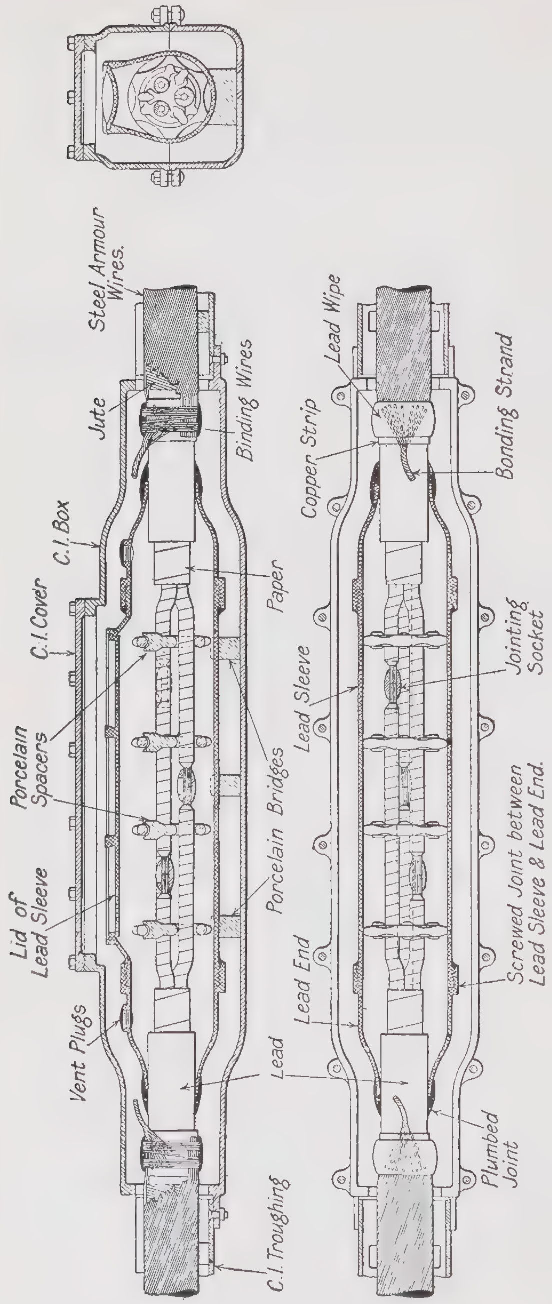


Fig. 84.—Joint for 33 kv. three-core cables.

Fig. 84 shows a somewhat similar joint box, for 33,000-volt three-core cables. The special jointing ferrules are designed with curved surfaces so shaped as to avoid concentration of stress, and thoroughly vitrified unglazed porcelain spacers are used between the cables in order to eliminate wrappings of tape or paper. The cast lead sleeve forming the bond between the lead sheathings is made in three parts to facilitate assembly.

Overheating.—Apart from the damage done to the insulation by very high temperatures, it is generally true that the dielectric loss increases with the temperature, and this is especially noticeable with paper-insulated cables. It is nevertheless important to operate underground cables at a reasonably high current density, otherwise the interest on capital expenditure will be greatly in excess of the annual cost of the losses, and the system will be economically unsatisfactory. The engineer is therefore faced with a problem of considerable magnitude if underground transmission is to be adopted on a larger scale than at present. General or local overheating of cables is perhaps the chief cause of service interruptions on underground systems, yet a liberal cross-section of conductor can hardly be considered as the proper solution of the problem. Attention to all the causes that may lead to local overheating is very important; but the matter is not one that lends itself to lengthy discussion in these pages.

Since submarine power cables will transmit much greater amounts of energy for a given size than underground cables, the possibility of flooding the ducts with water, as suggested by Mr. Harper[1] deserves consideration.

Electrolysis.—Chemical action, unassisted by electric currents, very rarely injures the lead sheath of underground cables; but electrolytic corrosion of the lead covering—resulting in perforations and damage to the insulation by the admission of moisture—is not uncommon. An underground cable laid within a few feet of an electric railway or car line, is liable to electrolysis; but in countries, such as England, where rules governing the maximum permissible potential difference between current-carrying rails and ground are rigidly enforced, trouble due to this cause is very rare. Perfect bonding of the lead sheath and also of the steel armoring will effectively prevent trouble which otherwise might be experienced.

[1] "Problems of Operation and Maintenance of Underground Cables," by J. L. Harper, *Trans.* A. I. E. E., p. 417, vol. xxxvi, 1917.

Leakage of current from the cables themselves, due to careless work and inefficient sealing at points where connections are made is another cause of electrolytic corrosion in underground distributing systems.

Electrolysis of lead and iron buried in the ground is usually caused by stray currents from electrically operated railroads or street-car lines which use continuous currents. The effects of alternating currents of frequencies between 15 and 60 cycles is practically negligible, being least with the highest frequency.

The actual amount of metal carried away from the surface of the anode (usually lead or iron) will depend not only upon the surface exposed and the density of the stray currents which cause the corrosion; but also on the nature of the soil and whether it is usually moist or dry.

The reader who desires to investigate more fully the subject of electrolytic corrosion is referred to the important contribution by Messrs. McCollum and Ahlborn.[1] Their paper includes references to the work of previous investigators, and brings out very clearly the importance of reversing the direction of the currents through the ground, even if the period of such reversal is measurable in hours or days. The shifting of loads on electric railway systems in cities usually produces large areas, called neutral zones, where the polarity of underground pipes reverses at more or less frequent intervals. Even when such intervals of reversal range from several minutes to one or two hours, the corrosion due to electrolysis is found to be less than would be expected if attributed to the average amount of current discharged from the pipes into the earth. It would seem as if the reversal of current actually causes metal to be redeposited on the corroded portions, even after the pipes have acted as anodes during a considerable lapse of time. The redeposited metal will probably have little effect in strengthening the pipe mechanically; but it will serve as an anode surface during the succeeding period of current discharge, and thus protect the uncorroded metal beneath, which otherwise would have been attacked. This action, due to comparatively slow reversals of current, will be interfered with by circulation of the electrolyte, and the action of air (oxygen and carbon dioxide) on the corroded metal.

[1] "Influence of Frequency of Alternating or Infrequently Reversed Current on Electrolytic Corrosion," by Burton McCollum and G. H. Ahlborn. Technological Paper No. 72 of the Bureau of Standards, Washington.

CHAPTER VIII

TRANSMISSION OF ENERGY BY CONTINUOUS CURRENTS

118. General Description of the Thury System.—In the Thury system of electric power transmission by continuous currents, the current is constant in value and the pressure is made to vary with the load. All the generators and all the motors are connected in series on the one wire, which may be in the form of a closed loop serving a wide area, or it may consist merely of the

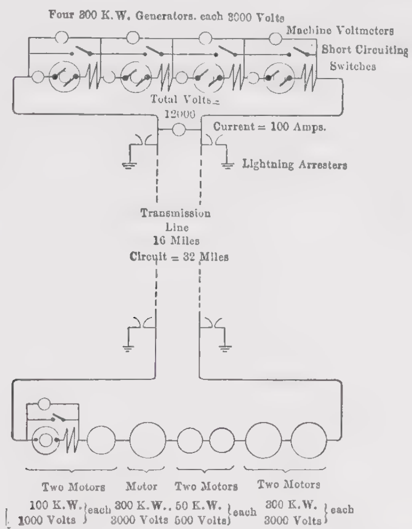


FIG. 85.—Diagram of connections—Thury series system.

outgoing and returning wires between a power station containing all the generators, and one or more substations, with motors, at the end of a direct transmission. The diagram, Fig. 85, shows a typical arrangement of machines for a small installation on the Thury system. In this example there are four generators at one end of the line, and seven motors of various capacities at the other end of the line, all the machines being connected in series and pro-

vided with short-circuiting switches. The connections are, however, so simple that the diagram is self-explanatory. The voltage at the terminals of any one dynamo is limited, as the necessity for a commutator renders it impossible to wind a continuous-current machine for so high a pressure as may be obtained from alternating-current machines. The limiting pressure per commutator on the existing Thury systems is 5000 volts, this being on the machines of the Metropolitan Electric Supply Company of London: the lowest is 1300 on a small two-generator plant in Russia.

In order to obtain the high pressures required for economical transmission over long distances, it is necessary to connect many generator units in series; the difficulty of insulation between machines and ground being overcome by mounting the dynamos and motors on insulators and providing an insulated coupling between the electric generators and the prime movers. An insulating floor is also provided.

When a machine, whether generator or motor, is not in use, it is short-circuited through a switch provided for this purpose. As the motor load varies, generators are switched in or out of circuit, thus varying the total voltage. When it is required to switch in an additional generator, the machine is brought up to speed until it gives the proper line current before the short-circuiting switch is opened to throw the machine in series with the line. To start up a motor, the short-circuiting switch is opened when the brushes are in the position of zero torque. The brush rocker is then gradually moved round, and the motor, starting from rest, increases in speed until the brushes are in the required position; the actual speed, for any particular position of the brushes, being dependent upon the load.

The motors may be distributed anywhere along the line, either on the premises of private users of power, where they may be directly coupled to the machinery to be driven, or in substations, coupled to constant pressure electric generators giving a secondary supply for lighting and power purposes. In most of the Thury undertakings in Europe, this secondary supply is three-phase alternating current.

A series-wound dynamo machine, with a current of constant value passing through field-magnet and armature windings, is essentially a constant torque machine. In the case of a motor, if the load is decreased, the motor will increase in speed and tend

to "run away"; with increase of load, the motor will slow down, and in time come to a stand-still. In regard to the generators, the ideal prime mover is one which will give a constant torque, such as a steam engine with fixed cut-off and constant steam pressure; and a single generator so driven would be practically self-regulating, and maintain constant current regardless of load, as the speed—and therefore the pressure at terminals—would adjust itself to suit the motor load. The generators are, however, usually driven by prime movers which are far from fulfilling the ideal conditions. Most of the Thury stations are driven by water turbines, which are most efficient as constant speed machines; while the maximum torque at low speeds is generally about twice the torque under conditions of highest efficiency at normal speed.

Just as various devices are provided, when working on the parallel system, to maintain constant pressure of supply, so in the series system, it is necessary to provide regulating devices to maintain a constant current. Regulators controlled by the main current, or by a definite fraction of the main current, passing through a solenoid, can be made to act on mechanism designed to vary the speed of the prime movers. This method is quite practicable, but, where the type of engine—such as a water wheel under constant head—is not suited to variable speed running, the machines may be run at constant speed, and the automatic device made to alter the magnetic field cut by the armature conductors, this being the only alternative means of varying the voltage generated. The alteration of the effective magnetic flux may be effected:

(1) By shunting a portion of the main current so that it shall not all pass through the field winding, and

(2) By shifting the position of the brushes on the commutator.

A combination of both methods appears to give satisfactory results. The method (1) alone is liable to lead to sparking troubles because of the relatively greater armature reaction due to the weakening of the field; and, in practice, it is found inadvisable to shunt more than one-third of the total current. It is, of course, understood that the large variations of voltage are obtained by connecting more or fewer generators in series on the line.

The motors, whether connected directly to the machinery of mills or factories, or used for driving sub-generators of the con-

stant pressure type, are usually required to run at constant speed. Their regulation is effected by a small centrifugal governor which rocks the brushes by acting on intermediate mechanism driven by the motor itself. The reversal of a motor is most simply effected by shifting the brushes round, through the no-voltage position, until the current reverses in the armature coils.

A short-circuit on a motor merely removes that portion of the total load from the system, and the regulators on the generators will readjust the pressure accordingly. If a short-circuit occurs on a generator, the prime mover may be protected from the shock by a slipping coupling, which is commonly provided. If, owing to the failure of a prime mover, a generator tends to reverse and be driven as a motor, it may be short-circuited by a switch that can very easily be made to operate automatically on reversal of current.

119. Straight Long-distance Transmission by Continuous Currents.—Although high-pressure direct current may be used on the loop system with any number of motors or motor substations distributed along the line—and, if desired, with any number of generating stations at suitable points on the loop—it will generally be found that a parallel constant-pressure system is preferable for covering a large industrial area, the simple reason being that, with the series system having a load more or less uniformly distributed along the loop, the system is a high-tension transmission at the start only, since the required voltage decreases with the distance from the generating plant. It is true that the cost of the insulation may therefore be less than for a system on which the pressure is high throughout, but that can be said of any low-tension system. The point is that, in the case of the series loop serving a wide district, with power taken off at intervals along the line, the average pressure at which power is supplied to the motors or substations is only about half that which is supplied to the line where it leaves the power station. It must not be concluded that the Thury system is not well adapted to supplying several motor substations. It is an easy matter, as previously mentioned, to connect any number of motors in series on the line, but in order to get the full benefit of the series system these substations should all serve a comparatively small district at the distant end of the transmission line.

Apart from these considerations, and notwithstanding the advantages of a long-distance straight transmission, the Thury

system would appear to be admirably adapted to serve as a link between otherwise isolated power plants in an industrial or thickly populated district of considerable area. By means of this system there is not the slightest difficulty in putting in series a number of generating stations on the one power line, and stations supplying alternating current of various voltages and frequencies can thus be linked together with the greatest ease and simplicity.

120. Insulation of Line when Carrying Continuous Currents.—The question of sparking distances and the behavior of insulating materials when subjected to continuous-current pressures of high values is of the greatest importance when considering the relative values of the Thury system and the more common three-phase high-tension transmission. On the assumption of the theoretical sine wave, the maximum instantaneous value of an alternating e.m.f. is $\sqrt{2}$ times the root-mean-square value, and comparisons between alternating-current and direct-current transmissions are usually made on this basis, which makes the allowable continuous-current pressure to ground or between wires, for the same insulation and spacing, $\sqrt{2}$ times the working pressure of an alternating-current system. The ratio should, however, be based on experimental data, and, with a view to obtaining definite and conclusive information on this point, Mr. Thury conducted some years ago a very complete set of comparative tests with high voltages, both continuous and alternating. The results of these tests are probably more favorable to the alternating-current systems than would have been the case had they been conducted on existing high-pressure power-transmission systems, because the experimental alternator used in the tests gave a rather flat-topped e.m.f. wave without any irregularities. The tests conducted to determine the comparative pressures at which various insulating materials would be punctured all tend to show that, with continuous currents, something more than twice the alternating pressure is required to puncture the insulation; and, in regard to sparking distances, the direct-current voltage necessary to spark over a given distance is, on the average, double the alternating-current voltage. In fact, this very complete series of tests seems to indicate that any existing transmission line designed for a definite maximum working pressure with alternating currents is capable of being used to transmit continuous currents at twice this pressure. It is also interesting to note that insulators which become hot when

subjected to high alternating-current voltages remain cool when tested with continuous currents. In fact, the leakage losses on the Thury transmissions are small. The total leakage loss over about 3000 insulators on the St. Maurice-Lausanne transmission (a distance of 35 miles), even in damp weather, is something of the order of 900 watts.

It is usual to employ two insulated wires for direct-current high-pressure transmission, but under certain conditions it might be quite satisfactory to use the earth as the return conductor. The arrangement with two wires and the entire electric circuit insulated from earth is usual for pressures up to 25,000 volts. It has the advantage over any grounded system that any point on the circuit may become grounded without causing a stoppage, and repairs can readily be carried out by temporarily grounding two more points, one on each side of the fault. The facility and safety with which repairs on the high-tension system can be carried out by grounding the point where the work is being done is another advantage of this arrangement.

If a ground connection is made at both ends of the two-wire transmission, the ground wire being so situated as to balance the load as well as possible, an arrangement equivalent to the ordinary three-wire system is obtained. The pressure between wires may then safely be doubled because the potential difference between any one wire and earth can never exceed half the maximum pressure of transmission. On the other hand, some of the advantages of the non-grounded system are lost.

A direct-current transmission to any economic distance by means of a single wire, using the earth as the return conductor, is by no means an impossible scheme. The ground resistance is practically zero, the loss of pressure being almost entirely in the immediate neighborhood of the grounding plates. Tests made on the St. Maurice-Lausanne line (35 miles) gave a total ground resistance of 0.5 ohm. Continuous currents of the order of 100 amp. returning through the earth do not appear to be objectionable in any way. By taking the ground connections to a considerable depth below the surface, the current density at ground level would everywhere be so small that interference with opposing interests would hardly be possible.

121. Relative Cost of Conductors: Continuous Current and Three-phase Transmissions.—In order to study the relative costs of conductor material required for the series direct-current

system and the more common three-phase alternating-current transmission, a basis of comparison is necessary, and the following assumptions will be made:

(A) Same distance of transmission; no tapping of current at intermediate points.

(B) Same total amount of power transmitted.

(C) Same power loss in conductors (losses due to leakage and capacity of lines are neglected).

(D) Same insulation used on both systems.

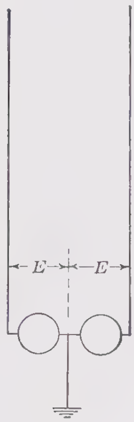


FIG. 86.

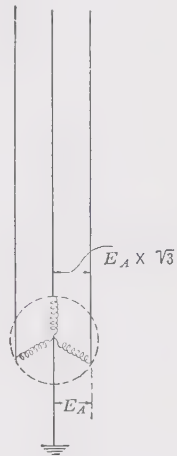


FIG. 87.

FIGS. 86 and 87.—Comparison of voltages on direct-current and three-phase systems.

This last condition is practically equivalent to stating that the maximum value of the voltage shall be the same. It is proposed to consider the following four conditions:

(a) Same maximum pressure above ground; the direct-current voltage being $\sqrt{2}$ times the alternating-current voltage (sine wave assumed).

$$\text{Ratio } \frac{E_a}{E} = \frac{1}{\sqrt{2}} \tag{92}$$

where E and E_a stand respectively for the continuous and alternating voltages to ground. (See Figs. 86 and 87.)

(b) Same as (a); but direct-current voltage double the alternating voltage.

$$\text{Ratio } \frac{E_a}{E} = \frac{1}{2} \tag{93}$$

(c) Same pressure between wires; the allowable direct-current pressure being $\sqrt{2}$ times the alternating-current pressure.

$$\frac{\sqrt{3}E_a}{2E} = \frac{1}{\sqrt{2}}, \text{ or ratio } \frac{E_a}{E} = \frac{\sqrt{2}}{\sqrt{3}} \tag{94}$$

(d) Same as (c); but direct-current pressure double the alternating-current pressure.

$$\frac{\sqrt{3}E_a}{2E} = \frac{1}{2}, \text{ or ratio } \frac{E_a}{E} = \frac{1}{\sqrt{3}} \tag{95}$$

To satisfy the condition of equal total power, the equation is,

$$2E \times I = 3E_a I_a \cos\theta \tag{96}$$

and for equal line losses,

$$2I^2R = 3I_a^2R_a \tag{97}$$

where I is the current per conductor in the direct-current transmission, and R the resistance per mile of single conductor; while I_a and R_a are the corresponding quantities for the three-phase transmission.

In either system the total weight (and cost) of the conductors is proportional to $\frac{\text{number of conductors}}{\text{resistance of each conductor}}$ which gives the relation,

$$\frac{\text{Cost of conductors, direct-current system}}{\text{Cost of conductors, three-phase system}} = \frac{2R_a}{3R} \tag{98}$$

but R_a can be expressed in terms of R thus:

By (97)

$$R_a = \frac{2I^2R}{3I_a^2} \tag{99}$$

and by (96)

$$I = \frac{3E_aI_a \cos\theta}{2E}$$

or

$$I^2 = \frac{9E_a^2I_a^2 \cos^2\theta}{4E^2}$$

which, when put for I^2 in formula (99), gives,

$$R_a = \frac{3E_a^2 \cos^2\theta \times R}{2E^2} \tag{100}$$

Thus the equation (98) becomes,

$$\frac{\text{Cost of conductors, direct-current system}}{\text{Cost of conductors, three-phase system}} = \frac{E_a^2}{E^2}\cos^2\theta$$

Assuming the very common value of 0.8 for the power-factor of the three-phase system, the numerical ratio for the four conditions previously stated would be:

(a) For same maximum pressure to ground, with sine wave assumption,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}} = \frac{\cos^2\theta}{2} = 0.32$$

(b) Same as (a), but allowable direct-current pressure assumed to be *double* the alternating-current pressure,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}} = \frac{\cos^2\theta}{4} = 0.16$$

(c) For same maximum pressure between wires, with sine wave assumed,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}} = \frac{2}{3}\cos^2\theta = 0.426$$

(d) Same as (c), but allowable direct-current pressure assumed double the alternating-current pressure,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}} = \frac{1}{3}\cos^2\theta = 0.213$$

The transmission line, apart from the cost of conductors, would be cheaper for the direct-current than for the three-phase scheme because there are fewer insulators required and only two instead of three conductors to string; and if a grounded guard wire is erected above the conductors, it is more convenient to arrange this over the two direct-current conductors than over the three alternating-current wires, and it would not necessitate the same total height of tower. The important saving is, however, in the conductors themselves. Taking the figure most favorable to the direct-current scheme (b), the alternating-current conductors to transmit the same power with the same loss would cost six and a quarter times as much as if direct-current transmission were used, and even under the assumption (c), most favorable to the three-phase scheme, the cost would still be 2.35 times the cost of the direct-current conductors. For the purpose of getting out preliminary estimates, it is certainly safe to assume that, if the power factor of the three-phase load may be taken as 0.8 the cost of conductors on a long-distance direct-current transmission would be only one-quarter of the cost of conductors with the alternating-current scheme on the assumption of equal I^2R losses.

If a ground return were used on a straight long-distance transmission—a perfectly feasible arrangement—the cost of copper for the same total I^2R loss would be only one quarter of the cost of a two-wire transmission, since the loss in the ground return would be negligible. The cost of copper would then be only about one-sixteenth of the cost of copper on the equivalent three-phase transmission, a point which suggests that the Thury system is worthy of more serious consideration than it has so far received outside of Europe. It is not suggested that a comparison on the basis of equal line losses is necessarily correct or justifiable on economic grounds; but this does not render the above comparisons less interesting or valuable.

122. Concluding Remarks on Direct-Current Transmission.—As an indication of what has been done in Europe since the introduction of the Thury system a quarter of a century ago, it may be stated that there are at present about 16 separate transmissions in operation, in Switzerland, Italy, France, Hungary, Spain, Russia and England. The shortest length of loop is 12.4 miles (Batoum, Russia), with a line pressure of 2600 volts. The longest is 248 miles (124 miles straight transmission), this being the Moutiers-Lyons line at a maximum pressure of 100,000 volts, the current being 150 amperes.

In England, the direct-current series system has been adopted by the Metropolitan Electric Supply Co. of London on their Western section. The plant has been in operation since March of 1911 and given entire satisfaction. The current is about 100 amp., but can be varied from 70 to 120 amp. without causing trouble through sparking on the commutators. The commutators measure 5 ft. in diameter and 6 $\frac{3}{4}$ in. in length. They have 1439 segments and run sparklessly at 5000 volts between brushes. The machines have six poles and only two sets of brushes.

An interesting account of the Thury system, by Mr. William Baum, with brief descriptions of the important European plants, will be found on page 1026 of the *General Electric Review* of Nov., 1915.

As an example of what might be done at the present time in the way of direct-current transmission on a large scale, it is clear that no difficulty need be experienced in building dynamos of a large size with 5000 volts on one commutator. Assuming a current of 300 amp., which would probably be transmitted by

two conductors connected in parallel, the output of each machine would be 1500 k.w. and two of these might be coupled to one prime mover. With two commutators per machine, the output would be 3000 k.w., and with four commutators, 6000 k.w. per unit. Six machines in series, each with four commutators, would have a total output of 36,000 k.w. at 120,000 volts. There would be practically no new or experimental engineering work in connection with such a scheme.

Electrical engineers on the American continent are rather inclined to the belief that when energy has to be transmitted from one place to another the one and only course open to them is to adopt the three-phase alternating-current system. It is not suggested that at the present time this may not, in the majority of cases, be the best system available; but undoubtedly there are conditions under which the continuous-current series system would prove more economical and reliable. Of course, first cost of plant and operating charges have to be taken into account when comparing different systems, and the most satisfactory way of doing this is to reduce all estimated costs to the common basis of annual charges. The cost of the direct-current generators must be set against the combined cost of alternators and exciters and step-up transformers with all intermediate switch gear.

In this connection the writer cannot refrain from quoting a paragraph which occurred in one of the leading articles in the *Electrical World* of New York, in which reference is made to the fact that transmission by continuous currents has received considerable attention in Europe.

"Any engineer who wanders through one of the large Thury stations and then calls to mind the usual long concrete catacombs bristling with high-tension insulators and filled with dozens of oil switches, scores of disconnecting switches, webbed with hundreds of feet of high-tension leads and spattered with automatic cut-outs, will stop and think a bit before he complacently sniffs at high-tension direct-current transmission."

In regard to reliability it is true that, on the Thury system, the generators have not the protection against lightning disturbances which the step-up transformers afford to the alternators on high-tension three-phase systems, and where thunderstorms are prevalent this must not be overlooked, as the cost of protective apparatus may prove excessive. In this connection it is interest-

ing to note that the charges of electricity in the upper atmosphere are always positive, and the negative wire will therefore tend to draw a lightning discharge away from the positive wire or grounded guard wire, but to how great an extent this would affect the proper disposition of the wires it is difficult to say.

This chapter will be concluded with a brief summary of the important points in favor of, and unfavorable to, the employment of continuous currents on the series system for the purpose of transmitting energy at comparatively high voltages from one place to another.

ADVANTAGES OF THE DIRECT-CURRENT SERIES SYSTEM

1. The power-factor is unity—a fact which alone accounts for considerable reduction of transmission losses.

2. Higher pressures can be used than with alternating current, the conditions, as shown by actual tests, being more favorable to direct-current transmission than is generally supposed. Without any alteration to insulation or spacing of wires, approximately double the working pressure can be used if direct current is substituted for alternating current. Moreover, the insulation is subjected to the maximum pressure only at times of full load, whereas on the parallel system the insulation is subject to the full electrical stress at all times.

3. There is no loss of power through "dielectric hysteresis" in the body of insulating materials.

4. The necessity for two wires only, in place of three, effects a saving in the number of insulators required and allows cheaper line construction.

5. Where it is necessary to transmit power by underground cables, continuous currents have great advantages over alternating currents. Single-core cables can be made to work with continuous currents at 100,000 volts. By using two such cables and grounding the middle point of the system it is, therefore, quite feasible to transmit underground at 200,000 volts.

6. The practicable distance of transmission, especially when the whole or a part is underground, is greater than with alternating currents.

7. There are no induction or capacity troubles and no surges or abnormal pressure rises due to resonance and similar causes, such as have been experienced with alternating currents. This

virtually makes the factor of safety on insulation greater than on alternating-current circuits, even when the working pressure is doubled.

8. A number of generating stations can easily be operated in series, and when the demand for power increases, a new generating station can be put up on any part of the line if it is inconvenient to enlarge the original power station.

9. The simplicity and relatively low cost of the switch gear is remarkable. A switch pillar with ammeter, voltmeter, and four-point switch is all the necessary equipment for a generator. The switch pillar for a motor includes, in addition, an automatic "by-pass" which bridges the motor terminals in the event of an excessive pressure rise. This compares very favorably with the ever-increasing—though in some cases unnecessary—complication and high cost of the switching arrangements in high-tension power stations on the parallel system.

10. With the Thury system any class of supply can be given, and the motors can be made to drive sub-generators capable of running in parallel with any local electric generating plant.

11. In hydraulic generating stations where the variations of head are considerable, as will generally be the case if there is no storage reservoir, a greater all-round efficiency can be obtained than if the machines had to be driven at constant speed.

12. For any industrial operation requiring a variable-speed drive at constant torque, the Thury motor, without constant-speed regulator, is admirably adapted. It might have a useful application in the driving of generators supplying constant current to electric furnaces in which the voltage across electrodes is continually varying.

DISADVANTAGES OF THE DIRECT-CURRENT SERIES SYSTEM

1. The necessity of providing insulating floors and mounting all current-carrying machines and apparatus on insulators. The highly insulated coupling required to transmit, mechanically, large amounts of power between prime mover and electric generator is also objectionable.

2. The smallness of the generators; the output of each generator being limited by the line current and the permissible voltage between the collecting brushes on the commutator. One prime

mover is usually coupled to two or more direct-current generator units. This, however, is necessarily more costly than if larger electric generators could be used; moreover, it practically limits the choice of hydraulic turbines to the horizontal type, since the coupling of several generators on the shaft of a vertical waterwheel would be difficult and unsatisfactory.

3. With constant current on the line, the line losses are the same at all loads, and the percentage power loss in conductors is inversely proportional to the load. This is exactly the reverse of what occurs on the alternating-current parallel system, in which the percentage line loss is directly proportional to the load. It should, however, be mentioned that on the Thury system the line current may be reduced about 30 per cent. at times of light load, except when the circuit feeds motors of industrial undertakings requiring constant current day and night. It must not be overlooked that large percentage losses at times of light load are of serious moment only where steam engines are used or where storage reservoirs are provided for water-power generating stations. In the case of water-power schemes without storage, the fact of the full-load line losses continuing during times of light load is not objectionable.

4. The series system is less suitable than the parallel system for distribution of power in the neighborhood of the generating station. It is essentially a transmission system, not a distributing system.

5. The water turbine working under constant head is not the ideal engine for driving constant-current machines.

6. Special regulating devices are necessary to maintain constant speed on the motors.

7. It is impossible to overload the motor, even for short periods. This would be a very serious objection to the use of these motors in connection with electric traction systems.

8. Greater liability to damage and interruption from the effects of lightning. It may be said that an overhead line, whether for alternating or direct current, is always liable to damage by lightning; but with the high-tension alternating-current system, the transformers and automatic oil switches will usually protect the generators themselves from serious damage, while with the Thury system there is always a path for lightning discharges through the generators and motors, and the damage done may be very great. This simply means that particular attention

should be given to the question of lightning protection on overhead direct-current lines, and the ease with which highly inductive choke coils can be introduced on a direct-current system, without opposing any obstacle (except ohmic resistance) to the passage of the line current, tends toward the attainment of increased safety.

CHAPTER IX

MECHANICAL PRINCIPLES AND CALCULATIONS—OVERHEAD CONDUCTORS

123. Introductory.—If a wire is stretched between two fixed points, A and B, lying in the same horizontal plane, and separated by a distance of l ft., there will be a certain *sag* of s ft. in the wire. This sag or deflection from the horizontal line AB, will be greatest at the center of the span, and its value, for a given length of span (l ft.), will depend upon the weight of the wire and the tension with which it has been drawn up. If the wire were perfectly uniform in cross-section and perfectly flexible, the curve ADB (Fig. 88) would be a catenary. It should be observed, however, that in this and subsequent diagrams, the sag s is shown much larger relatively to the span l than it would

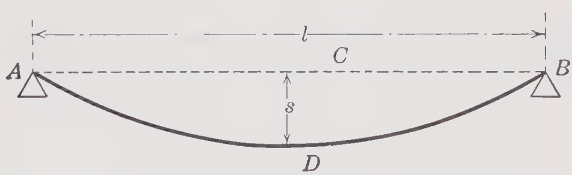


FIG. 88.—Wire hanging between two supports at the same elevation.

be on most practical transmission lines; and as the span l is generally very little shorter than the length of the wire between the suspension points A and B, no appreciable error is introduced by assuming the weight of the wire to be distributed uniformly along the horizontal line ACB instead of along the curve ADB. On this assumption the curve ADB becomes a parabola, and as the calculations are more easily made on the assumption of a parabolic curve than with the possibly more correct catenary, it is customary to use the formulas relating to the parabola for the solution of sag-tension problems. On very long spans, even with fairly large conductors, the actual curves assumed by the wires hanging in still air under the influence of their own weight only, will approximate more nearly to the catenary than to the

parabola, and if it is desired to introduce refinement of calculation, the catenary is the more correct curve to work to on long spans,[1] but owing to the many more or less arbitrary assumptions that must necessarily be made in all calculations on the mechanical features of a transmission line, such refinements are, in the writer's opinion, unnecessary and only justifiable when they involve no additional time or labor in the calculations.

124. Graphical Statics Applied to Transmission-line Calculations—General Problem.—Instead of assuming a definite shape of curve to represent the form taken by a wire which is free to hang between two supports a known distance apart, it is proposed to develop the necessary formulas for sag and tension calculations by applying the well-known principles of graphical statics.

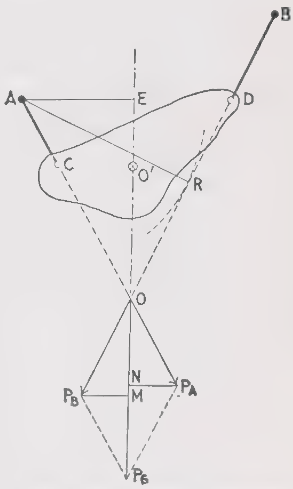


FIG. 89.—Mass suspended from two points—diagram of forces.

Consider a mass of any irregular shape, the weight of which may be represented by the vertical vector OP_G passing through its center of gravity O' as shown in Fig. 89. This mass is suspended by two perfectly flexible ties from the fixed points A and B. Except for the special case of parallel forces, the resultant P_G and the components P_A and P_B acting in the direction of the suspension cords, AC and BD, will pass through the common point O. The system of forces is in equilibrium and the conditions to be fulfilled are therefore,

1. That the vectorial sum of all forces and reactions shall be zero, and

2. That the sum of all moments taken about any point shall be zero.

[1] In his paper read at the annual Convention of the A. I. E. E. at Chicago, June, 1911, Mr. W. Le Roy Robertson works out solutions of sag and span problems with the aid of the catenary. A more recent contribution by Mr. F. K. Kirsten in *Trans.* A. I. E. E., p. 735, vol. xxxvi, 1917, contains a complete mathematical analysis of the mechanical problems on the assumption that the wires take the form of the catenary curve.

The following is an explanation of the manner in which a force diagram similar to Fig. 89 may be constructed.

Known Data.—1. The position of suspension points A and B.

2. The magnitude and position relatively to points A and B of the force of gravity (*i.e.*, the length of the vector PO_G and the distance AE of this vector from the point A).

3. The magnitude, but not the direction, of the force P_B acting through the suspension point B.

Required.—1. The direction of the force P_B.

2. The magnitude and direction of the force P_A.

3. The horizontal and vertical components of the forces at the points of support.

Taking moments about the point A gives the equation

$$P_B \times AR = P_G \times AE$$

thus

$$AR = AE \times \frac{P_G}{P_B}$$

From the point A as a center, use this radius to draw an arc of circle the tangent to which, passing through the point B, will locate the point O. Draw OP_G to the proper scale to represent the force of gravity and complete the parallelogram of forces. The vertical component of the reaction at point A is NO, and at the point B it is MO. The horizontal reactions at the points of support are P_AN and P_BM respectively. These are obviously equal, but opposite in direction, in every conceivable case of a body in equilibrium subject only to the force of gravity acting, as it always does, vertically downward.

125. Stretched Wire. Supports on Same Level.—In Fig. 90, a wire weighing w lb. per foot is stretched between the supports A and B lying in the same horizontal plane. The maximum tension in the wire is P_B lb. This is the tension at the points of support, and, owing to the symmetry of the figure, it is the same in amount at A as at B. The assumption is now made that the total weight of wire is equal to the weight per foot multiplied by the straight line distance between the points A and B. Thus

$$P_G = w \times l$$

where l is the distance in feet between A and B. This assumption is allowable on all except spans of extraordinary length, because the actual length of the wire differs only by a very small

amount from the shortest distance between the points of suspension.

Draw the vector OP_G to represent the total downward force wl. It will lie on a vertical line midway between A and B. Let it be bisected by any horizontal line such as AB. From O lay off OP_B at such an angle that the head P_B of the vector lies

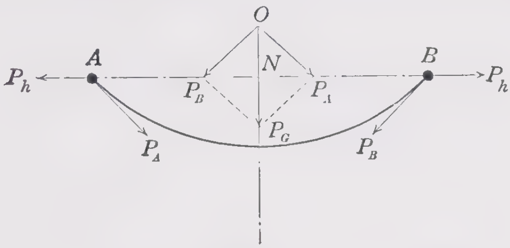


FIG. 90.—Diagram of forces—suspended wire with supports on same level.

on the horizontal line bisecting OP_G. Complete the parallelogram of forces. Then ON represents the vertical component at each point of support, and NP_B or NP_A is the horizontal force P_h acting at each support; it is also the total tension in the wire at the center or lowest point of the span.

Referring to Fig. 91, which shows a span of length l, we can now calculate the sag s at the center. Consider the moments

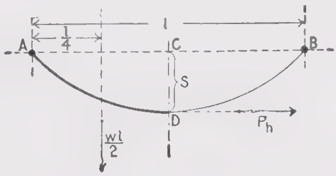


FIG. 91.—Diagram of forces for sag calculation—supports on same level.

about the point A due to the half span AD. These must balance, and the equation is

$$s \times P_h = \frac{l}{4} \times \frac{wl}{2}$$

whence

$$s = \frac{wl^2}{8P_h} \tag{101}$$

which is the well-known formula for sag calculations when the parabolic assumption is made.

Given an overhead conductor hanging in still air from supports

at the same level, under the influence of its own weight only, the tension in the wire at the center of the span will therefore be

$$P_h = \frac{wl^2}{8s} \tag{102}$$

and in practical work, except perhaps on exceptionally long spans, the maximum tension in the wire will be so nearly equal to the horizontal component of the pull that the tension at the points of support (where the maximum pull occurs) may be assumed the same as the horizontal component as calculated by formula (102). The tension at any point in the span can, however, be very easily calculated. Thus, let θ in Fig. 92 be the angle which the tangent to the curve makes with the horizontal

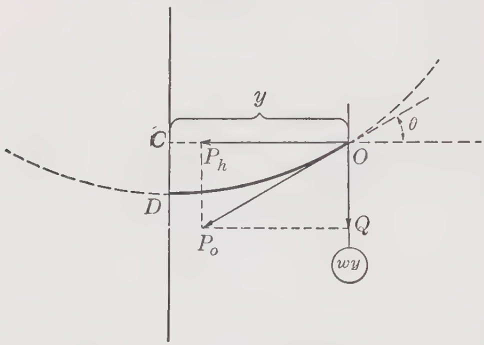


FIG. 92.—Vector diagram of forces for calculating tension at any point in the span.

at any point O distant y feet from the vertical plane normal to the wire at the lowest point. On the assumption previously made that the weight is distributed uniformly over the straight-line distance between the two points of support, the weight of the section considered in Fig. 92 is $y \times w$ where w is the weight of the conductor per foot length. This acts vertically downward at the point O, and we may write

$$\tan \theta = \frac{wy}{P_h}$$

Substituting for P_h the value given by formula (102), we have,

$$\tan \theta = y \times \frac{8s}{l^2} \tag{103}$$

which becomes a maximum at the points of support, where its value is

$$\tan \theta = \frac{4s}{l} \tag{104}$$

The wire will take the stress in the direction of the tangent to the curve, and the resultant tension at the point O is therefore

$$P_o = wy\frac{OP_o}{OQ}$$
$$= wy \times \text{cosec } \theta \tag{105}$$

which is readily solved with the aid of trigonometrical tables, since the angle θ is given by formula (103).

The use of tables can be avoided by putting the expression in the form

$$P_o = \frac{w}{8s}\sqrt{l^4 + 64\ s^2y^2} \tag{106}$$

the maximum value of which occurs at the points of suspension of the wire where it becomes

$$P_{max} = \frac{wl}{8s}\sqrt{l^2 + 16s^2} \tag{107}$$

A simpler formula, which is a very close approximation to the correct formula, is

$$P_{max} = P_h + ws$$
$$= \frac{wl^2}{8s} + ws \qquad (108)^1$$

[1] This formula is based on the fact that when the quantity a is small relatively to A, it is permissible to write,

$$\sqrt{A^2 + a^2} = A + \frac{a^2}{2A}$$

Thus, the total or resultant force at the point of support is,

$$P_{max} = \sqrt{P_h{}^2 + \left(\frac{wl}{2}\right)^2}$$
$$= \frac{wl^2}{8s} + \left(\frac{wl}{2}\right)^2 \times \frac{8s}{2wl^2}$$
$$= \frac{wl^2}{8s} + ws.$$

The ratio of the maximum tension (at point of support) to the tension at center of span is,

$$\frac{P_{max}}{P_h} = \frac{Formula(107)}{Formula(102)} = \frac{\sqrt{l^2 + 16s^2}}{l} \qquad (109)$$

$$\text{which, approximately} = 1 + \frac{8s^2}{l^2} \qquad (110)$$

Seeing that s is usually very small in relation to l, the quantity given by formula (109) or (110) is generally so nearly equal to unity that no serious error is introduced by using the simple formula (102) for calculating the tension in the conductors; in other words, the wire, as previously mentioned, is so nearly horizontal throughout its entire length that the horizontal component of the tension, instead of the resultant, may be considered as the tension acting at any point on the wire.

The length of the parabolic curve ADB (Fig. 88) is,

$$\lambda = l\left(1 + \frac{8s^2}{3l^2} - \frac{32s^4}{5l^4} +, \text{etc.}\right)$$

but it is usual to omit all except the first two terms of the series. This gives,

$$\lambda = l + \frac{8s^2}{3l} \qquad (111)$$

126. Supports at Different Elevations.—Many students and some engineers appear to have trouble in understanding the distribution of forces in overhead lines carried up a steep grade: the idea that the poles carry considerably increased loads as they occupy positions higher up the hill-side is not uncommon, and is sometimes put forward in explanation of the troubles which are not infrequent with poorly constructed lines carried up steep inclines. Such troubles as have occurred in the past were perhaps not entirely unconnected with the fact that the designing engineer, or the construction engineer, or both, had no clear understanding of the distribution of forces in such a line. The problem is very simple if the component forces necessary to the state of equilibrium are studied as in the preceding article wherein the supports were supposed to be at the same elevation.

In Fig. 93, the difference in elevation of the supports is h feet. The span measured horizontally is l feet, and the straight-line distance between the supports A and B is l' feet. If θ

is the angle which the line AB makes with the horizontal, we may write

$$\tan \theta = \frac{h}{l}$$

and

$$\cos \theta = \frac{l}{l'}$$

The weight of wire per foot is w lb., and it is assumed that the total weight is wl' lb. This force acts through the point C midway between A and B. The other known quantity is the magnitude (but not the direction) of the maximum tension in the wire: this is the force P_B acting through the highest point of support (B).

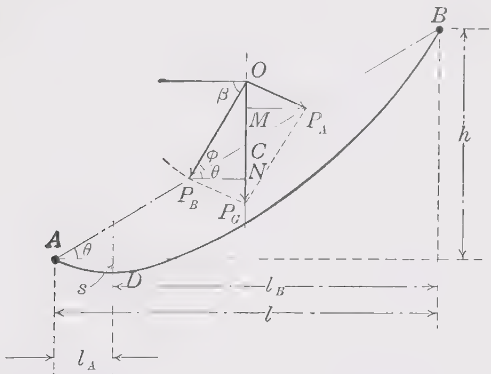


Fig. 93.—Diagram of forces—supports at different elevations.

Draw the vertical line OP_G to represent the total force of gravity (wl'). Through its center C draw the line AB (or a line parallel to AB) making an angle θ with the horizontal. Then from O as a center describe an arc of radius OP_B equal to the known force P_B. Its intersection with AC locates the head of the vector OP_B. Complete the parallelogram of forces, and drop the perpendiculars P_AM and P_BN on to the vertical OP_G. These are a measure of the (equal) horizontal components of the reactions at the two points of support, and also of the total tension in the wire at the lowest point D. The length MO is the vertical component of the reaction at the lower support A; while NO is the vertical reaction at B. Their sum is, of course, equal to OP_G. The angle β which the vector P_B makes with the horizontal is the slope of the wire where it leaves the point B.

The construction as above described satisfies the fundamental conditions of equilibrium, namely, that all vertical forces shall balance; that all horizontal forces shall balance, and that the sum of all moments taken about any point shall be equal to zero.

In practice it is not convenient to solve such problems by actual measurement of quantities plotted to scale on drawing paper, because the horizontal components of the forces are usually very much greater than the vertical components. A trigonometrical solution is therefore desirable.

In Fig. 93, the known quantities are the two sides OP_B and OC of the triangle OCP_B of which the angle OCP_B is also known, being equal to $\theta + 90$ degrees.

The angle OP_BC or φ may be calculated from the relation

$$\sin \varphi = \frac{OC}{P_B} \sin (90 + \theta) = \frac{wl'}{2P_B} \cos \theta$$
$$= \frac{wl}{2P_B}$$

and the angle β which the wire at the upper support makes with the horizontal is therefore

$$\beta = \theta + \sin^{-1}\left(\frac{wl}{2P_B}\right) \tag{112}$$

With the aid of trigonometrical tables or a slide rule, the values of $\sin \beta$ and $\cos \beta$ can be obtained, and the other components of the force diagram readily calculated. Thus, the horizontal component at either point of support, which is also the total tension in the wire at the point (if any) where the slope is zero, is

$$P_h = P_B \cos \beta \tag{113}$$

The vertical component of the reaction at the highest point B, where this reaction is greatest, is

$$P_{BV} = NO = P_B \sin \beta \tag{114}$$

The weight supported by the lower pole is

$$P_{AV} = MO = wl' - P_{BV} \tag{115}$$

This may, obviously, be a negative quantity, *i.e.*, the wire may exert an upward pull on the lower support. In that case the sag s below the point A, will be zero, and this would correspond to the usual condition on a steep incline, or on a moderate incline if the spans are short.

Position of Lowest Point of Span. Supports at Different Elevations.—The position of the lowest point D in the span will be determined by the vertical weight carried by each of the two supports. Thus, referring to Fig. 93, the vertical component of the forces acting at B is simply the weight of that portion of the wire comprised between the support B and the point D where the tension in the wire has no vertical component. The horizontal distance of the point D from B (in feet) is

$$\begin{aligned} l_B &= l \times \frac{ON}{OP_G} \\ &= \frac{lP_{BV}}{wl'} \\ &= \frac{P_{BV} \cos \theta}{w} \end{aligned} \tag{116}$$

This may give a distance for l_B which is greater than l. In that case the support A would be the lowest point in the span. The horizontal distance from the lower point of support, A, to the point D is $l_A = l - l_B$; which shows that l_A will be a negative quantity when l_B is greater than l.

If it is desired to express these formulas in terms of the horizontal component (P_h) of the tension, a reference to the force diagram in Fig. 93 will show that we may write

$$P_G = 2\ (P_{BV} - P_h \tan \theta)$$

and since $P_G = wl'$, the vertical component of the tension at the upper support is

$$\begin{aligned} P_{BV} &= \frac{wl'}{2} + P_h \tan \theta \\ &= \frac{wl'}{2} + \frac{hP_h}{l} \end{aligned} \tag{117}$$

Substituting in formula (116), we have:

$$l_B = \frac{l}{2} + \frac{hP_h}{wl'} \tag{118}$$

Similarly

$$l_A = \frac{l}{2} - \frac{hP_h}{wl'} \tag{119}$$

These formulas can be solved without reference to trigonometrical tables, because, although $l' = \frac{l}{\cos \theta}$, it can also be expressed as $l' = \sqrt{l^2 + h^2}$.

17

Formula (119) shows that if $\frac{hP_h}{wl'}$ is equal to $\frac{l}{2}$, the lowest point of the wire coincides with the lower support A, while, if the second term in the equation is greater than the first, l_A is negative, and there is a possibility of the resultant *upward* pull on the lower insulator at A being greater than the downward pull due to the weight of the wires in the adjoining span. It is well to bear this point in mind when considering an abrupt change in the grade of a transmission line.

127. Calculation of Sag with Supports on an Incline.—The formula (101) as calculated for spans with supports on the same level may be used for spans on an incline provided the distance l_B of Fig. 93 is considered as half of a level span of which the sag is $(s + h)$. Thus,

$$(s + h) = \frac{wl_B{}^2}{2P_h \cos \theta} \tag{120}$$

Similarly

$$s = \frac{wl_A{}^2}{2P_h \cos \theta} \tag{121}$$

On a steep incline, where A is the lowest point of the span, it is more useful to know the maximum deflection of the wire from the straight line AB as observed by sighting between the points A and B. This maximum deflection will occur at the center of the span. A careful study of the force diagram in Fig. 93 will make it clear that, just as OP_B is the slope of the tangent to the wire at the point B, and P_BN is the slope of the tangent to the wire at the lowest point D, so P_BC is the slope of the tangent to the curve at the middle of the span. This is therefore the point of maximum deflection from the straight line AB.

Consider now Fig. 94. The maximum deflection of the line on the slope is s', and, by taking the sum of all the moments about A and putting this sum equal to zero, the value of this deflection is found to be,

$$s' = \left(\frac{wl'}{2} \times \frac{l}{4}\right) \times \frac{\cos \theta}{P_h}$$

$$= \frac{wl^2}{8\,P_h} \tag{122}$$

which indicates that the maximum deflection from the straight line, of a conductor strung between supports on a slope, as measured at the center of the span, is exactly the same as the maxi-

mum sag s of the same conductor strung between points on the same level; provided the span measured horizontally and the horizontal component (P_h) of the tension are the same in both cases.

If P is the tension $\left(\frac{P_h}{\cos\theta}\right)$ in the direction AB, the formula (122) can be written

$$s' = \frac{wl^2}{8P\cos\theta} \tag{122a}$$

Length of Wire with Supports at Different Elevations.—The length of wire between the two supports A and B may be considered as the sum of two distinct portions of the parabolic curve ADB

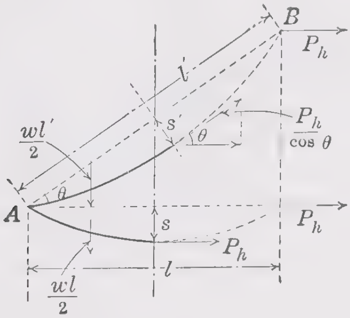


FIG. 94.—Diagram of forces for sag calculation—supports at different elevations.

(Fig. 93); the one, AD, the length of which, according to formula (111), is

$$\lambda_A = l_A + \frac{4s^2}{3 \times 2l_A}$$

and the other, DB, of length

$$\lambda_B = l_B + \frac{4(s+h)^2}{3 \times 2l_B}$$

The sum of these two quantities is

$$\lambda = l + \frac{2}{3}\left[\frac{(s+h)^2}{l_B} + \frac{s^2}{l_A}\right] \tag{123}$$

128. Example Illustrating Use of Formulas.—Consider a span of 485 ft. measured horizontally, with a difference of level of 40 ft. between points of support. The wire is No. 2-0 B. & S. gauge aluminum, weighing 647 lb. per mile. Calculate vertical and horizontal components of forces, also sag and position of lowest point of span, if the maximum tension in the wire is 185 lb.

The known quantities are:

$$l = 485$$
$$h = 40$$
$$w = 0.1226$$
$$P_B = 185$$

The unknown quantities are calculated as follows:

$$\tan\theta = \frac{40}{485} = 0.0825$$
$$\theta = 4^\circ\, 43'$$
$$\cos\theta = 0.9966$$
$$\sin\varphi = \frac{485 \times 0.1226}{2 \times 185} = 0.161$$
$$\varphi = 9^\circ\, 15'$$

By formula (112) $\beta = \theta + \varphi = 13^\circ\, 58'$

$$\cos\beta = 0.971$$
$$\sin\beta = 0.2413$$

By (113) $P_h = 185 \times 0.971 = 180$ lb.

By (114) $P_{BV} = 185 \times 0.2413 = 44.6$ lb.

By (116) $l_B = \dfrac{44.6 \times 0.9966}{0.1226} = 364$ ft.

By (120) $(s + h) = \dfrac{0.1226 \times (364)^2}{2 \times 180 \times 0.9966} = 45.2$ ft.

Thus the sag below the bottom support will be 5.2 ft. when the maximum tension is 185 lb.

If it is desired to avoid the use of trigonometrical tables, the procedure would be as follows; but the assumption now made is that the known tension is the horizontal component, or $P_h = 180$ lb.

The straight-line distance between the two points of support is $l' = \sqrt{l^2 + h^2} = \sqrt{(485)^2 + (40)^2} = 486.5$ ft. (approx.).

By formula (117)

$$P_{BV} = \frac{0.1226 \times 486.5}{2} + \frac{40 \times 180}{485} = 44.65 \text{ lb.}$$

By formula (118)

$$l_B = 242.5 + \frac{40 \times 180}{0.1226 \times 486.5} = 363 \text{ ft. (approx.)}$$

and

$$l_A = 485 - 363 = 122 \text{ ft.}$$

The sag s measured from the lowest support is given by formula (121) wherein $\cos\theta$ can be replaced by $\dfrac{l}{l'}$. Thus,

$$s = \frac{0.1226\,(122)^2 \times 486.5}{2 \times 180 \times 485} = 5.1 \text{ ft.}$$

which is probably more accurate than the dimension 5.2 obtained in the preceding calculation by subtracting h from the total sag (measured from the upper support). No high degree of accuracy is claimed for the results of these numerical examples, the calculations being made on the slide-rule.

129. Conclusions—Overhead Lines on Steep Grade.—Except for the fact that the pole foundations must be wisely chosen, and special attention paid to the setting of the poles, there are no engineering difficulties in the way of running an overhead line up a steep incline. It may be well to reduce the length of span measured horizontally so that this is equal to about $l' \cos \theta$, where l' is the distance between supports on level ground, and θ is the angle which the average slope of the line makes with the horizontal. This will ensure that the vertical load to be supported by the insulators does not exceed the vertical load on the level runs; but no difficulty need be experienced in arranging for each pole to take its proper share of the weight of the wire (with or without ice loading).

The stringing of the wires may be done with the aid of a dynamometer; or the deflection, as calculated by formula (122), can be measured by sighting from pole to pole. If the poles or suspension insulators of a line carried on an incline are deflected from the vertical owing to the unbalancing of horizontal forces, at the time when the wires are strung, the effect must be attributed to incompetence on the part of the engineer in charge of construction, and not to any unexplained flaw in the known laws of equilibrium. If the conductor is tied to the insulator at the proper point, the horizontal components of the forces at this point will balance, and the only forces which the insulators will have to resist, when the conductors are hanging under normal conditions in still air, are those due to gravity acting vertically downward: there should be no unbalanced forces tending to move the point of support in the direction of the line, either up or down the hill-side.

130. Effect of Temperature Variations on Overhead Wires.—If a is the temperature coefficient of the material of the conductor (see the table of constants in Chapter IV, page 75), then the length of the wire when the temperature is raised t degrees is,

$$\lambda_t = \lambda_o (1 + at) \tag{124}$$

in which λ_o is the original length of the wire.

This formula assumes that the wire is unstressed, or that the stress remains unaltered notwithstanding the increase in temperature. This, however, is not the case with an overhead conductor. As indicated by formula (102), the tension in a wire suspended horizontally between two fixed supports is almost exactly proportional to the square of the span, and inversely proportional to the sag at center of span. The effect of temperature variation is to alter the length of wire and therefore the amount of sag and tension. With a reduction of temperature, the length of wire will decrease; this will cause an increase in the tension, but owing to the fact that the wire will stretch under the influence of the increased tension, the sag at the lower temperatures will be somewhat greater than it would be if there were no elongation of the wires with increase of stress. All sag-temperature calculations, whatever the method adopted, must therefore take into account not only the effect of elongation with increase of temperature, but also the effect of the elastic contraction of the wire with increase of sag.

If P is the tension in a wire of cross-section A, and if M is the elastic modulus (as given for various materials in the Table of Article 41, Chapter IV, p. 75), then the elongation of the wire due to the tension P is,

$$\lambda_e = \frac{P \times \lambda}{A \times M}$$

the original length of the wire being λ.

If instead of $\frac{P}{A}$ the letter S be used to denote the stress in pounds per square inch of cross-section, the formula becomes

$$\lambda_e = \lambda \times \frac{S}{M} \tag{125}$$

It is customary to assume that the material of the conductors is perfectly elastic up to a certain critical stress known as the elastic limit; that is to say, if the application of a certain stress produces a strain represented by λ_e, it is assumed that, on the removal of the stress, the conductor will contract to its original length λ, and that this process of elongation and contraction follows a straight-line law. This is not scientifically correct, because, on removal of load, the amount of contraction is not directly proportional to the decrease of stress; but the departure from the straight-line law is not considerable, and no serious error is introduced by disregarding refinements of this nature.

A matter of greater importance is the fact that, when stranded conductors are used, the ratio between stress and strain is not correctly given by the modulus M as calculated from tests made on solid wires. The modulus for stranded cables will depend upon the number of strands, the "lay" of the strands, whether the central core is of metal or hemp, etc., and it should be determined by actual tests on samples of the completed cable; but since it will depend largely on the stress to which the cable has been subjected, and will ultimately differ little from the coefficient for solid wires, it is usual in sag calculations to use the same value of M for both solid and stranded conductors. Great accuracy in sag-temperature calculations is only necessary in the case of very long spans (1000 feet, and over).

A very simple and convenient diagram may be constructed, by which the elongation of a conductor owing to change of stress or temperature, acting either singly or jointly, can readily be ascertained. Fig. 95 shows such a diagram, constructed for stranded conductors of either copper or aluminum. Referring to the vertical scales from left of diagram to center, if a straight line be drawn from a point on scale 1 corresponding to the stress in a *copper* conductor, to a point on the center scale corresponding to the temperature, the point at which this line crosses scale 2 will indicate the increase in length above the condition of zero stress and zero temperature. This increase in length is expressed as a percentage, such as feet per hundred feet of the conductor.

As an example, an increase in length of 0.1 per cent. of a copper conductor will occur

(a) When the stress remains constant and the temperature increases 104°.

(b) When the temperature remains constant and the stress increases 15,000 lb. per square inch.

(c) When the stress *increases* from 12,000 to 15,500 lb. per square inch at the same time as the temperature falls from 137 to 6° F.

The vertical scales 4 and 5 are used in a similar manner with the common temperature scale for determining the changes in length of stranded aluminum conductors.

The value of M for stranded aluminum conductors which has been used in constructing the diagram Fig. 95, is 7,500,000 instead of the value $M = 9{,}000{,}000$ which is more frequently used in sag-

temperature calculations. If it is desired to use the larger figure for this modulus, the stress as read off the center scale for *Aluminum*, must be multiplied by 1.2.

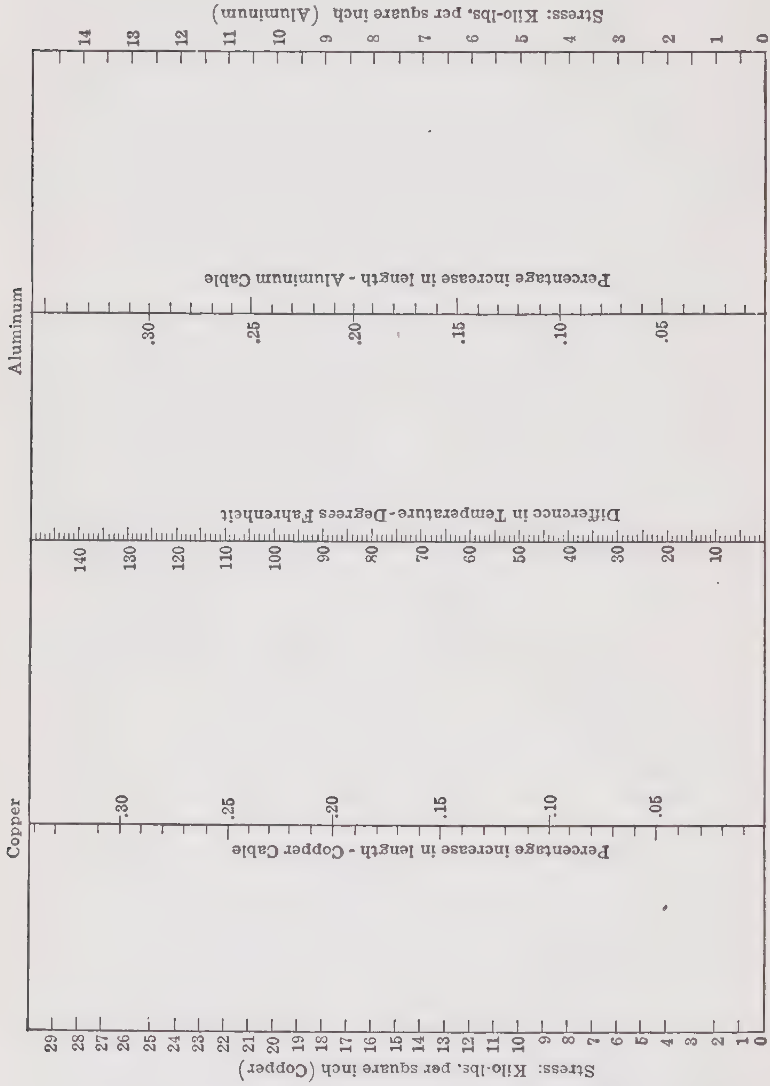


FIG. 95.—Chart for calculating changes in length of overhead conductors.

131. Abnormal Stresses in Wires Due to Wind and Ice.—When a wire hangs between horizontal supports in still air under

the influence of its own weight only, the tension at center of span for a definite distance between supports and a definite sag at center will be proportional to the weight per foot length of the wire. When loaded with sleet or ice, or subject to wind pressures, or a combination of both these additional loads, the calculations of the proper sag to allow are based on the assumption that the weight per foot is no longer that of the wire only, but n times this amount; where n is a factor depending upon the character and amount of the abnormal loading that the wire is likely to be subjected to.

Ice and Sleet.—The effect of snow and sleet adhering to the wires, and forming an ice coating of variable thickness, is to add to the dead weight of the wires and offer an increased surface to wind blowing across the line. Sleet will generally collect with slightly greater thickness near the lowest point of the span, but it is usual to assume that the extra vertical loading is uniformly distributed over the whole length of the span. Mr. Max N. Collbohm[1] says that in the winter of 1908–9, in Wisconsin and neighboring States, the snowstorm of January 28 covered nearly all overhead wires with sleet and snow 2½ in. to 4 in. diameter. The temperature went down to 4° F. below zero, while a wind velocity of 40 miles per hour was recorded. These conditions were, of course, exceptional; at the same time an average coating of sleet and snow weighing ½ lb. per foot of wire is not unusual. A coating of ice ½ in. thick on wires running through districts where sleet and low temperatures are common is generally allowed for in calculations. Sometimes this is increased to a radial thickness of ¾ in. Sleet deposits 6 in. in diameter have actually been observed on some of the steel conductors of the Central Colorado Power Company. The weather conditions in some of the passes through which this line is carried are, however, particularly severe.

Taking the weight of sleet at 57.3 lb. per cubic foot, the total weight per foot of the loaded conductor is $w + 1.25t\,(d + t)$ where w is the weight of the wire; t, the radial thickness of sleet (assumed to be of circular section), and d, the diameter of the uncoated wire; these dimensions being expressed in inches. Thus, in the case of a No. 0 B. & S. gauge wire, d equals 0.325 in., and assuming t equals 0.5 in., the weight of the ice coating is $1.25 \times 0.5 \times 0.825 = 0.516$ lb., or about ½ lb. per foot run,

[1] *Electrical World*, New York, March 25, 1909, p. 734.

which is a common allowance to make for ice loads on wires of average size.

Effect of Material of Conductor on Sleet Deposits.—It is frequently contended that sleet does not deposit readily on aluminum owing to the greasy character of the oxide which forms on the surface of aluminum conductors. The experience of many engineers does not, however, confirm this. Mr. W. T. Taylor says that copper collects a little more sleet than aluminum. He bases this statement on observations on long-distance telephone lines in California, where copper and aluminum conductors run side by side. Mr. E. H. Farrand[1] has observed copper wire accumulate snow to the extent of 3 in. in diameter in a few minutes, while iron wire took a coating ½ in. to 1 in. thick; but after a short time the wires of both materials had accumulated snow to the same diameter. Messrs. R. B. Matthews and C. T. Wilkinson[2] have made many observations in the sleet districts of the United States, and they are of opinion that an aluminum conductor will collect as much sleet and ice as copper or steel wires.

When wires are hung vertically one above the other, in the manner frequently adopted in the earlier constructions for double-circuit lines, there is the possibility of sleet falling off one of the lower wires, while the upper wire remains heavily loaded and with considerable sag. This might cause the lower wire to rise into contact with the upper wire. With this possibility in mind, the disposition or spacing can be made so that short-circuits due to this cause are not likely to occur.

Sleet storms are not infrequently followed by low temperatures and high winds. The loading—especially when the conductors are of small diameter—is then liable to be so great that it may be false economy to guard against it by increasing the factor of safety. It is probable that means will be adopted in the future to prevent the formation of sleet deposits on overhead power conductors by passing sufficient current through them to raise the temperature above that at which the deposit will form. On many systems it is possible to increase the current in the lines by controlling the power factor of the load. Users of power might assist in maintaining service during severe sleet storms by under exciting synchronous motors connected to the transmission

[1] *Journal* of the Inst. E. E., p. 659, vol. 46 (1911).

[2] "Extra High Pressure Transmission Lines." *Jour.* Inst. E. E., p. 573, vol. 46 (1911).

system. If sufficient current cannot be obtained by such means, the procedure on systems where there is a duplicate line would be to throw all the load on one line and pass a suitable current through the spare line which would be disconnected from the load and short-circuited at the receiving end.

The watts per square inch of cooling surface necessary to raise the temperature sufficiently to prevent sleet deposits will depend largely upon whether or not there is much wind at the time of the sleet storm; but something under $\frac{1}{4}$ watt per square inch should suffice. This is a problem in connection with which data obtained from various districts where sleet deposits are common would be valuable.

Wind Pressures.—The pressures due to winds of high velocity acting on poles and wires in a direction approximately at right angles to the transmission line are of great importance, both where sleet formation is possible, and in districts where sleet cannot form. In the latter case, the velocity of the wind is frequently greater than in the colder districts; but, on the other hand, a moderate wind acting on the larger diameter of ice-coated wires will generally lead to the greatest stressing of the conductor material. The maximum wind velocities rarely occur at the lowest temperatures, but falling temperatures and rising wind are not unusual after sleet storms. In mountainous districts a transmission line may be subjected at certain points to gusts of wind blowing almost vertically downward; the pressure in such a case, being directly added to the weight of wire and the ice load, may lead to more serious results than an even stronger wind blowing horizontally across the line.

Numerous observations have been made on wind pressures, and it is found that the pressure exerted on small surfaces is proportional to the square of the wind velocity. It will also depend to some small extent upon the density of the air, or the barometric pressure; but the correction for barometric pressure is usually not worth making.

Wind Velocity.—It is well to distinguish between indicated and true wind velocities. The United States Weather Bureau observations are made with the cup anemometer, and wind velocities over short periods of time are calculated on the assumption that the velocity of the cups is one-third of the true velocity of the wind, for great and small velocities alike. This assumption is not justifiable, and a correction must therefore be

made in order to convert the Weather Bureau recorded velocities into true velocities. The actual wind velocities corresponding to definite indicated velocities, as given by the U. S. Weather Reports, are as follows:

Wind Velocity: Miles per Hour

Indicated	Actual
10	9.6
20	17.8
30	25.7
40	33.3
50	40.8
60	48.0
70	55.2
80	62.2
90	69.2
100	76.2

Unless otherwise stated, when wind velocity is referred to, this must be understood to be the true velocity.

Formulas for Wind Pressures.—The formula proposed by the U. S. Weather Bureau (Professor C. F. Marvin), giving pressure in pounds per square foot on small flat surfaces normal to the direction of the wind is:

$$F = 0.004 \frac{B}{30} V^2 \tag{126}$$

where B is the barometric reading in inches, and V is the wind velocity in miles per hour. Other formulas are:

Langley	$F = 0.0036V^2$
Smeaton, Buck	$F = 0.005V^2$

In the case of cylindrical wires, the pressure per square foot of projected area is less than on flat surfaces. The formula proposed by Mr. H. W. Buck,[1] and generally conceded to be correct is:

$$F = 0.0025V^2 \tag{127}$$

where F is the pressure per square foot of projected surface of a cylindrical wire. A more convenient form of expression for this relation is:

$$p = \frac{dV^2}{5000} \tag{128}$$

[1] In paper read at the World's Fair, St. Louis, 1904.

where p is the pressure per foot length of the wire and d is the diameter of the wire in inches; and the denominator, although not exactly as obtained from Mr. Buck's equation, is an easily remembered round number, which is very close to the average of many experimental results. The upper curve in Fig. 96 has been plotted from Mr. Buck's formula for cylindrical wires; while the lower curve for pressures on flat surfaces gives the relation between pressure and wind velocity according to Professor Langley's formula.

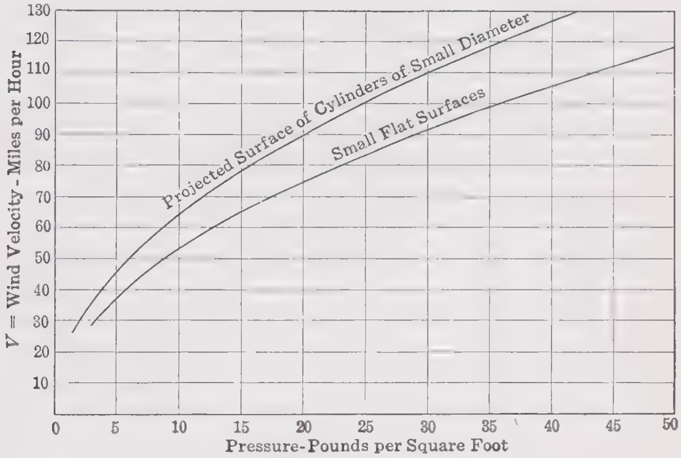


FIG. 96.—Curves giving relation between wind velocity and resulting pressure.

Relation between Wind Velocity and Height above Ground.—Owing to the resistance offered by the ground surface, the force of the wind is not so great near the ground as at higher altitudes, and greater maximum wind pressures on wires must be allowed for when the line is carried on high steel structures than in the case of the average wood pole line, as used for the lower voltage transmissions.

Mr. F. F. Fowle[1] has given valuable particulars based on maximum wind velocities at different elevations, observed in Chicago, from which the curve of Fig. 97 has been plotted. It will be seen that, at the average elevation of transmission line wires (25 to 45 ft.), the probable maximum wind velocity is less

[1] "A Study of Sleet Loads and Wind Velocities." *Electrical World*, New York, October 27, 1910.

than half what it would be at 100 ft. above ground, and only about one-third of what may be expected at an elevation of 300 ft.

Mr. Fowle suggests that overhead line calculations might be based on a maximum wind velocity of 47 miles per hour for ordinary steel tower construction, and 40 miles for wood pole lines. These are probably safe limits, especially in climates where this maximum wind pressure is considered as acting on ice-coated wires. In exposed positions, and where the line runs

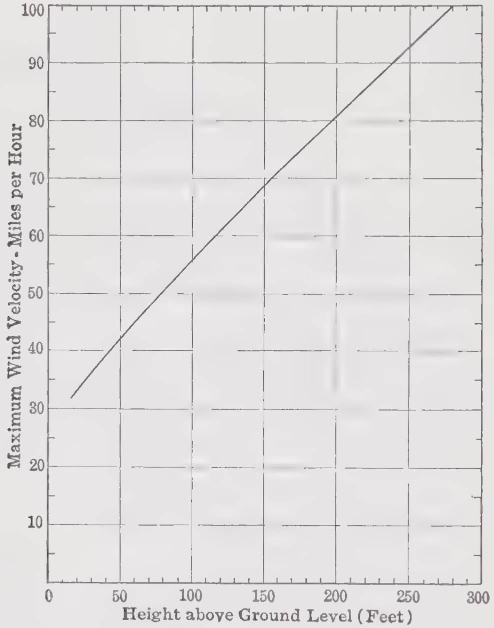


FIG. 97.—Maximum wind velocities at different elevations.

through wide stretches of open country, it is well to allow a maximum of 60 for steel tower lines, and 50 for wood pole lines.

According to Mr. Fowle, the wind velocity very rarely exceeds 50 miles in Chicago, the maximum recorded during a period of thirty-six years being 84, while 90 miles per hour has been recorded in Buffalo. Wind velocities of 53, and on one occasion 60 miles, have been recorded during sleet storms; but such velocities are exceptionally high.

The revised British Board of Trade requirements for the calculation of pole lines are a limit of 25 lb. per square foot on flat surfaces, and $25 \times 0.6 = 15$ lb. on the projected surface of

cylindrical poles and wires. No allowance for ice coating is made; but since it is specifically stated that no allowance is to be made for the elasticity of the wires, and the factors of safety insisted upon are 5 for wires, 6 for steel poles, and 10 for wood poles, there is very little danger of electric power lines being damaged by storms in England. Whether or not such stringent regulations are necessary, or likely to encourage the development of power transmission schemes, is another question, the answer to which is generally known.

Calculations based on British B. O. T. requirements are made for a temperature of 22° F. At this temperature, the sag of a No. 2/0 copper conductor on a 600-ft. span would be 24.6 ft. to satisfy the requirements. At 122° F. (the approximate maximum summer sun temperature in England) the sag of the same wire would be 33 feet. With aluminum conductors the conditions would be worse.

In America, where the weather conditions are more severe, the larger and more economical spans are very common; but there are few interruptions to service due to broken wires notwithstanding the smaller factors of safety as usually employed on this side of the Atlantic. The fact that an overhead transmission line is a very flexible and elastic structure appears frequently to be overlooked by the inventors whose genius finds an outlet in the framing of rules and regulations.

The Committee on Overhead Line Construction, appointed by the National Electric Light Association of New York, assumes a half-inch ice coating for all sizes of conductors, and maximum wind velocities of 50 to 60 miles per hour. This committee states that 62 miles is a velocity not likely to be exceeded during the cold months.

Three classes of loading are considered by the Joint Committee on Overhead Crossings, particulars of which are as follows:

Class of loading	Vertical component	Horizontal component, lb. per sq. ft. of projected area
A.	dead	15
B.	dead + ½-in. ice	8
C.	dead + ¾-in. ice	11

For the class *B* loading the ordinary range of temperature is given as −20 to 120° F. For the calculation of pressures on

supporting structures the requirements are 13 lb. per square foot on the projected area of closed or solid structures, or on $1\frac{1}{2}$ times the projected area of latticed structures. The same Joint Committee allows a safe stress on copper 50 per cent. of ultimate breaking stress—that is to say, the wires may be stressed to a point very near to the elastic limit.

The proposed National Standards, as formulated by the Bureau of Standards, U. S. Dept. of Commerce[1] cover three grades of overhead construction:

(a) power lines crossing railways;
(b) power lines crossing unimportant railways;
(c) power lines in country districts where the risk of accident from falling wires is small.

Three classes of loading are also considered:

(1) *Heavy loading:* the resultant, at a temperature of 0° F., due to dead weight of wire plus $\frac{1}{2}$ in. radial thickness of ice, with horizontal wind pressure of 8 lb. per sq. ft. of the projected surface of the ice-coated wire.

(2) *Medium loading:* at a temperature of 15° F., a total load equal to $\frac{2}{3}$ of (1) with a minimum of 1.25 times the dead weight of the conductor (without ice covering).

(3) *Light loading:* at a temperature of 30° F., a total load equal to $\frac{2}{3}$ of (2) or $\frac{4}{9}$ of (1) with a minimum of 1.25 times the dead weight of the conductor (without ice covering).

The Board of Railway Commissioners for Canada specifies for H. T. wires crossing railways, a factor of safety of 2 when wires are coated with ice or sleet to a depth of 1 in., and subject to wind pressure of 100 miles per hour. This combination of abnormal loads being more or less imaginary, it is probable that the factor of safety is actually about 6, which appears to be unnecessarily high.

A more reasonable specification for railroad crossings—although probably unduly severe, and therefore leading to unnecessary capital outlay—is that of the New York Central & Hudson River Railroad, which provides for $\frac{1}{2}$ in. ice coating with a wind pressure of 20 lb. per sq. ft. of projected area, and a limiting stress equal to $\frac{4}{10}$ of the ultimate stress of the wire.

[1] Taken from the *National Electric Safety Code*, second edition, Nov. 15, 1916.

132. Swaying of Wires in Strong Winds.—If a transmission line is well designed and constructed, all the wires of one span will generally be found to swing synchronously in any wind. Under exceptional conditions, however, trouble is liable to occur through wires swinging together, even when all details of design and construction have received careful attention. Troubles of this description are more likely to be met with when the spans are long and the sag in the wires necessarily large, and for this reason the spacing between wires must increase with increase of span length, irrespective of voltage considerations. Copper conductors are decidedly less likely to swing out of synchronism than aluminum conductors; not only because the latter have usually to be strung with a greater sag, but also because of the lightness of the material. Aluminum conductors of small section are easily shaken by sudden gusts of wind, and a little difference in sag will in all probability lead to non-synchronous swinging. It must not be overlooked that wires, after erection, do not always remain equally taut. This may be due to many causes, such as a slight slipping in the ties, straining of insulator pins on cross-arms, unequal ice loading, or local faults in the wires themselves. Again, it has been observed that during snow storms all the wires do not always become coated to an equal extent, and such a want of uniformity in the ice coating may well lead to wires being blown together in a strong wind.

On the high-tension transmission system of the Central Colorado Power Company, with spans averaging 730 ft., the lines cross some very exposed positions at the openings to canyons, and the excessively strong winds that occur at such points have been known to mix up the conductors. It was found necessary to dead-end the line at each tower, guy the towers, and increase the tension in the wires to a point near the elastic limit of the material, steel being used where necessary in lieu of copper.

133. Calculation of Total Stress in Overhead Wires.—The formula (101) of Article 125 gives the relation between tension, sag, and weight for a wire strung between supports a known distance apart. Thus, if the tension, P, is known or assumed, the sag can readily be calculated. We shall now consider how this tension can be computed, not only when the wire carries an increased vertical load in the form of ice deposit; but also when the effect of wind blowing across the line increases the total stress.

If w_r is the total loading in lb. per foot run of the wire, and w

is the weight in lb. per foot length of the unloaded wire, then $w_r = nw$, where n is a multiplier which takes account of the extra load on the wire under the most severe weather conditions likely to be encountered in the district where the transmission line is erected.

It is usual to assume that the wind pressure acts in a horizontal direction and that the total load on a conductor is the resultant of two forces, one acting vertically downward due to weight of wire together with added weight of sleet or ice, if any, and one acting horizontally due to the wind pressure. These forces are indicated in Fig. 98 where Op represents the wind pressure, Ow the weight of the conductor, and ww_1 the added weight of ice. The resultant pressure Ow_r is equal to $\sqrt{p^2 + w_1^2}$. If the line runs through a country where sleet does not form on the wires the maximum resultant pressure is Or instead of Ow_r if the assumed maximum force due to wind is the same in both cases.

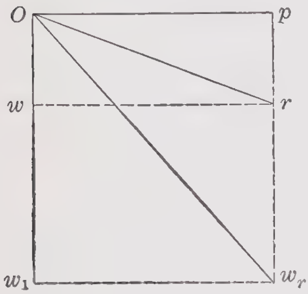


FIG. 98.—Vector diagram of forces acting on overhead wires.

The diagram Fig. 99 gives values of the multiplier n (*i.e.*, the ratio $Or \div Ow$ of Fig. 98) corresponding to various wind velocities for standard sizes of solid copper conductors on the assumption that there can be no ice formation on the wires, while Fig. 100 gives values of n (*i.e.*, the ratio $Ow_r \div Ow$) for copper conductors when the weight is increased by a coating of ice 0.5 in. thick with a correspondingly greater wind effect due to the increased diameter. The curves of Figs. 101 and 102 give similar relations but for conductors of aluminum instead of copper.

The formula used for the calculation of wind pressure in connection with these diagrams is

$$p = dV^2 \div 4820$$

where d is the diameter in inches of the conductor or of the ice coating, as the case may be; V is the actual wind velocity in miles per hour, and p is the wind pressure in pounds per foot length of conductor.

This is the more correct form of the formula (128) already given. When using the diagrams, it should be noted that the

distances plotted horizontally represent the squares of the wind velocities, and the sizes of the conductors are expressed in equivalent B. & S. gauge numbers or in circular mils for the larger sizes.

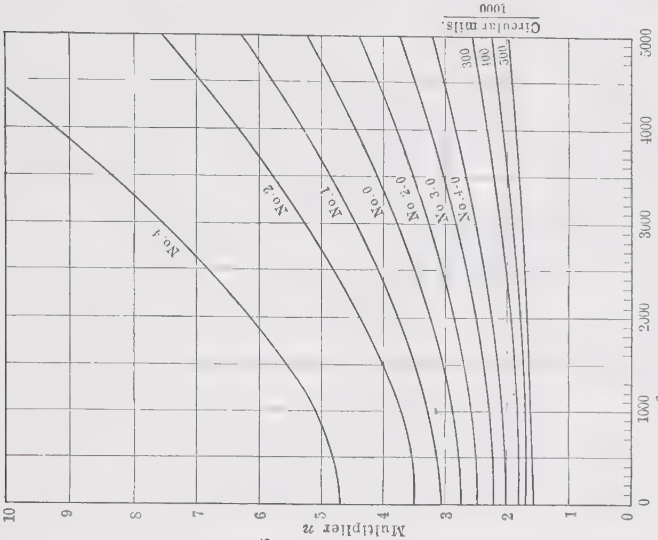


FIG. 100.—Chart giving factor n for ice-coated copper conductors.

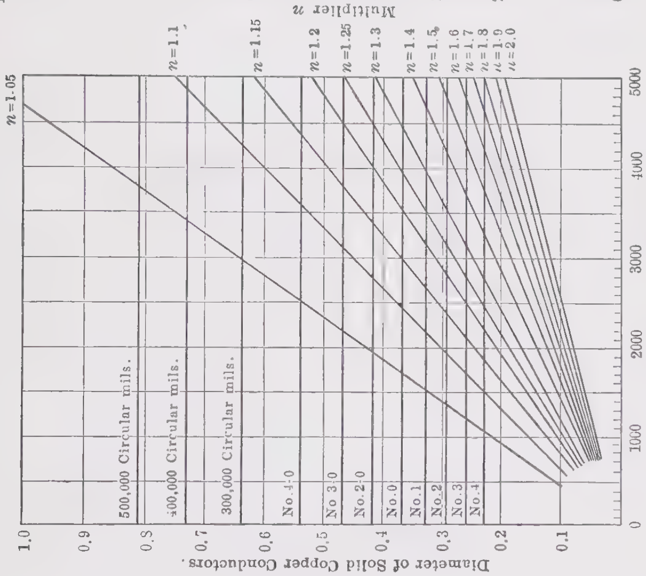


FIG. 99.—Chart giving factor n for copper conductors without ice loading.

The curves of Fig. 99 are correct for solid copper wires: the values of n for stranded conductors would be somewhat greater because of the larger surface presented to the wind for the same

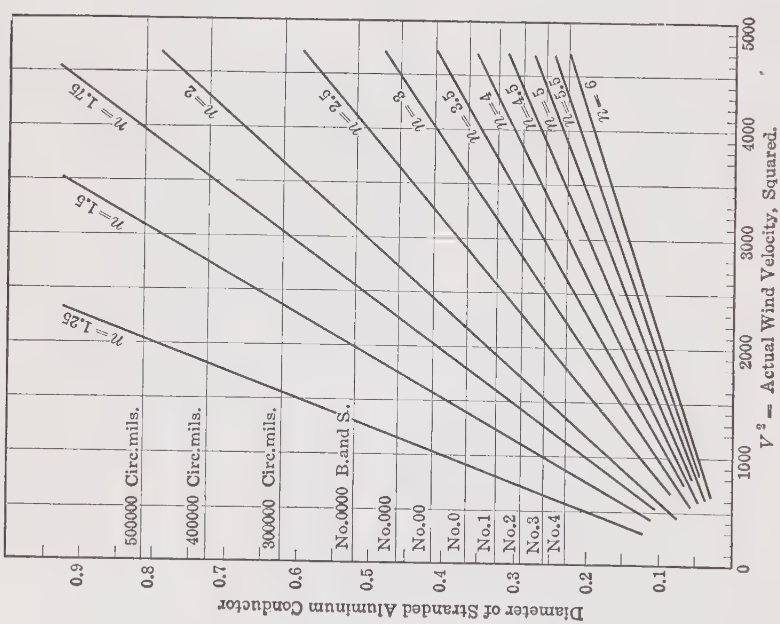


FIG. 101.—Chart giving factor n for aluminum conductors without ice loading.

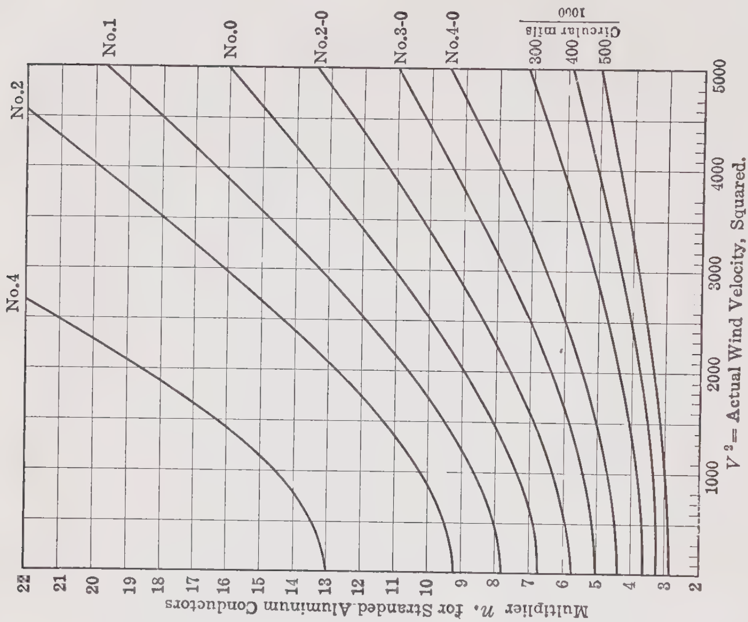


FIG. 102.—Chart giving factor n for ice-coated aluminum conductors.

vertical loading. The error introduced by using Fig. 99 for stranded cables is not of great practical importance.

The curves of Fig. 101 are approximately correct for stranded aluminum conductors; but since the actual diameter will vary with the method of stranding, these charts are intended only for the use of practical engineers who are not interested in mathematical niceties. The calculations, on the basis of the assumptions previously made, are, however, very simple. Thus, if θ stands for the angle wOr or w_1Ow_r as the case may be (see Fig. 98), we may write:

$$\tan\theta = \frac{\text{horizontal loading}}{\text{vertical loading}}$$

whence θ, and therefore $\sec\theta$ can be obtained from trigonometrical tables. This last quantity, being the ratio $\frac{\text{resultant loading}}{\text{vertical loading}}$ is the required factor n when ice loading is not considered. The correction to be made when the vertical load includes ice deposit is simple and obvious.

The special case of solid wires without ice coating can be treated as follows:

Wind pressure per foot length. . . . . $p = \frac{dV^2}{4820}$

Vertical load per foot length (copper). . . . . $w = 3.02d^2$

Vertical load per foot length (aluminum), $w = 0.92d^2$

$$n = \frac{\sqrt{w^2 + p^2}}{w} = \sqrt{1 + \frac{p^2}{w^2}} \tag{129}$$

whence, for *solid* wires *without ice deposit*,

$$n \text{ (for copper)} = \sqrt{1 + \frac{V^4}{d^2 \times 21{,}200 \times 10^4}} \tag{130}$$

$$n \text{ (for aluminum)} = \sqrt{1 + \frac{V^4}{d^2 \times 1965 \times 10^4}} \tag{131}$$

Example: What is the ratio of the resultant load with actual wind velocity of 70 miles per hour, to normal load with wire hanging in still air, in the case of a No. 6 B. & S. copper conductor?

The diameter is $d = 0.162$ in., and by formula (130)

$$n = \sqrt{1 + \frac{(7)^4 \times 10^4}{(0.162)^2 \times 21{,}200 \times 10^4}} = 2.3$$

134. Effect of Temperature Variations on Sag and Stress.—Assuming the tension at all parts of the wire to be the same, and equal to P lb., the formula (102) of Article 125 may be written:

$$P = \frac{wl^2}{8s} \tag{132}$$

where l and s are in feet, and w is the weight per foot of the wire. If wind or ice or both result in a loading per lineal foot equal to to nw, it follows that the tension in the loaded wire is now n times as great. If the symbol S be used to denote the stress or tension in the wire *per square inch of cross-section*, the formula giving the relation between tension and sag may be written,

$$S = kn\frac{l^2}{s} \tag{133}$$

in which k is constant for a given material; it is equal to $\frac{w}{8A}$ where A is the cross-section of the conductor in square inches. The numerical value of k is therefore one-eighth of the weight in pounds of 12 cu. in. of conductor material, or 1.5 times the weight of a cubic inch. The values of k, together with ultimate and working values of the stress S, will be found in the table of physical constants in Article 41 of Chapter IV (p. 75). The length of wire between fixed supports of equal height bears a definite relation to the span and sag. This relation, as already given, is:

$$\lambda = l + \frac{8s^2}{3l} \tag{111}$$

where l is the distance between supports and s the sag at center, both expressed in feet.

The increase in length due to stress will be directly proportional to the tension per square inch (S) provided the elastic limit of the material is not exceeded. The approximate elastic limit for conductor materials is given in the table of Chapter IV. The formula for elastic stretching, as already given, is

$$\lambda_e = \lambda\frac{S}{M} \tag{125}$$

where λ is the length of wire when $S = 0$, λ_e is the elongation due to the stress S, and M is the elastic modulus.

The increase of length due to rise of temperature, on the assumption that stress remains unaltered, is:

$$\lambda \times a \times t \tag{134}$$

where a is the temperature-elongation coefficient.

Values of k, a, M and the maximum safe working stress S, taken from the data in Chapter IV, are as follows:

For copper	$k = 0.485$; $a = 0.0000096$; $M = 15{,}000{,}000$; $S = 28{,}000$
For aluminum	$k = 0.146$; $a = 0.0000128$; $M = 9{,}000{,}000$; $S = 13{,}000$
For copper-clad steel (40 per cent.)	$k = 0.447$; $a = 0.0000067$; $M = 20{,}000{,}000$; $S = 40{,}000$
For iron wire	$k = 0.423$; $a = 0.0000066$; $M = 25{,}000{,}000$; $S = 26{,}000$
For Siemens-Martin steel	$k = 0.427$; $a = 0.0000066$; $M = 29{,}000{,}000$; $S = 33{,}000$

It is a simple matter to calculate by means of formula (132) or the modified formula (133) the sag corresponding to any span (l), load (nw) and tension (S), and by using either of these formulas, the minimum allowable sag, for the safe limiting tension when the wire is subject to the greatest expected load in the matter of ice and wind pressure, should be determined in the first instance. Having determined the amount of this sag—which should be that corresponding to the lowest expected temperature—the length of the wire in the span can readily be calculated by means of formula (111).

The calculation of the sags and corresponding tensions at other temperatures and under other conditions of loading is not by any means so simple a matter, because the alteration in the length of the conductor depends not only upon the temperature, but also upon the tension. If the extra load on the conductor due to wind pressure and ice (if any) be removed, the sag will adjust itself until the formula (132) is again satisfied, the weight w being in this case that of the wire only. A further condition is that the length of wire shall be equal to the length as originally calculated for the loaded wire, less the elastic contraction due to the reduction in the tension. If the temperature be now supposed to rise, the length of the wire will increase, but not in direct proportion to the temperature rise as indicated by formula (134) because so soon as there is any increase in length leading to an increased sag, the tension in the wire is immediately relieved, and since it is assumed that the elastic limit has not been exceeded there will be a reduction in length which could be calculated by formula (125) *if the amount by which the tension is relieved were known.*

A mathematical formula which expresses the required length or sag under normal conditions, in terms of the length corresponding to minimum temperature and heaviest loading, is very complex and difficult of solution. Mr. H. J. Glaubitz[1] has evolved an equation in which the first and third power of the unknown quantity (the deflection or sag) appear simultaneously. The solution of such an equation is tedious and is usually accomplished with the assistance of more or less scientific guesswork. A graphical method, which is probably in more general use, consists in plotting two curves, one showing the relation between sag and tension for the selected span when the wire hangs naturally under its own weight only, and another curve calculated for a definite constant temperature and giving the relation between sag and tension when a wire of definite known length under a known tension is subjected to various assumed changes in the tension. The point where the two curves cross will indicate the required conditions of sag and tension. This process is a lengthy and laborious one and has to be repeated for every assumed change of temperature.

Graphical methods of calculating sags or tensions under various conditions of temperature and load are quite accurate enough for practical purposes, and since the material published in this connection is so abundant and varied that there is room for individual choice in the selection of a particular method, or chart, or combination of charts and diagrams, the needs of the engineer who prefers graphical methods to the more lengthy analytical processes, are easily satisfied.

For one who is working constantly on the same kind of problem, diagrams or charts are usualy of very great assistance; but if a method involving any but the simplest diagrams is not made use

[1] *Electrical World*, March 25, 1900, p. 731. The reader may also refer to the *Electrical World* of July 13, 1912, p. 101, where Mr. H. V. Carpenter evolves a formula containing the third and first powers of the unknown quantity, and proposes a chart to assist in arriving at the solution. An excellent article on Sags and Tensions in wire spans from the pen of Dr. Harold Pender appeared on page 604 of the *Electrical World* of Sept. 28, 1907. Among more recent contributions there is an article by Mr. K. L. Wilkinson in the *Electrical World* of Feb. 6, 1915, and a paper by Mr. A. T. Arnall on p. 360 of vol. cci (June, 1916) of the *Proceedings of the Institution of Civil Engineers* (England). Practical suggestions for solving the equations containing the third power together with the first or second power of the unknown quantity are made in these papers.

of very frequently, mistakes are likely to occur, or else the time spent in re-studying the method of procedure leaves the graphical treatment without advantage over the more tedious process involving the solution of mathematical equations.

The writer has always found charts or diagrams of great assistance in practical engineering work, and in the first edition of this book an accurate method of solving sag-temperature problems by means of two superimposed diagrams was described; but, for the reasons above given and also to avoid the introduction of unnecessary material, the graphical method of procedure is omitted from this edition. The analytical method about to be described does not require the solution of equations containing the third and other powers of the unknown quantity: it is accurate, and does not include any difficult mathematical processes.

135. Calculation of Sags and Tensions under any Conditions of Load and Temperature.—This method of calculation requires the knowledge of the sag and tension corresponding to one particular temperature. A different sag is then assumed, and the temperature at which this sag will occur is calculated by means of a simple formula. The manner of obtaining the preliminary data will be explained later.

It is assumed that the conductor is strung between two fixed supports on the same level, and that the material of the conductor is not strained beyond the elastic limit.

The meaning of the symbols used is as follows:

l = the length of span, or horizontal distance between points of support, in feet.

s = the vertical sag at center of span, in feet, when wire hangs in still air under the influence of its own weight only.

P = the tension in the conductor at the lowest point of span, in pounds.

S = the stress in the conductor at the lowest point of span, in pounds per square inch of cross-section.

λ = the length of conductor measured between the two points of support.

t = temperature, in degrees Fahrenheit.

s_c, S_c, λ_c = known values of sag, stress and length corresponding to a definite temperature t_c, when wire

hangs in still air under the influence of its own weight only.

a = the coefficient of linear expansion of the conductor per degree Fahrenheit.

M = the modulus, or coefficient, of elasticity of the conductor, being the ratio of stress in pounds per square inch to extension per unit length.

w = the weight of conductor in pounds per foot of length.

w_r = the resultant or total load in pounds, per foot, including wind pressure and ice (if any).

n = a multiplier depending on the material of the conductor and weather conditions, being the ratio w_r/w, when supports are on the same level.

k = a constant depending upon the material ofthe conductor, being 1.5 times the weight in pounds of a cubic inch of the conductor material.

The well-known formula giving the relation between sag, length of span, horizontal load, and tension is:

$$s = \frac{l^2 w}{8P} \tag{101}$$

The approximate formula for the length of a parabolic curve (which is quite sufficiently accurate for all practical purposes) is:

$$\lambda = l + \frac{8s^2}{3l} \tag{111}$$

It is assumed that the sag s_c, and therefore the corresponding stress S_c and length λ_c are known for the particular temperature t_c, which may be fairly high so that another value, s, of the sag, arbitrarily chosen, shall be smaller than s_c; and it is proposed to calculate the temperature t which will correspond to this assumed sag, s.

With the reduction in the amount of sag, there must of necessity be a reduction in the length λ_c of the conductor and an increase in the tension P_c or stress S_c. The amount by which the length has decreased is not directly proportional to the reduction in temperature, because the increase in tension causes an elastic elongation of the conductor, and the reduction in length is actually the difference between the amount by which the wire would contract with the lowering of the temperature if the tension were to remain constant, and the amount by which the wire

would be extended, due to the increased tension, if the temperature were to remain constant; although, from a strictly scientific point of view, the argument may be inaccurate. The decrease in length due to temperature reduction is:

$$\lambda_c \times a \times (t_c - t) \tag{135}$$

and the increase in length due to additional tension is:

$$\frac{\lambda_c}{M}\left(\frac{s_c}{s}S_c - S_c\right),$$

or

$$\frac{\lambda_c}{M}S_c\left(\frac{s_c}{s} - 1\right) \tag{136}$$

Therefore,

$$\lambda_c - \lambda = \lambda_c \times a \times (t_c - t) - \frac{\lambda_c}{M}S_c\left(\frac{s_c}{s} - 1\right)$$

The lengths λ and λ_c can be eliminated by substituting their values in terms of sag and length of span, as given by formula (111). This leads to the equation:

$$t_c - t = \frac{8s_c^2 - 8s^2}{(3l^2 + 8s_c^2)a} + \frac{S_c}{aM}\left(\frac{s_c}{s} - 1\right) \tag{137}$$

This formula is very simple to use, because for a given material and size of wire, it may be written:

$$t_c - t = \frac{K_1 - 8s^2}{K_2} + K_3\left(\frac{s_c}{s} - 1\right) \tag{138}$$

where K_1, K_2, and K_3 are constants, the values of which are:

$$K_1 = 8s_c^2 \tag{139}$$

$$K_2 = (3l^2 + 8s_c^2)a \tag{140}$$

$$K_3 = \frac{S_c}{aM} \tag{141}$$

Moreover, since $8S_c^2$ is always very small in comparison with $3l^2$, the constant K_2 may, for nearly all practical purposes, be written:

$$K_2 = 3al^2 \quad \text{(approx.)} \tag{142}$$

This value of K_2 may be used for spans up to 500 ft. if the multiplier n does not exceed 12, and for spans up to 1000 ft. if n does not exceed 6. In the case of longer spans, in which the sag is relatively large, or if a closer approximation is required, the more exact expression (140) should be used. The only unknown quantities in equation (137) or (138) being the sag s

and temperature t, it follows that, by inserting any numerical value for s in the equation, the change of temperature, and therefore the actual temperature t corresponding to the assumed value of the sag, can be readily calculated. In order that this method of calculation may be of practical utility, it is necessary that the sag s_c and the stress S_c at the particular temperature t_c shall be known. The fundamental data on which all line calculations are based must include the limiting or maximum allowable value of the stress and the conditions of maximum loading under the most severe weather conditions. The maximum load per foot being w_r and the weight of the unloaded wire being w, it follows that the ratio $\frac{w_r}{w} = n$ will be greater as the wind conditions, either without ice or combined with a coating of ice on the wires, are the more severe. The wires will generally be subject to the greatest stress at times when strong winds, with or without a coating of ice or sleet, occur at a low temperature, because the lowness of the temperature alone will account for a considerable increase in the tension.

If the extra load on the wire due to wind and ice combined is great in proportion to the weight of the wire, the maximum deflection will usually occur under winter conditions; but there will be a higher temperature at which the sag of the unloaded wire hanging in still air, subject to its own weight only, will be exactly the same as the deflection under winter conditions when subject to wind pressure and extra load of ice (if the line runs through a district where sleet and ice formation is possible). This temperature, which may be called the critical temperature for the material of the conductor when the maximum winter loading has been determined, is easily calculated; and its numerical value, together with the known value of the sag under conditions of maximum load, and of the tension corresponding to this sag, may be used in equation (137) or (138) for the known quantities t_c, s_c and S_c.

Calculation of Critical Temperature t_c

Let S_m be the stress in the wire under the most severe conditions of load, and t_o the temperature at which this stress occurs. The tension S_c will be equal to S_m divided by n, because, at the critical temperature t_c, the sag is the same as the maximum deflection of the loaded conductor, but the weight per foot of length is in

the ratio $\frac{w}{w_r}$ or $\frac{1}{n}$. (Refer to formula (101), bearing in mind that, for any given size of conductor, the stress S is proportional to the tension P.) With an increase of temperature from t_o to t_c the reduction in stress is

$$S_m - S_c = S_m - \frac{S_m}{n} = S_m\left(1 - \frac{1}{n}\right)$$

and the reduction in length of the wire due to this difference of tension is

$$\frac{\lambda_c}{M} S_m\left(1 - \frac{1}{n}\right)$$

It is required to calculate the temperature rise $(t_c - t_o)$ which will produce an *extension* exactly equal to this elastic contraction, in order that the length λ_c of the wire, and consequently the sag s_c, shall remain as before.

The extension due to temperature rise is

$$\lambda_c \times a \times (t_c - t_o)$$

and the required equation is:

$$\lambda_c \times a \times (t_c - t_o) = \frac{\lambda_c}{M} S_m \left(1 - \frac{1}{n}\right)$$

or

$$(t_c - t_o) = \frac{S_m}{M \times a}\left(1 - \frac{1}{n}\right) \tag{143}$$

The curves in Fig. 103 give the relation between the ratio $\frac{w}{w_r}$ (being the reciprocal of n) and the temperature rise $(t_c - t_o)$ for stranded cables of different materials. The values of M and a, which have been adopted for the purpose of drawing the diagram, are:

For hard-drawn copper,	$M = 15 \times 10^6$, $a = 9.6 \times 10^{-6}$
For hard-drawn aluminum,	$M = 9 \times 10^6$, $a = 1.28 \times 10^{-5}$
For galvanized steel,	$M = 25 \times 10^6$, $a = 6.5 \times 10^{-6}$

Having determined the critical temperature t_c at which—it is interesting to note—the tension in the wires, if correctly strung, will be the same whatever the length of span, the sag s_c can be calculated by the formula (101), or by the more convenient formula:

$$s_c = \frac{kl^2}{S_c} \tag{144}$$

which can be put in the form

$$s_c = \frac{knl^2}{S_m} \tag{145}$$

In this manner the numerical values of the quantities t_c, s_c, and S_c for use in formula (137) or (138), are obtained.

Example.—Construct a sag-temperature chart for the use of the construction engineers in the field, based on the following data:

Horizontal span = 485 ft. with rigid supports on the same level.

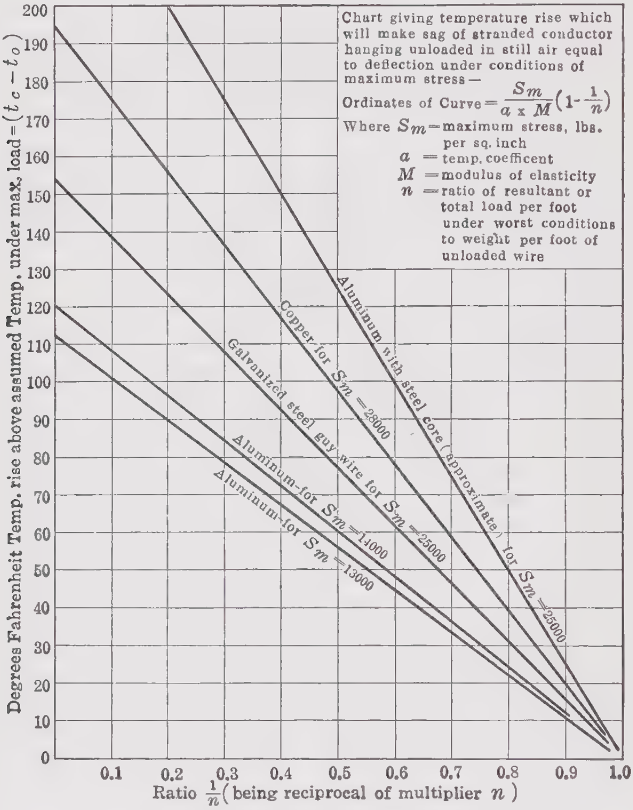


FIG. 103.—Chart for determining "critical temperature."

Conductors: stranded aluminum No. 2/0 B. & S.

Stress not to exceed S_m = 14,000 lb. per sq. in. with a combined load of 0.5 in. ice coating and 47-mile wind at a temperature t_o = − 20° F.

From Fig. 102, the value of n for $V^2 = (47)^2 = 2210$ is 8, whence

$$\frac{1}{n} = 0.125.$$

From Fig. 103 $(t_c - t_o) = 106$
whence $t_c = 106 - 20 = 86°$ F.
this being the "critical temperature" at which the vertical sag of the unloaded conductor hanging in still air will be the same as the maximum deflection of the loaded conductor under the specified conditions.

Other required values are $S_c = \dfrac{14{,}000}{8} = 1750$, and $k = 0.146$.

By formula (144), $S_c = \dfrac{0.146 \times (485)^2}{1750} = 19.6$ ft.

By formula (139), $K_1 = 8 \times (19.6)^2 = 3070$

By formula (142), $K_2 = 3 \times 485 \times 485 \times 1.28 \times 10^{-5} = 9$

By formula (141), $K_3 = \dfrac{1750 \times 10^5}{9 \times 10^6 \times 1.28} = 15.2$

By formula (138), $(86 - t) = \dfrac{3070 - 8s^2}{9} + 15.2\left(\dfrac{19.6}{s} - 1\right)$

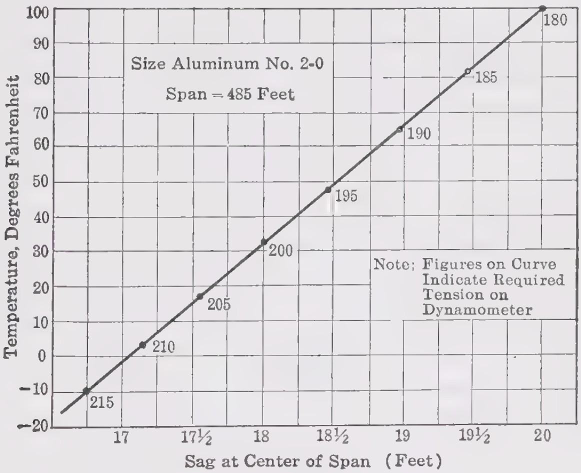


Fig. 104.—Sag-temperature curve for use when stringing wires.

By choosing values for s not very different from s_c the corresponding temperatures are easily calculated. Thus

when $s = 16\frac{1}{2}$ ft., $t = -\ 15.8°$ F.
when $s = 17\frac{1}{2}$ ft., $t = +\ 15.4°$ F.
when $s = 18\frac{1}{2}$ ft., $t = +\ 48.4°$ F.
when $s = 20\frac{1}{2}$ ft., $t = +\ 120°$ F.

With the aid of these figures a curve such as Fig. 104 is readily plotted. This curve gives the men in the field all necessary

information for the correct stringing of the conductors, whatever may be the temperature when the work is carried out.

136. Tensions in Conductors when Spans are of Different Lengths.—It is well to keep the consecutive spans in a transmission line as nearly as possible of the same length, because, although it is possible to string the wires so that the tension shall be the same in all spans at the time of stringing or under specified conditions of load and temperature, there will be an unbalancing of the tensions in adjoining spans with every change of temperature. If properly strung, the wires in long and short spans should be subjected to the same maximum tension under the severest conditions of loading, and the condition of equal tensions will repeat itself at the higher temperature—previously referred to as the critical temperature—when the sag of the unloaded conductor is the same as the deflection of the loaded conductor at the lower temperature; but, except when the deflection at center of span remains unaltered,[1] the pull on each side of a supporting insulator will be unbalanced. It is partly for this reason that extra long spans are usually "dead-ended" on guyed poles or strain towers. When calculating sags in spans of different lengths, it is therefore not correct to assume that the sag is always directly proportional to the square of the length of span; because when the wire hangs in still air subject to its own weight only, this proportionality exists for no other temperature but the critical temperature as determined for use in the sag-temperature calculations.

137. Tension in Different Sized Wires on the Same Span.—It may be questioned whether, having calculated the sag-temperature conditions for a conductor of diameter d_1, there is not a short cut by which similar relations can be arrived at for a wire of diameter d_2 when the length of the span, l, remains unaltered. There does not appear to be a quick way of obtaining the required results; but there is one condition that holds:

[1] The condition is that the quantity $\frac{n(t_1 - t_o)}{n - n_1}$ shall remain constant. This expression is derived in the same manner as the formula (143): there must obviously be some particular value of the wind or ice loading (corresponding to the factor n_1) which, in conjunction with a rise of temperature $(t_1 - t_o)$ will cause the deflection to be the same as when the temperature is t_o and the (maximum) loading is n times that due to the weight of the wire acting alone. The necessary relation between t_1 and n_1 is given by the above formula.

Let n be the load coefficient $\left(\frac{w_r}{w}\right)$ for conductor d_1, and n_2 the load coefficient for conductor d_2 then, since l^2 is assumed constant, it follows from the formula (145) that

$$s \propto \frac{n}{S}$$

and

$$s_2 = s_1 \times \frac{n_2}{n_1}$$

which is true only when the stress (S) per square inch of cross-section is the same for both sizes of conductor (the material of the conductors being assumed the same).

138. Further Example Illustrating Temperature-sag Calculations.—Although the problem may not be of great practical utility, it will serve as a further illustration of the methods of calculation explained in Article 135.

Determine the reduction of temperature which will make the tension in the unloaded conductor the same as in the loaded conductor when the resultant load is n *times the weight of the wire.*

When calculating the "critical temperature," the formula (143) was developed. This enables us to calculate the rise of temperature necessary to make the *length* of the unloaded conductor hanging in still air equal to that of the same conductor at a lower temperature, but with an extra load due to wind or ice, or to both combined. The problem now before us is concerned with the *stress,* which must be the same for the loaded and unloaded conditions. Since the stress remains unaltered, the change in length of the conductor corresponding to the (necessarily) smaller sag is caused by reduction of temperature and not by elastic contraction.

If s = maximum deflection under loaded condition, and

s_1 = sag of the unloaded wire when the temperature has fallen $t°$ F. and the stress S is again equal to what it was under the loaded condition, then,

$$s_1 = \frac{s}{n}$$
$$= \frac{kl^2}{S}$$

The reduction in length is,

$$\lambda_1 \times a \times t = \lambda - \lambda_1$$
$$= l + \frac{8s^2}{3l} - l - \frac{8s_1{}^2}{3l}$$

which resolves itself into

$$\lambda_1 \times t = \frac{8s^2(n^2 - 1)}{3n^2 \times a \times l}$$

No appreciable error will be introduced, when dealing with spans of moderate length, if the length of wire λ_1 is replaced by the length of span l. Hence,

$$t = \frac{8s^2(n^2 - 1)}{3n^2 \times a \times l^2} \tag{146}$$

which, if it is desired to eliminate s, can be written

$$t = \frac{8(n^2 - 1)k^2l^2}{3 \times a \times S^2} \tag{147}$$

It may be observed that, for a given material and limiting tension, the required reduction in temperature is proportional to $(n^2 - 1) \times l^2$, or $t = \frac{(n^2 - 1)l^2}{K}$, where K is a numerical constant.

By using the data for materials previously given and assuming maximum allowable tensions corresponding to $S = 28{,}000$ for copper; 13,000 for aluminum; and 25,000 for steel guy wire, the calculated value of K is,

For copper	$K = 12{,}000$
For aluminum	$K = 38{,}000$
For steel	$K = 8{,}250$

139. Sag-temperature Calculations with Supports at Different Elevations.—We shall consider (1) the case of a small difference of elevation between supports, which will cause the lowest point in the span to be below the level of the lower point of support, as illustrated in Fig. 105; and (2) the case of a line running up a steep incline, which will cause the lower support to be the lowest point in the span (see Fig. 106). The problem will be studied by working out numerical examples.

Data for numerical examples:

Wire; No. 0 B. & S. solid copper.

Diameter of wire, $d = 0.325$ in.

Cross-sectional area of wire, $A = 0.083$ sq. in.

Weight of wire per foot run, $w = 0.32$ lb.

Distance between points of support (measured on incline), $l' = 240$ ft.

Breaking stress (say) 55,000 lb. per sq. in.

In these examples, we shall neglect ice-coating on the wires,

but allow for a high wind velocity (78 miles per hour) giving 15 pounds per square foot of projected surface of the wire, and we shall also adopt the abnormally high factor of safety of 5 as called for in the regulations issued by the British Board of Trade. The maximum allowable stress, at a specified temperature $t_o = 22°$ F., is therefore

$$S_{\text{max.}} = \frac{55{,}000}{5} = 11{,}000 \text{ lb. per sq. in.}$$

Maximum tension in wire, $P_{\text{max.}} = 11{,}000 \times 0.083 = 915$ lb.

The wind pressure per foot run of the wire is $p = 15 \times \frac{0.325}{12} = 0.406$ pounds, and the value of the factor n as defined in Article 133, is

$$n = \frac{\sqrt{(0.32)^2 + (0.406)^2}}{0.32} = 1.58$$

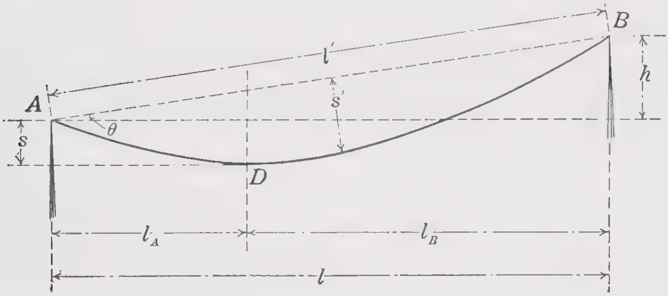


FIG. 105.—Wire hung between supports at different elevations.

For the calculation of the *critical temperature* (see Article 135) we have, $M = 15 \times 10^6$ and $a = 9.6 \times 10^{-6}$, whence, by formula (143),

$$(t_c - t_o) = \frac{11{,}000 \times 10^6}{9.6 \times 15 \times 10^6}\left(1 - \frac{1}{1.58}\right) = 28$$

and $t_c = 28 + 22 = 50°$ F. It is at this temperature that the length of the wire hanging in still air will be the same as the length under the condition of maximum load when the stress is n times as great. It follows that the tension in the wire at a temperature of 50° F. (without wind) will be

$$P_c = \frac{P_{\text{max.}}}{n} = \frac{915}{1.58} = 578 \text{ lb.}$$

Case (1). *Small Difference of Elevation.* (See Fig. 105.) For the additional data required, assume:

Difference in elevation of points of support, $h = 13$ ft., whence $\sin\theta = \frac{13}{240} = 0.0542$, and $\cos\theta = 0.9985$, which indicates that the span measured horizontally, being $l = l' \cos\theta$, is so nearly equal to l' that no appreciable error will be introduced if we neglect to take account of this difference. In this particular example it is therefore a matter of indifference whether we put l or l' in the formulas.

Now choose two or three values of tension in addition to the tension $P_c = 578$ at the "critical" temperature of 50° F., and calculate the corresponding temperatures by the step by step method as followed in the accompanying table.

This method of procedure is correct except for the fact that formula (123) used for obtaining item (f) does not include all the terms necessary for the exact calculation of the length of a parabolic arc. With a difference in level of only 13 feet with supports spaced 240 feet apart, the error is probably negligible; but with a greater difference in elevation between the points of support (as in Case (2) about to be considered) additional terms would have to be included in the equation, thus adding to the time required for the calculations.

The maximum deflection (item (c)) of the wire from the straight line AB joining the points of support is always small relatively to the distance $l' = AB$; and in computing the length of a curve which approximates to a straight line no appreciable difference will be observable whether we consider the curve to be part of a parabola, or catenary, or ellipse, or circle. For our purpose it will be most convenient to calculate the length (λ) of the wire by using formula (122) of Article 127, merely substituting s' (item (c)) for s, and l' (see Fig 105) for l. This has been done in the example, Case (1), the results being given under items (f)′ and (g)′ at the end of the accompanying table. The change in length ($\lambda - \lambda_c$) is seen to be the same whichever method of calculation is used,[1] and since the latter method of calculating λ is much shorter than the former, it should be adopted when making sag-temperature calculations for spans on an incline.

Case (2). *Large Difference of Elevation.* (See Fig. 106.) For additional data required, assume:

[1] The very small difference in the two sets of figures may be attributed to errors in reading the slide rule.

NUMERICAL EXAMPLE, CASE (1)—DIFFERENCE OF LEVEL, $h = 13$ FEET

(a)	Assumed tension, P, in direction AB	650	$P_c = 578$	500	400
(b)	Horizontal component, $P_h = P \cos \theta$ (in this example $P_h = P$ approximately)	650	578	500	400
(c)	Maximum deflection from straight line AB. By formula (122) $s' = \frac{wl^2}{8P_h} = \frac{2310}{P_h}$ feet	3.55	3.99	4.61	5.76
(d)	By formula (119) $l_A = 120 - \frac{13 \times P_h}{0.32 \times 240}$ $= 120 - 0.1693P_h$ feet	10	22	35.3	52.3
(e)	By formula (121) $s = \frac{0.32}{2 \times 0.9985} \frac{l_A{}^2}{P_h}$ $= 0.16 \frac{l_A{}^2}{P_h}$ feet	0.0246	0.134	0.399	1.095
(f)	By formula (123) $\lambda = 240 \cos \theta + \frac{2}{3}\left[\frac{(s+h)^2}{l - l_A} + \frac{s^2}{l_A}\right]$ $= 240 \cos \theta +$	0.492	0.527	0.587	0.720
(g)	$\lambda - \lambda_c$ (feet)	− 0.035	0	0.060	0.193
(h)	Elongation due to stress variation. From formula (125), approximately $\frac{l'}{M}\left(\frac{P - P_c}{A}\right) = \frac{P - P_c}{5180}$	0.014	0	−0.015	−0.034
(i)	Elongation due to change of temperature = (g) − (h)	− 0.049	0	0.075	0.227
(j)	$(t - t_c) = \frac{\text{item } (i)}{a \times \lambda} = (i) \times 433$ (approximately)	−21.2	0	32.5	98.5
(k)	$t = t_c + (j) = 50 + (j)$	28.8	50	82.5	148.5

ALTERNATIVE METHOD

(f)′	Substituting s' for s, and l' for l, in formula (111), $\lambda = 240 + \frac{(s')^2}{90}$ $= 240 +$	0.140	0.177	0.236	0.369
(g)′	$\lambda - \lambda_c$	− 0.037	0	0.059	0.192

Difference in elevation of points of support, $h = 140$ feet, whence $\sin\theta = \dfrac{140}{240} = 0.583$, and $\cos\theta = 0.8124$.

The span measured horizontally is $l = l' \cos\theta = 195$ feet.

If the formula (119) is used for calculating the distance l_A from the lower support, A, to the point where the wire becomes horizontal, this quantity will be negative. It will determine the shape and position of an imaginary parabolic curve as indicated by the dotted line of Fig. 106. The vertical component of the tension at the point B will be equal to the weight of the wire in

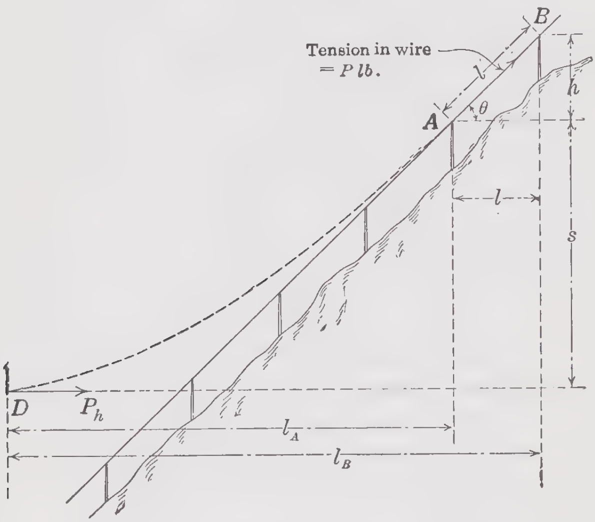


FIG. 106.—Transmission line on steep incline.

the parabolic arc BD; while the vertical component at A will be the weight of the portion AD. The horizontal component (P_h) of the tension will be $P\cos\theta$, where P stands for the tension at the middle of the span AB.

Temperature-stress calculations can be made by considering the length of wire in the span AB to be the difference between the imaginary half spans BD and AD, the procedure being exactly as indicated in the foregoing table illustrating the method for small differences of level. This method is objectionable because the required lengths are very small differences between comparatively large quantities, and unless several additional terms are added to the approximate formula for the length of the parabolic

curve, serious errors may be introduced. We shall therefore adopt the alternative method, and make the necessary calculations on what may be thought of as an equivalent horizontal span.

Fig. 107 is an enlarged view of the span AB of Fig 106. The loading of the wire in a direction perpendicular to its length is no longer w lb per foot, but $w \cos \theta$ lb. per foot, and the fundamental equation $s = \frac{l^2w}{8P_h}$ (see formula (101) of Article 125) becomes

$$s' = \frac{(l')^2 w \cos \theta}{8P} \tag{148}$$

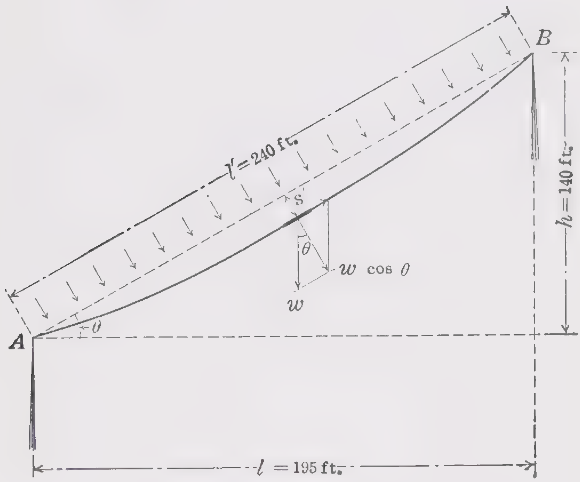


FIG. 107.—Enlarged sketch of span AB of Fig. 106.

which can also be written

$$s' = \frac{l^2 w}{8P \cos \theta}$$

this last being the formula (122)[a] as developed in Article 127.

The slope of the wire being considerable, it will be advisable to re-calculate the value of the factor n which, in Case (1), was taken to be the same as if the span were horizontal. The wind pressure at right angles to the wire is the same as in the preceding example, namly 0.406 lb. per foot; and by definition (see Article 133) the value of n is therefore

$$n = \frac{\sqrt{(0.32 \cos \theta)^2 + (0.406)^2}}{0.32 \cos \theta} = 1.85$$

instead of the previously calculated value of 1.58 which is correct for a horizontal or nearly horizontal span.

The maximum allowable tension being the same as before, we have

$$P_c = \frac{915}{1.85} = 494 \text{ lb.}$$

For the "critical" temperature, we have, by formula (143)

$$(t_c - t_o) = \frac{11{,}000 \times 10^6}{9.6 \times 15 \times 10^6}\left(1 - \frac{1}{1.85}\right) = 35$$

whence $t_c = 35 + 22 = 57°$ F.

We are now in a position to proceed as in *Case* (1) (alternative method), or if preferred we can use the formulas of Article 134 which will give exactly the same results.

Let the assumed values of tension, in addition to the "critical" value $P_c = 494$ lb., be $P_1 = 600$ lb., and $P_2 = 400$ lb. The corresponding maximum deflections from the straight line, as calculated by formula (148) are,

$$s'_c = 3.79 \text{ ft.}$$
$$s'_1 = 3.12 \text{ ft.}$$
$$s'_2 = 4.68 \text{ ft.}$$

By formula (139), $K_1 = 8\ (3.79)^2 = 115$

By formula (142), $K_2 = 3 \times 9.6 \times 10^{-6} \times (240)^2 = 1.66$

By formula (141), $K_3 = \dfrac{494}{0.083 \times 9.6 \times 15} = 41.5$

By formula (138), putting $s = 3.12$ ft., we have,

$$(57 - t_1) = \frac{115 - 8(3.12)^2}{1.66} + 41.5\left(\frac{3.79}{3.12} - 1\right)$$

whence $t_1 = 24.7°$ F.

Similarly, when $s = 4.68$

$$t_2 = 101° \text{ F.}$$

The curve marked (2) in Fig. 108 has been plotted from these results, while curve (1) refers to the previously calculated *Case* (1). The correction for n, and therefore for the "critical" temperature t_c, need not be made when the slope is small. It is only when the difference between the horizontal spacing (l) and the actual distance between supports (l') is appreciable that the correction need be made. The procedure here recommended

for constructing sag-temperature curves for lines on an incline consists in replacing the actual span by an "equivalent" span with supports on the same level. This equivalent span may be defined as a horizontal span of the same length (l') measured between points of support as the actual span, the loading being $w \cos \theta$ lb. per foot run, where w is the weight per foot of the wire, and θ is the angle between the direction of the line and the horizontal.

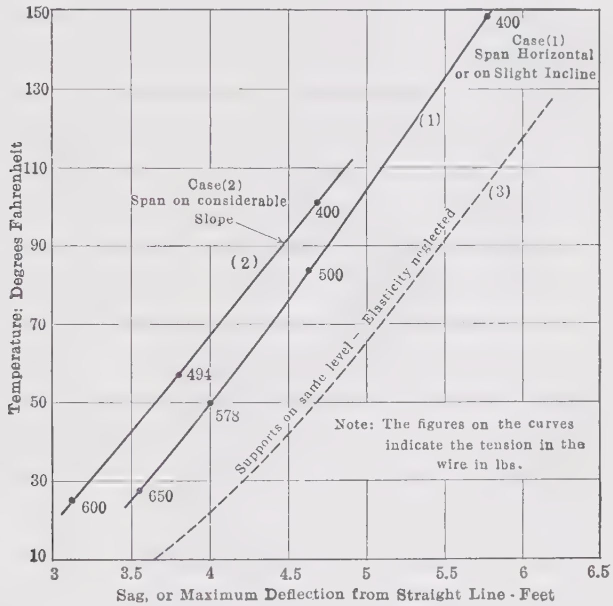


FIG. 108.—Sag-temperature curves illustrating numerical example.

The dotted curve marked (3) in Fig. 108 is plotted from the same data as curve (1) except that the effect due to elasticity of the wire has been neglected (item (h) in the table on page 293). The omission is often permissible, especially on long spans of aluminum wire; but, on comparatively short spans of copper wire, as in the present instance, it is seen to give entirely inaccurate results. The point is mentioned here because of the peculiarity that in England—where the stringent regulations involving large factors of safety necessitate uneconomical construction with short spans—it is by no means uncommon to neglect the elasticity of the wire; while in America—where the longer spans

often cause the inclusion of this item to become an unnecessary refinement—it is customary to take account of the changes in length due to variation of load.

140. Length of Spans. Conductor Materials—Copper; Aluminum; Iron.—When the wires of a transmission line are supported on single wood poles, the span may be anything from 120 to 250 or even 300 ft.; but extra long poles, specially selected to withstand the greater stresses, are required when the span is appreciably in excess of 200 ft.

On steel tower lines, very much greater spans may be used. The length of span is determined not only by strength considerations, but also by considerations of economy (see Articles 15 and 16 in Chapter III). The requirements in the matter of supporting poles or structures (which depend largely on length of span) will be referred to in the following chapter.

Whether copper or aluminum should be used on a given transmission line cannot be determined on general principles. Conductor materials were discussed in Chapter IV, and, apart from the physical properties of these materials, the relative cost, which is a variable quantity, must be taken account of when deciding upon the material best suited for the work.

Generally speaking, the deflections or sags of aluminum conductors on spans of moderate length will be about 30 per cent. or 35 per cent. greater than with copper conductors. The difference will be more marked with the smaller sizes of wires and the shorter spans. With extra long spans in the neighborhood of 1000 ft., it will be found that the maximum sag of aluminum and copper conductors will be about the same, *if storm and abnormal winter conditions are neglected;* that is to say when the factor n (defined in Article 134) is unity. Under this condition it will even be found that copper has a *greater* sag than aluminum on the very long spans. The reason for this is that the greater temperature elongation of aluminum is inappreciable in the case of long spans with necessarily large sags, while it is an important factor in the comparison of the two metals when the spans are short. The above statement is made rather as being of scientific interest than of practical utility, because, under storm conditions (when n has a large value), aluminum will be found to be an unsatisfactory material to use on long spans.

Although the tension in a conductor can always be kept reasonably small by allowing sufficient sag, it is obvious that a large sag

involves higher and more costly supporting structures if the clearance between ground and lowest point of the conductor is to remain the same. When crossing open country, the clearance above ground need not be very great; but in crossing roads, a clearance of 21 feet should be allowed, and over foot paths the clearance should never be less than 15 ft. For mechanical reasons, it is a general rule that no conductor for the transmission of electric energy shall have a lower ultimate strength than a No. 6 B. & S. gauge hard-drawn copper wire.

Although it is convenient in sag calculations to assume rigid supports at each end of the span, the deflection, under heavy load, of pole or steel mast as used on the shorter spans, may be regarded to some extent as a factor of safety. Refinements of calculation are out of place when figuring on short span lines: the tendency is to string wires of short-span transmission lines too slack rather than too tight. On spans not exceeding 150 ft., if the wires are strung at comparatively low temperatures, it is almost impossible to draw them up too tight.

The writer is of the opinion of Mr. H. W. Buck and one or two other eminent and experienced engineers, who hold that most transmission lines are too slack. The fact that the natural stretch which takes place in all overhead conductors during the few months following erection, increases the sag which is usually excessive in the first instance, is frequently overlooked; or, if it is recognized, the additional sag is erroneously supposed to increase the factor of safety. It is not good engineering practice to have the wires hanging in festoons between the supports, and the danger of short circuits resulting from slack conductors being thrown together in stormy weather is unquestionably greater than the risk of breakages on a well-constructed line with wires strung tightly between towers of sufficient strength to withstand the maximum loads that they may be subjected to under the worst weather conditions.

Examples of Extra Long Spans

Where power lines cross rivers or other navigable waterways, exceptionally long single spans are sometimes necessary. The longest in the world crosses the St. Lawrence river about 20 miles from Three Rivers, Quebec, and measures 4800 feet. The ends of the span are attached to steel towers 350 feet high. Construction details will be found in the paper by Mr. S. Svenningson,

Proc. A. I. E. E., Nov., 1918. Another long single span is the Missouri river crossing of the Mississippi River Power Co. which is 3182 feet long: the conductors consist of copper wires of a total cross-section of 0.162 sq. in. laid on a central steel core of 0.275 sq. in. cross-section.

The Pacific Light and Power Corporation have a 2871-foot span at Sunland, Cal.: in this case the conductors are of aluminum wire of 0.475 sq. in. total cross-section laid on a central steel core of 0.062 sq. in. cross-section.

The Great Western Power Co. at Oakland, Cal., have a 2740-ft. span over the navigable waterway: it consists of copper-clad steel conductors, of 0.132 sq. in. cross-section. The Niagara River crossing of the Canadian Niagara Power Co. is 2192 ft. long: it consists of copper-clad steel conductors 0.155 sq. in. in cross-section.

The longest existing span using aluminum cables without steel core is at Piedra, Cal., on the system of the San Joaquin Light and Power Co.: it is 1700 feet long; the conductor cross-section being 0.132 sq. in.

The possibilities of iron wire as a conducting material for overhead power lines was considered in Articles 46 to 49 of Chapter IV. It is only when the price of copper and aluminum is abnormally high that iron as a conductor material need be seriously considered; but in certain cases—such as short lines transmitting small amounts of energy—the saving resulting from the use of iron wire may be considerable.[1] This is due partly to the fact that, with conductors of 1/4 in. or 5/16 in. galvanized steel strand cable, the spans can safely be made longer than when the equivalent copper wire is used. Thus, for the purpose of transmitting from 50 to 80 kw. a distance of 20 to 30 miles by a single three-phase line, the average span on a straight run across country might be 300 feet with 30-ft. poles, 500 ft. with 35-ft. poles, or (preferably) 600 to 700 feet with 40- to 45-ft. poles; and the cost of such a line—excluding the cost of wire—might with care and proper supervision be no more than two-thirds of the figure indicated by the lower curve of Fig. 18 (Chapter III).

Wood poles will usually be found satisfactory for this cheap

[1] Useful mechanical data referring to iron and steel conductors will be found in an article by Messrs. Oakes and Sahm in the *Electrical World* of Aug. 10, 1918, vol. lxxii, p. 249.

type of construction, but with the longer spans it is probable that steel poles or masts handled and erected in one piece, with or without concrete settings, will frequently prove more economical than wood. Double-galvanized wire or cable should be used. For very long spans it may be advisable to use special high-grade steel in order to avoid excessive sag and spacing between wires, and to pay particular attention to the guying of corner poles.

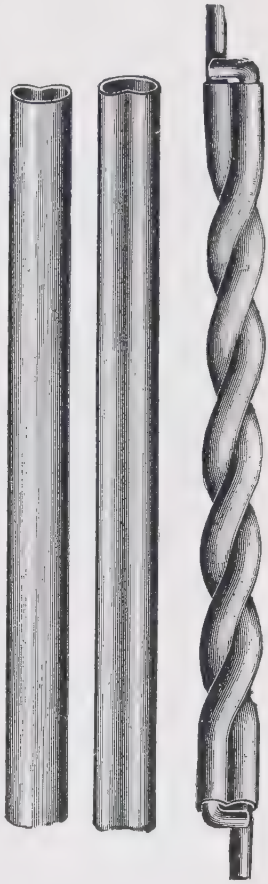


FIG. 109.—Type of joint for overhead conductors.

141. Factors of Safety. Joints and Ties.—The factors of safety usually adopted when calculating the permissible tension in overhead wires have already been referred to in Article 131, and little remains to be added here. In America, the factor of safety for conductors is about 2: this means that, under the worst assumed conditions of loading, the material of the conductor may be stressed to very nearly its elastic limit. It is usual to assume a wind velocity of 70 miles per hour combined with a sleet deposit ½ in. thick at a temperature of 0° F. For guy wires, a factor of safety of 3 to 3½ is generally allowed.

In Great Britain, where higher factors of safety are used, the Board of Trade calls for a maximum tension not exceeding one-fifth of the breaking load, on the assumption of a temperature of 22° F. and a wind velocity corresponding to a pressure of 15 lb. per square foot of the projected surface of the wire. Possible accumulations of snow and ice are ignored.

On the Continent of Europe, the net factor of safety for wires under the worst conditions of loading is about 2½.

When the "flexible" type of tower construction is used, it is customary to allow a somewhat higher factor of safety for the conductors than when the towers are of the so-called rigid type.

Joints in wires can and should be made of the same strength as the wire itself. A discussion of the various types of joints as used in practice would be out of place in these pages. A very common type is the McIntyre joint, illustrated in Fig. 109. It is claimed that when the sleeve is long enough to allow of three complete turns, the strength of this joint is equal to that of the cable itself. Under favorable weather conditions it is possible to make very good butt-welded joints on aluminum conductors with the aid of a blow lamp. Such joints would probably not be satisfactory on conductors of large cross-section (more than ½ in. diameter), and they should be made at the insulator where they will not be subjected to the maximum tensile stress.

The tie wire secures the conductor to the insulator, usually in such a way as to prevent as far as possible the creeping of the conductor from one span to another. In some cases, however, it is advisable to allow the conductor to slip, or the tie wire to break, before the tension in the cable is great enough to damage insulator or cross-arm. Soft or semi-hard wire is generally used for ties, and the size should not be less than No. 4 B. & S. copper, or No. 2 B. & S. aluminum. The tie wire should be of the same material as the conductor.

Some forms of tie include a serving of the tie wire extending an appreciable distance on each side of the insulator; the object being to afford some protection to the conductor in the event of an arc striking over the insulator.

A long sleeve over line wires at point of support is objectionable. It is true that breakages of wires caused by too great rigidity at points of support are of rare occurrence; but where mechanical forms of cable clamps are used, as with the suspension type of insulator, and in many of the larger pin type insulators, the method of clamping the conductor to avoid mechanical injury or weakening due to vibration or to swaying of the wires in a wind, should receive careful attention.

CHAPTER X

TRANSMISSION-LINE SUPPORTS

142. General Considerations. Types of Transmission-line Supports.—The supporting poles or structures for overhead electric power transmission lines are of various kinds. Where the ordinary wood telegraph pole or a larger single pole of similar type is not suitable, double poles of the "A" or "H" type, or even braced wooden towers of considerable height and strength, may sometimes be used with advantage. Under certain conditions it may be economical to use steel poles of the tubular type, or light masts of latticed steel, even for comparatively short spans; and poles of reinforced concrete have much to recommend them. But for long spans, and the wide spacing of wires necessary with the higher pressures, steel towers, either of the rigid or flexible type, will generally be required.

The use of wood poles is therefore limited to comparatively low pressures, and spans of moderate length. It is probable that, in rough country where suitable timber is plentiful, and the cost of transporting steel towers would be high, wooden supports of the "A" or "H" type might be used economically for voltages up to 60,000, even if two three-phase circuits were carried on one set of poles; but for higher voltages the steel tower construction with fairly long spans, would in almost every case, be preferable. The case of a 100,000-volt three-phase line supported on wood poles was mentioned in Chapter I as an example of economic wood-pole construction for a high voltage transmission. Another example of a wood structure, erected where steel would have been adopted by many engineers as being the only material available, occurs at a river crossing between Astoria and Flavel, Ore. At this point a 1611-foot span is supported on wood-pole towers 135 ft. high, a description of which will be found in the *Electrical World* of Sept. 11, 1915.

The decision as to the best type of line to adopt is not easily or quickly arrived at. The problem is mainly an economic one, and the decision will depend, not only on the first cost of the

various types of line construction, but also on the probable life of the line and the cost of maintenance.

It is necessary to make up many preliminary estimates of the completed line, and these must obviously include, not only the cost of the various types of supporting structure delivered at points along the line, but also the cost of foundations and erection. Again, even if a suitable kind of wood is readily available in the district to be traversed by the transmission line, it is possible that the cost of seasoning the poles, and treating them with preservative compounds to ensure a reasonably long life, may render the use of steel structures more economical even for comparatively low pressures. The use of latticed steel poles, from 30 to 40 feet high, capable of being shipped and handled in one piece, appears to be gaining favor in districts where ultimate economy over wood poles can be shown to result from the adopcion of these light steel structures. The life of a steel tower line depends somewhat on climatic conditions. In Great Britain the dampness of the climate, together with the impurities in the atmosphere in the neighborhood of manufacturing and populous districts, render light steel structures less durable than in America (except, perhaps, on the Pacific coast, where special precautions are required to guard against rapid corrosion due to the prevalence of fogs and moisture). Not only has the iron-work protected by paint to be repainted on the average every three years, but the spans must usually be short, as the private ownership of valuable property renders the construction of a straight transmission line with long equal spans almost impossible in the United Kingdom. These conditions are all in favor of the employment of selected and well creosoted wood poles, the life of which may be 30 years or more.

When making comparisons between wood and steel for transmission line supports, it is not only the matter of first cost that has to be considered. Steel structures have the advantage of being invulnerable to prairie and forest fires; moreover, owing to the longer spans rendered possible by the stronger and taller supports, there is less chance of stoppages owing to broken insulators, and less leakage loss over the surface of insulators. A fact that is often overlooked is that the size of conductor limits the practical length of span; for instance, with a small conductor such as a No. 4 B. & S., it would not be wise to have spans much above 250 or 300 ft. This suggests what is frequently found to be

the case, namely, that the total cost of a line may be reduced by using a conductor of rather larger section than the electrical calculations would indicate as being necessary, because the stronger cable permits of a wider spacing of the supporting towers.

The economic length of span on steel tower lines usually lies between 500 and 700 ft., but very much longer spans can be used where the character of the country would render their use economical or where rivers have to be crossed.[1] On the transmission system supplying Dunedin City, New Zealand, with electric energy at 35,000 volts, there is a span 1700 ft. long where the line crosses the ravine near the power station. The peculiarity of this span is the great difference in level between the two supports, the upper tower, which is a special steel structure, being 650 ft. above the lower tower.

143. Wood Pole Lines. Kinds of Wood Available.—Among the varieties of straight-growing timber used for pole lines on the American continent may be mentioned cedar, chestnut, oak, cypress, juniper, pine, tamarack, fir, redwood, spruce, and locust. In England the wood poles are usually of Baltic pine or red fir from Sweden, Norway, and Russia. The woods used for the cross-arms carrying the insulators include Norway pine, yellow pine, cypress or Douglas fir, oak, chestnut, and locust.

Probably the best wood for poles is cedar; but chestnut also makes excellent durable poles. Much depends, however, on the nature of the soil, and, generally speaking, poles cut from native timber will be more durable than poles of otherwise equally good quality grown under different conditions of soil and climate.

With the more extended adoption of preservative treatments (to be referred to later), the inferior kinds of timber which under ordinary conditions would decay rapidly, will become of relatively greater value, and with the growing scarcity of the better kinds of timber, it is probable that poles of yellow pine, tamarack, and Douglas fir will be used more extensively in the future.

The trees should be felled during the winter months, and after being peeled and trimmed should be allowed to season for a period of at least twelve months.

A brief specification covering wood poles for power transmission lines will be found in Appendix II at the end of this book.

[1] See Article 140, Chapter IX.

144. Typical Wood Pole Lines.—For a single three-phase line transmitting power at 20,000 to 22,000 volts single poles having a top measurement of about 8 in. would be suitable. The distance between wires would be about 3 ft.; the arrangement being as shown on the sketch, Fig. 110, with pole-top details as in Fig. 111. This shows an arrangement without overhead guard wire, but with some or all poles protected by a grounded light-

Fig. 110.—Typical wood pole line for pressures up to 22,000 volts three-phase.

ning rod. In exposed positions, and at angles, pieces of bent flat iron may be fitted with advantage on the cross-arm near the insulators, as shown by the dotted lines in Fig. 111. These pieces serve the double purpose of hook guard in case of the wire slipping off insulator, and of additional protection against lightning. A discharge from the line tends to leap across to this grounded metal horn over the surface of the insulator, thus frequently preventing the piercing or shattering of insulators.

Fig. 112 shows the pole-top arrangement for a 33,000-volt

line of the Central Illinois Company, while the double insulator construction adopted by the same company for corner poles is illustrated in Fig. 113. These two illustrations are reproduced here by kind permission of the *Electrical World.*

A simple "A" frame construction for a duplicate three-phase line operating at 11,000 volts is shown in Fig. 114. Another type of construction for duplicate three-phase line is shown in Fig. 115, where the standard single wood pole is used. This is the arrangement adopted by the Central Illinois Company when it is desired to carry two circuits on the one pole line.

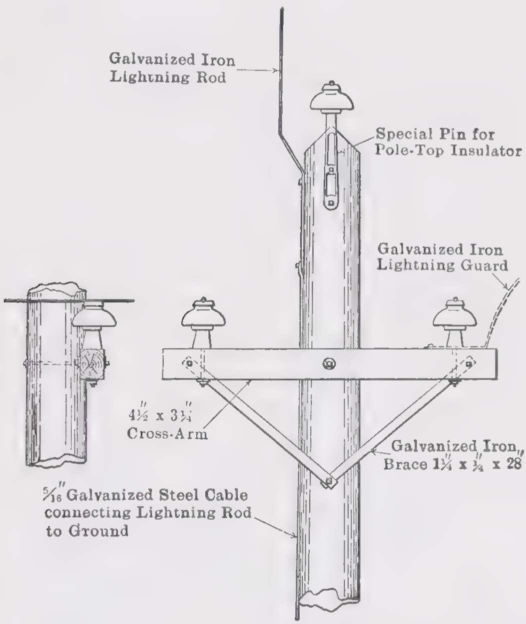


FIG. 111.—Pole-top details.

145. Life of Wood Poles Preservative Treatment.—It is not easy to estimate the probable life of poles, because this will depend not only on the kind of timber, but also on the nature of the soil, climatic conditions, the time of seasoning, whether or not the poles have received treatment with preservative compounds, and the nature of such treatment.

In England the life of well-seasoned, creosoted poles may be about 35 years in good soil, and from 18 to 20 years in poor

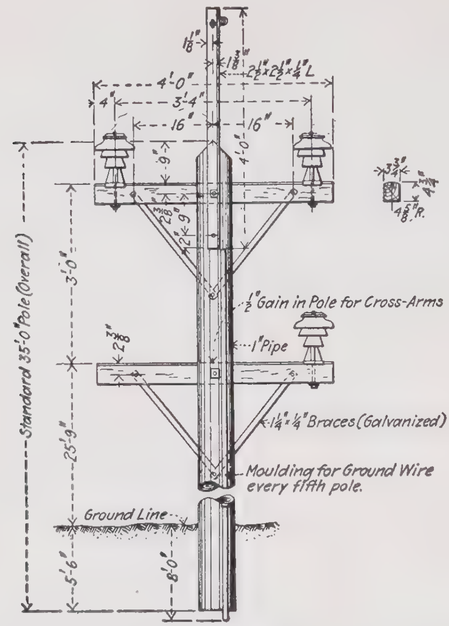


FIG. 112.—Single-circuit construction for three-phase overhead transmission.

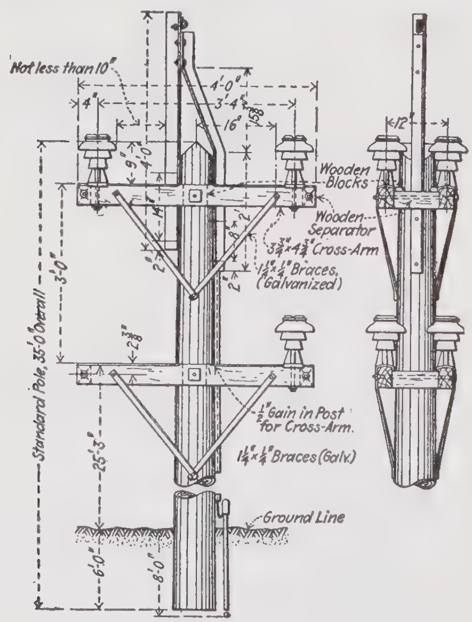


FIG. 113.—Double-arm corner construction.

soil. On the American continent, where untreated poles have been used in large numbers, the average life is probably about 12 years. The better woods, such as cedar and chestnut, might last on the average 14 to 16 years, while juniper and pine might have to be replaced in 6 to 10 years. On certain lines where untreated poles of unsuitable timber have been erected in poor soil, or where destructive insects are particularly active, the poles have had to be replaced in less than 4 years. The creosoted poles, as used in England, will usually stand best in moist or clayey

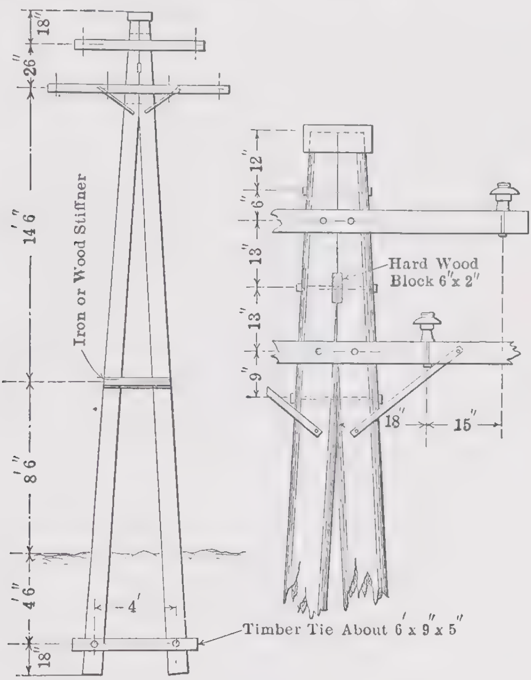


FIG. 114.—Typical "A" frame construction for duplicate three-phase line.

ground; there is a tendency for the creosote to run out and be absorbed into the ground when the soil is loose and sandy, with the result that the poles deteriorate rapidly just below the ground level. Marshy soil is generally bad for wood poles, also ground that is alternately wet and dry.

Preservative Treatment of Poles.—Many chemical solutions and methods of forcing them into the wood have been tried and used with varying success; but it is generally conceded that

treatment with coal-tar creosote oil gives the best protection against decay; and its cost is probably lower than that of any other satisfactory treatment.

There are three recognized methods of applying oil:

(a) The high-pressure treatment (Bethel system).
(b) The open-tank treatment.
(c) Brush treatments.

In Europe the treatment known as the Rüping process is largely used; it is less costly than the Bethel system. In France

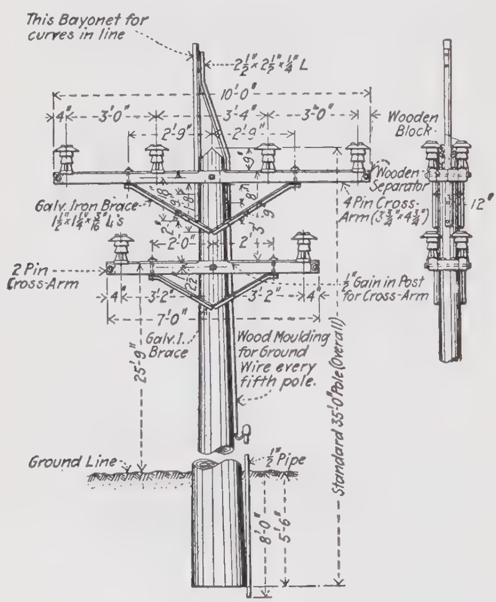


Fig. 115.—Double-circuit construction.

copper sulphate is used extensively as a preservative (Boucheric process), but the results are not entirely satisfactory.

HIGH-PRESSURE TREATMENT

This is undoubtedly the best, but it is also the most costly. The poles, after being trimmed and framed, are placed in large

treating cylinders capable of being hermetically closed. If the poles are green or wet they are first subjected, in these cylinders, to a steaming process from three to eight hours, the steam being admitted under a pressure of 12 to 20 lb. The steam is then blown off, and the treating cylinder is exhausted, the vacuum being maintained for a period of one to two hours. Immediately afterward the creosote is forced in under pressure at a temperature of 140° to 200° F. Seasoned timber is not subjected to the steaming process, but the temperature inside the treating cylinder is raised by means of heating coils to about 150° F. prior to the filling process.

The poles will absorb from a minimum of 10 lb. to a maximum of 15 lb. of oil per cubic foot. The softer and more porous woods will absorb the most oil; but on the other hand, the benefit such woods derive from the treatment in the matter of increased life is more marked than in the case of the closer grained timber.

OPEN-TANK TREATMENT

The butts of the poles are placed in the creosote oil, which is preferably heated to a temperature of 200° F. to 220° F. They are maintained in this bath for a period of one to three hours, after which they are placed in cold oil for another period of one to three hours. This process will permit of a complete penetration of the sapwood to a height of about 2 ft. above ground level. When properly carried out it is capable of giving very satisfactory results. The open-tank process is specially applicable to the treatment of the more durable kinds of timber, such as cedar and chestnut.

BRUSH TREATMENT

The oil is applied hot with hard brushes, a second coat being applied after the first has soaked in. The temperature of the oil should be about 200° F. This method of application of the oil is the cheapest and the least effective, but it affords some protection when the wood is well seasoned and dry. There is little advantage to be gained by the external application of preservative compounds to green timber; indeed, the sealing up of the surface of such timber, by enclosing the fermentative juices, may even lead to more rapid decay. The brush treatment cannot be applied to poles which are set in the winter months in cold cli-

mates, as the frost would so harden the surface of the poles that there would be no absorption of the preservative liquid.

The quality of oil used, whatever the method of application, is a matter of importance. In circular No. 98 of the United States Forest Service, Department of Agriculture, issued May 9, 1907, the concluding statement is to the effect that light oils boiling below 400° F. will not remain in the timber; but the heavy oils, containing a high percentage of anthracene oil, will remain almost indefinitely, and afford excellent protection against decay and boring animals.

The reader who desires to go further into the important question of wood pole preservation, is referred to the excellent book by Howard F. Weiss, "The Preservation of Structural Timber" (McGraw-Hill Book Company).[1] It seems hardly necessary to point out that the saving effected by prolonging the life of the poles will usually justify the cost of the special treatment. Given reliable data regarding costs and probable life of treated and untreated poles, the necessary calculations are easily made. As an example; if the cost of a pole after treating by the brush process is $24 and the average life 10 years, while the extra cost of open-tank treatment is $1 with an added 10 years to the life of the pole, it is easy to show that the tank treatment will be the more economical in the long run.

Inflammability of Treated Timber

Poles and cross-arms treated with creosote oil are less liable to destruction by fire than untreated timber of the same kind. This appears to be due to the fact that the free carbon deposited by the burning oil on the surface of the timber affords some protection from the action of the fire. A committee appointed by the National Electric Light Association of New York City conducted a series of experiments on similar specimens of treated and untreated short-leaf pine, and proved conclusively that the latter suffered considerably more damage from the effects of fire than the specimens that had been impregnated with creosote oil.

Reinforcing Pole Butts.—Wood poles usually decay most rapidly at or near the ground surface. It is not always necessary

[1] See also "Preservative Treatment of Telephone Poles," by F. L. Rhodes and R. F. Hosford, *Trans.* A. I. E. E., vol. xxxiv (part 2), p. 2549, Oct., 1915.

to replace poles which may be otherwise sound, but which have been weakened locally by decay just below the ground surface. They may be reinforced by means of steel rods about ½ in. diameter, pointed at both ends and driven into the pole above and below the damaged portion. Concrete is then filled in around the pole, extending at least 12 in. above the ground level. This and similar methods of prolonging the life of poles have proved satisfactory. The cost may be from $3 to $4 per pole, and the life may be prolonged 5 to 10 years.

146. Insulating Qualities of Wood Poles.—One advantage which may be claimed for wood poles is the possibility of working on live wires with little danger to life when the conditions are favorable. In the Black Hills in the mining district of South Dakota, where the climate is dry, it is usual to work on live wires at 24,000 volts, supported on ungrounded poles, without using any insulating devices. In this instance, the separation between wires is unusually large, being 5 ft., which affords additional safety. On the other hand, it may be argued that accidental contact with a wood pole carrying high-tension conductors may be a danger to the public, which is practically absent in the case of grounded steel poles or towers. Tests have been made to determine, if possible, the nature of charges likely to pass to ground through a person touching ungrounded wooden poles of a high-tension transmission. These tests show that it is possible for poles to become dangerously charged, but not probable. A grounded metal ring or wire placed around the pole from 6 to 7 ft. above ground eliminates all possibility of accidents from this cause.

147. Weight of Wood Poles.—For the purpose of estimating the costs of transport and handling of poles, the weight may be calculated on the assumption that the pole is of circular section and of uniform taper, such that the diameter D at the bottom is equal to the diameter d at the top *plus* a quantity tH, of which H is the distance between the two sections considered, and t is a constant depending on the taper and therefore on the kind of wood. Some approximate (average) values of t together with average weight per cubic foot of various kinds of *dry* timber will be found in the accompanying table, from which the value of t for cedar is seen to be 0.0165 (the height H being understood to be expressed in *inches*), and the weight per cubic foot, 35 lb.:

CONSTANTS FOR WOOD POLES

Kind of wood	Wt. per cubic foot, lb. approx.	Natural taper t, average	Modulus of rupture S[1]	Modulus of elasticity M[2]
Juniper	..	......	3700	
American Eastern white cedar	35	0.0165	4000	700,000
Spruce	27	......	4500	1,300,000
White pine	26	......	4500	1,000,000
Red pine	34	......	5000	
Douglas fir[3]	34	......	6500	1,400,000
Norway pine	..	......	7000	1,400,000
Redwood	..	......	7000	700,000
Idaho cedar	23	0.01	6000	
Chestnut	42	0.016	6000	900,000

[1] Being stress in pounds per square inch at moment of rupture under bending conditions.

[2] Inch units. Average figures, which must be considered approximate only.

[3] This name is intended to cover yellow fir, red fir, Western fir, Washington fir, Oregon fir, North-west and West-coast fir.

The *volume* of a frustrum of right circular cone is:

$$Volume = \frac{H}{3} \times \frac{\pi}{4}(D^2 + Dd + d^2)$$

but $D = d + tH$, and the formula becomes:

$$Volume = \frac{\pi H}{12}(3d^2 + t^2H^2 + 3dtH)$$

By using this formula and putting for H the value $65 \times 12 = 780$ in., and for d the value 7 in., the weight of a pole of American eastern white cedar measuring 7 in. diameter at top and 65 ft. over-all length works out at 2410 lb.

148. Strength and Elasticity of Wood Poles.—Apart from the dead weight to be supported by the poles of a transmission line—which will include not only the fixtures and the conductors themselves, but also the added weight of sleet or ice in climates where ice formation is possible—the stresses to be withstood include the resultant pull of the wires in adjoining spans, and the wind pressure on poles and wires. It is customary to disregard the dead weight or column loading, except when the spans are large and the conductors numerous and heavy. A formula for approximate calculation of loads carried by poles when acting as

struts or columns will be given later. The pull due to the conductors on corner poles is usually met by guying these poles, by which means the pull tending to bend the pole is largely converted into an increased vertical downward pressure; but even on straight runs there may be stresses due to unequal lengths of span which would cause a difference in the tensions on each side of the pole. The most important stresses to which the poles are subjected, apart from such accidents as are due to falling trees or the severing of all wires in one span, are those caused by strong winds blowing across the line. The resulting pressure at pole-top due to strong winds acting on long spans of ice-coated wires may be very great, and the poles must be strong enough to resist this.[1]

For the purpose of making strength and deflection calculations, the pole may be considered as a truncated cone of circular section, firmly fixed in the ground at the thick end, with a load near the small end in the form of a single concentrated resultant horizontal pull. The calculation is therefore exactly the same as for a beam fixed at one end and loaded at the other. Such a beam, if it exceeds a certain length depending upon the amount of taper, will not break at the point where the bending moment is greatest (*i.e.*, at the ground level), because, in a beam of circular section and uniform taper, the stresses in the material are not necessarily greatest at this point, as will be shown later. The ordinary telegraph or electric lighting pole usually breaks at a point about 5 ft. above ground unless the butt has been weakened by decay.

Calculations on strengths and deflections of wood poles cannot be made with the same accuracy as in the case of steel structures; and the constants in the table of Article 147 are averages only for approximate calculations. The factor of safety generally used on the American continent is 6, both for poles and cross-arms. The maximum wind pressure is taken at 30 lb. per square foot of flat surface, or 18 lb. per square foot of projected surface of smooth cylinders of not very large diameter. In England the factor of safety for telegraph poles is 8, and for power lines 10. The latter figure would seem to be unnecessarily high: it suggests a want of confidence either in the strength calculations or in the tests and load assumptions on which the calculations are based.

[1] See Article 131 in Chapter IX.

149. Calculation of Pole Strengths.—The relation between the externally applied load and the stresses in the fibers of the wood is:

Bending moment = stress in fibers most remote from neutral axis $\times$ *section factor*, or $M_B = S \times Z$.

If P is the force in pounds applied at a point distant x in. from the cross-section A (see Fig. 116), then:

$$M_B = Px \text{ lb.-in.}$$

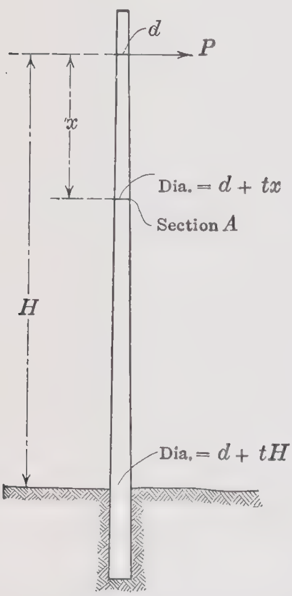


FIG. 116.—Wood pole with horizontal load near top.

And if the stress S is expressed in pounds per square inch, and the section is assumed circular:

$$Px = S \times \frac{\pi d^3}{32}$$

But it is assumed that the diameter at any point x in. below the section of diameter d is $d \times tx$, therefore:

$$S = \frac{32P}{\pi} \times \frac{x}{(d + tx)^3} \qquad (149)$$

In order to find the position of the cross-section at which the pole is most likely to break—that is to say, where the fiber stress is a maximum—it is necessary to differentiate the last equation with respect to x, and find the value of x which makes this differential equal to zero. This gives

$$x = \frac{d}{2t}$$

for the point where the stress S is a maximum. The position of this cross-section is evidently not always at ground level. If this value of x is greater than H, then the maximum fiber stress will be at ground level, and it is calculated by substituting H for x in formula (149).

The diameter of the pole at the weakest point is:

$$\begin{aligned} d_w &= d + tx \\ &= d + t\left(\frac{d}{2t}\right) \\ &= 1.5\, d \end{aligned}$$

and it is only when the diameter at ground level is greater than one and a half times the diameter where the pull is applied that the pole may be expected to break above ground level.

If the stress S, the taper t, and the pole-top diameter d are known, the load P is readily calculated as follows:

$$\text{Bending moment} = P \times x$$

$$\text{Resisting moment} = S \times \frac{\pi d^3_w}{32}$$

But $x = \frac{d}{2t}$ and $d_w = 1.5d$

therefore

$$\frac{Pd}{2t} = S \times \frac{\pi \times (1.5d)^3}{32}$$

or

$$P = \frac{2t\ S\pi \times 3.375 \times d^3}{32 \times d}$$

$$= 0.662 \times S \times t \times d^2 \qquad (150)$$

Similarly, if the pull P is known, the pole-top diameter should be:

$$d = \sqrt{\frac{P}{0.662 \times S \times t}} \qquad (151)$$

Example of Strength Calculation

Consider a pole of Eastern white cedar designed to sustain a pull of 500 lb. applied 26 ft. above ground level. The average breaking stress (from table of constants) is 4000 lb. per square inch, and assuming a factor of safety of 6 the safe working stress is $S = 660$ lb. per square in. The other numerical values are:

$$P = 500 \text{ lb.}$$
$$H = 26 \text{ ft.}$$
$$t(\text{from table}) = 0.0165$$

By formula (151):

$$d = \sqrt{\frac{500}{0.662 \times 660 \times 0.0165}}$$
$$= 8.33 \text{ in.}$$
$$d_w = 1.5 \times 8.33 = 12.5 \text{ in.}$$

The distance below point of application of load of the section where fiber stress is a maximum is:

$$x = \frac{d}{2t} = 252 \text{ in.} = 21 \text{ ft.}$$

Therefore this pole, if subject to a load about six times greater than the maximum working load, may be expected to break $26 - 21 = 5$ ft. above ground level.

Double pole supports of the type illustrated in Fig. 114 will be twice as strong as each of the component poles in resisting stresses applied in the direction of the line; but they will be able to withstand about five times as great a load as the single pole when the stresses are in a direction at right angles to the direction of the line. When loaded in this manner up to the breaking point, these double poles of the "A" type usually fail through the buckling of the member in compression due to initial want of straightness. The strength of both the "A" and the "H" type of pole structure can to some extent be increased by judicious and rigid bracing.

150. Deflection of Wood Poles.—It is now generally recognized that there are advantages in having transmission-line supports with flexible or elastic properties. The ordinary single wood pole is very elastic, and will return very nearly to its original form after having been deflected considerably by abnormal stresses. The figures given for the elastic modulus in the tal le previously referred to are subject to correction for different qualities and samples of the same timber. It is well to make a few experiments on the actual poles to be used if accuracy in calculated result is desired. The double-pole structures of the "A" or "H" type will have about half the deflection of the single poles in the direction of the line, and, of course, very much less in a direction at right angles to the line. An "A" pole of usual construction with the two poles subtending an angle of 6½ degrees will deflect only about one-fiftieth of the amount of the single-pole deflection under the same transverse loading. The movement is usually dependent upon the amount of slip between the two poles at top, which again depends upon the angle subtended by the poles.[1] If this angle is as much as 10 degrees there will be practically no likelihood of the poles slipping at the top joint; but this large angle is unsightly, and probably makes a less economical structure than the more usual angle of about 6½ degrees.

151. Calculation of Pole Deflections.—Assume the pole to be fixed firmly in the ground, and that there is no yielding of founda-

[1] Much useful information on the behavior of "A" poles under test is to be found in Mr. C. Wade's paper read before the Institution of Electrical Engineers on May 2, 1907.

tions. The load P being applied in a horizontal direction at the top end, as indicated in Fig. 117, the pole may be considered as a simple cantilever, the deflection of which, *if the section were uniform throughout the entire length,* would be:

$$\delta = \frac{PH^3}{3MI}$$

where δ and H are in inches; I is the moment of inertia of the section, and M is Young's modulus (pounds per square inch). For a circular section $I = \frac{\pi d^4}{64}$ where d is the diameter of the (cylindrical) pole in inches. The formula then becomes:

$$\delta = 6.78 \frac{PH^3}{Md^4} \tag{152}$$

If P is evenly distributed, as would be the case with a uniform wind pressure on the pole surface, regardless of other loads, the deflection would be:

$$\delta = \frac{PH^3}{8MI}$$

but it is best to consider the wind pressure on pole surface as a single equivalent load concentrated at pole-top and added to the load due to wind pressure on the wires. When estimating the probable value of this equivalent load, it should be remembered that the wind pressure is not evenly distributed along the length of the pole, since the wind velocity at ground level is comparatively small and increases with the height above ground surface.

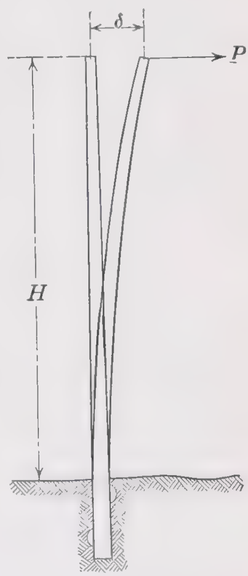


FIG. 117.—Deflection of wood pole.

The formula (152) assumes a constant diameter throughout length of pole, and the question therefore arises as to where the measurement of diameter should be made on an actual pole. Mr. S. M. Powell has shown that, on the assumption of a uniform taper, the quantity d^4 in formula (152) should be replaced by $(d_g{}^3 \times d_1)$ where d_g is the diameter at ground level and d_1 is the diameter where the force P is applied.

Example of Calculation of Pole-top Deflection

Using the same figures as in the example of strength calculations:

$$P = 500 \text{ lb.} \qquad H = 26 \times 12 = 312 \text{ in.}$$
$$d_1 = 8.33 \text{ in.} \qquad t = 0.0165$$
$$d = 8.33 + (0.0165 \times 312) = 13.48 \text{ in.}$$
$$M = 700{,}000$$

then

$$\delta = \frac{6.78 \times P \times H^3}{M\ (d_g{}^3 \times d_1)}$$
$$= \frac{6.78 \times 500 \times (312)^3}{700{,}000 \times (13.48)^3 \times 8.33}$$
$$= 7.2 \text{ in.}$$

When possible it is well to make tests on a few actual poles; then for similar poles of the same material subject to the same loading:

$$\delta \propto \frac{H^3}{d_g{}^3 \times d_1}$$

152. Pole Foundations.—A permanent deflection of the pole when the stresses are abnormal may occur owing to the yielding of the earth foundation; but this is unusual if the poles are properly set in good ground.

The diagram Fig. 118 has been drawn to show the depth to which poles of various heights are usually set. These depths are such as would be adopted on a well-designed pole line, and need not be exceeded except in special cases. In marshy or otherwise unsatisfactory ground, special means must be adopted to provide a reasonably good setting for the pole butts.

Loam and gravel, and even sand, or a mixture of these, provides a firm foundation for poles. A pole that is properly set should break before the foundations will yield to any appreciable extent. Even if there should be a movement of the pole butt in the ground with excessive horizontal load at pole-top, this will result in a firmer packing of the earth, which will then be better fitted to resist any further movement.

Firm sand, gravel, or loam, will withstand a pressure of about 4 tons per square foot; but only half this resistance should be reckoned on in the case of damp sand, moist loam, or loose gravel.

Proper supervision is necessary to ensure that the earth shall

be packed firmly around the pole when refilling the holes. This matter of tightly packing the dirt around the pole butt is referred to in Appendix II. Although no attempt has been made to treat adequately, in this book, of practical details such as the best method of digging holes, it may not be out of place to mention that the use of dynamite for digging post holes appears to have met with success where the method has been carefully studied and intelligently applied. (Refer to the *Electrical World*, June 8, 1912 and Feb. 7, 1914.)

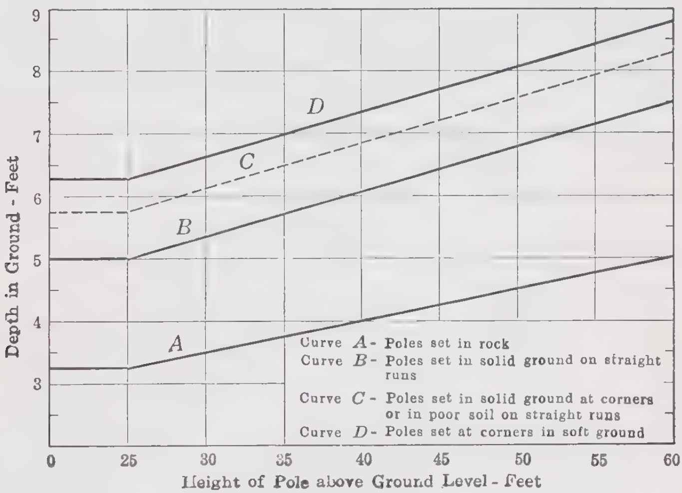


FIG. 118.—Chart giving approximate depth of holes for wood poles.

153. Spacing of Poles at Corners—Guy Wires.—In order to reduce the stresses, not only on the pole itself, but also on the insulator pins and cross-arms, it is usual to shorten up the spans on each side of the corner pole. The reduction in length of span will depend upon the amount by which the direction of the wires departs from the straight run. A rough and ready rule is to reduce the span length $1\frac{1}{2}$ per cent. for each degree of deviation from the straight line. For angles less than 5 degrees, it is not necessary to alter the span.

It is not advisable to turn more than 25 degrees on one pole, and whenever the side strain is likely to be excessive, double cross-arms and insulators should be used. By giving proper attention to the matter of guying and to the mechanical con-

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struction generally, it is not difficult to meet all requirements at points where a change of direction occurs.

A safe plan is to assume that a corner pole must carry the full load without breaking if the guy wire or wires should fail to take their proper share of the load: but all corner poles should be propped or guyed for extra safety, and to avoid the unsightly appearance of poles bent under heavy side stresses or set at an angle with the vertical.

Sometimes when sharp corners have to be turned, the spans on each side are "dead-ended" on poles with double fixtures. Such poles are head-guyed, and the span adjoining the guyed pole is usually shortened, being not more than three-fifths of the average spacing. For further particulars of common practice in guying poles in special positions, the reader is referred to the sample specification for wooden pole line in Appendix II.

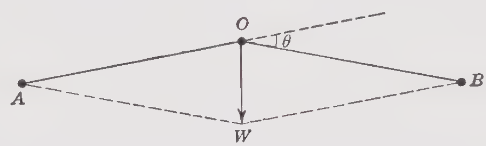

Fig. 119.—Diagram of stresses at corner pole.

The non-synchronous swaying of wires in a high wind, although uncommon, sometimes occurs on wood pole lines, being aggravated by the difference in the natural period of oscillation of poles and wires. This trouble can generally be cured by guying one or more of the poles at the place where the wires have been found to swing non-synchronously.

Guy wires should be of galvanized stranded steel cable, the breaking strength of which should preferably not exceed about 34 tons per sq. in. The reason for this limitation of strength is that the high-strength steel is usually too hard to allow of proper handling and finishing off.

154. Load to be Carried by Corner Poles.—If P is the total tension in pounds of all the wires on each side of the corner pole, and if θ is the angle of deviation as indicated in Fig. 119, then the resultant pull in the direction OW at the pole top will be,

$$W \text{ (lb.)} = 2P \sin\left(\frac{\theta}{2}\right) \qquad (153)$$

The stress in the guy wire is readily calculated when the angle

α (Fig. 120) which the wire makes with the vertical is known. If W is the side pull as calculated by formula (153), then

$$\text{Tension in guy wire} = \frac{W}{\sin \alpha}$$
$$= \frac{W \times OC}{CD} \qquad (154)$$

155. Props or Struts—Wood Poles in Compression.—Sometimes it is difficult or impossible to provide guy wires in certain locations; or impurities in the atmosphere may render the use of props or push braces preferable to guy wires. In such cases it is necessary to know approximately what load a wooden pole will support in compression, that is to say when used or considered as a column. Instead of using the values of unit stress, S, as

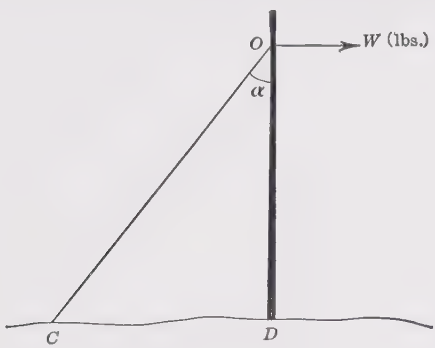

FIG. 120.—Diagram of stress in guy wire.

given in the Table on p. 314, the ultimate stress which a wood column will stand in compression should be calculated by the empirical formula:

$$\text{Stress in compression (lb. per sq. in.)} = S\left(1 - \frac{l}{60d}\right) \qquad (155)$$

where l is the length in inches, and d is the diameter or least thickness at the center of the strut.

Example.—Calculate the safe load for a prop of Douglas fir, 8 in. diameter and 10 feet long, assuming a safety factor of 6.

By formula (155) the breaking stress will be

$$6500\left(1 - \frac{120}{60 \times 8}\right) = 4870 \text{ lb. per sq. in.}$$

and the maximum safe load will be

$$\frac{4870}{6} \times \frac{\pi}{4} \times 64 = 41{,}000 \text{ lb.}$$

156. Reinforced Concrete Poles.—As substitutes for wood poles supporting overhead wires, steel poles of the tubular form and latticed steel masts are used. The full advantage of the galvanized or painted steel structure is best realized in the high towers with extra wide spacing, such as are used for the transmission of electric energy at high pressures. The use of Portland cement for moulded poles of moderate height is by no means new; the experimental stage has long ago been passed, and with the deplorable but no less rapid depletion of our forests and the incomparably longer life of the concrete poles, these will probably be used in increasing numbers during the next few years.

There is much to be said in favor of the wood pole when the right kind of timber, properly seasoned and treated, is used; but, apart from the general unsightliness of wood poles in urban districts, their life is uncertain and always comparatively short. In Switzerland the experiment has been tried of covering the ordinary wood pole with concrete mortar about 1 in. thick. The strength, and especially the life, are greatly increased thereby, as the decay which so frequently occurs at ground level will be largely, if not entirely, prevented; but it is doubtful whether the system will, in the long run, prove satisfactory or economical. The ideal material to use for reinforcing concrete is undoubtedly steel or iron. Longitudinal rods or bars of iron can be placed exactly where required to strengthen those parts of the pole section that will be in tension, and the concrete, filling up the spaces between the reinforcing rods, takes the place of all bracing and stiffening members of the ordinary steel structure in an almost perfect manner. It is probably at this time generally admitted that iron embedded in cement will last almost indefinitely without suffering any deterioration. When excavating for the foundations of the new General Post Office in London, England, some old Roman brickwork was discovered in which the hoop-iron bonds were still bright and in perfect condition. The life of a concrete pole is, in fact, almost unlimited, a consideration which should not be overlooked when estimating the relative costs of different kinds of supporting structures. It requires no painting and practically no attention once it is erected. If any small cracks should at any time develop, they can readily be filled with cement.

While referring to the advantages of the cement pole it may be

added that every pole is virtually a lightning rod, an advantage which it shares with the steel pole or tower. On lines where both timber and concrete poles have been used and where many wood poles have been shattered by lightning, the concrete poles have rarely been struck. There is an instance of a concrete pole of the Marseilles (Ill.) Land & Water Company having been struck, but the only damage done was the chipping out of a small piece at the top of the pole and one at the bottom where the current entered the ground after following down the steel reinforcing bars inside the pole.

157. Weight and Cost of Concrete Poles.—The weight of concrete poles is necessarily considerable, and unless the poles are made near the site where they will be erected the cost of transportation will generally be prohibitive. Concrete poles usually measure 6 in. at the top with a base width of 10 in. to 14 in. depending on the height. It is, however, quite permissible to use poles with 5 in. top measurement, in which case the base measurement might be about 9 in. for poles not exceeding 25 to 30 feet overall length.

The weight of concrete is about 150 lb. per cubic foot, and the cost of poles will range between 35c. and 70c. per 100 lb. Should it be found that conditions of labor, transportation, etc., are such that the cost would be in excess of 70c. per 100 lb. it is probable that steel or wood supports would prove more economical than reinforced concrete.

On the basis of 50c. per 100 lb. weight of the finished pole, the following figures indicate roughly the approximate cost of concrete poles. The lengths given are the overall lengths, including the portion buried in the ground. The weights are such as might be expected for poles designed with hollow cores.

Length, ft.	Weight, lb.	Cost
30	1800	$9.00
35	2200	11.00
40	3600	16.00

The great weight of concrete poles is probably the most serious objection to their more general adoption in the place of wood poles, where the latter are not readily obtainable or where their appearance is unsightly.

It is probable that the concrete poles of cross-country transmission lines are usually made somewhat heavier than the strength requirements necessitate because, being moulded on site, not always with the best and most convenient appliances, they are made solid throughout or through a large part of their length, whereas a hollow construction would have been adopted had suitable collapsible cores been available.

Poles up to 35 ft. in length are usually moulded in a horizontal position, the forms being removed after three or four days. After a period of seasoning lasting from two to three weeks they are erected in the same manner as wood poles.

Poles longer than 35 ft. are often moulded in a vertical position. The forms are set up immediately over the hole previously prepared for the pole base. They are set truly vertical and temporarily guyed, the reinforcing inside the form being held together and in position by whatever means of tying or bracing may be adopted. Sometimes iron wire is used, but more uniform results are obtained by using specially designed iron distance pieces with the required spacing between them. The concrete is raised to the top of the mould by any suitable and economic means (preferably direct from the concrete mixer by an arrangement equivalent to the ordinary grain elevator) and is dropped in. By this means the hole in the ground is entirely filled with concrete. No tamping is required, a firm hold being obtained, since the ground immediately surrounding the concrete base has not been disturbed.

The best quality of crushed stone and sand should be used, the usual proportions being: cement, one part; sand, two parts; crushed stone, three or four parts, not too large to pass through a ¾-in. screen. The mixture used for the poles on the Pennsylvania Railroad is 1.5 :2 :4. When gravel is used the mixture may be one part of Portland cement to five parts of gravel, provided that the latter is graded, including sand, and with the largest pieces of a size to pass through a ¾-in. screen.

The cost of concrete poles, when the long life and other advantages are taken into account, does not compare unfavorably with that of other types; but it must not be overlooked that the cost of materials and labor required to manufacture the poles does not represent the cost of the finished pole erected in position. Much valuable information on the costs of manufacture and handling of concrete poles, together with practical details relating to

methods of manufacture, will be found in Mr. R. A. Lundquist's book on Transmission Line Construction.[1] The reader is also referred to an article by Mr. J. G. Jackson who describes in the *Electrical World* of Jan. 17, 1914, how the concrete poles used on the Toronto Hydro-electric system were manufactured. The article by Mr. R. D. Coombs, in the *Electrical World* of Feb. 6, 1915, and a more recent article entitled "Concrete poles carry 22,000-volt power line," in the *Electrical World* of Feb. 9, 1918 (Vol. 71, p. 296) should also be consulted by those desiring further information on concrete-pole transmission lines.

As an example of a concrete-pole line, the transmission line of the Northern Illinois Light and Traction Company, of Marseilles, Ill., may be mentioned. This company transmits three-phase energy at from 30,000 volts to 33,000 volts. Most of the poles used are about 30 ft. high, spaced from 125 ft. to 132 ft. apart. The section is square, with 6-in. sides at the top of the pole and 9 in. at the base. The reinforcing consists of six ½-in.-square steel bars through the entire length of the pole. Many of the concrete poles on this line have now been in position over nine years, and they have given entire satisfaction.

158. Strength and Stiffness of Concrete Poles.—When designing a concrete pole to withstand a definite maximum horizontal load applied near the top, the pole is treated as a beam fixed at one end and loaded at the other. The calculations are very simple if certain assumptions are made, these being as follows:

(1) Every plane section remains a plane section after bending.

(2) The tension is taken by the reinforcing rods.

(3) The concrete adheres perfectly to the steel rods.

(4) The modulus of elasticity of concrete is constant within the usual limits of stress.

The ultimate crushing stress of the concrete may be taken at from 2000 to 2600 lb. per square inch. The reinforcing bars should be covered with concrete to a depth of not less than 1 in. The effect of keeping the reinforcing bars under tension while the concrete is poured in the mould and until it has hardened sufficiently to support the strain itself has been tried and found to improve the performance of the poles, but it is doubtful whether the extra apparatus and labor required are justifiable on economic grounds. When subjected to excessive load a concrete pole will generally yield by the crushing of the material in the base near

[1] McGraw-Hill Book Co.

ground level; but, unless it is pulled out of its foundations, it will not fall to the ground.

The comparative rigidity of concrete poles cannot be said to be a point in their favor, as the flexibility and elasticity of wood poles and some forms of steel structures are features of undoubted advantage under certain conditions. On the other hand, the degree of deflection of concrete poles before breaking is remarkable. The elastic limit is variable, and no exact figure can be given for the elastic modulus of cement concrete; it may be as low as 1,000,000 but for a 1:2:4 mixture 2,000,000 may be taken as a good average figure for approximate calculations.

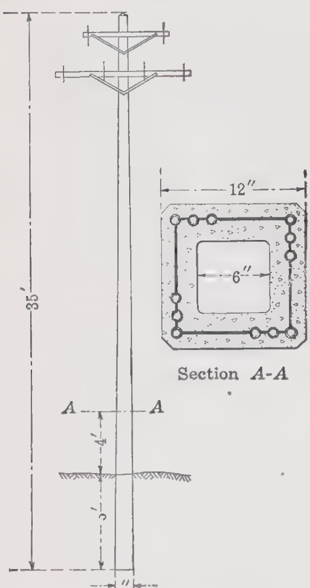

Fig. 121.—Concrete pole of hollow section.

Some tests made on 30-ft. concrete poles gave deflections of from 3 in. to 4 in. at a point near the top of pole, when subjected to a test load equal to about double the maximum working load.[1] Another series of tests made recently in England on some 44-ft. poles of hollow section, 17 in. square at the base and 8 in. square at the top (inside dimensions 13 in. and 4 in. respectively), with loads applied 38.5 ft. above ground level, gave a deflection of 66 in. under a horizontal load of 10,500 lb., and the permanent set on removal of load was 21 in. The pole did not fail completely until the deflection was 78 in.

The illustration Fig. 121 shows a typical concrete pole of hollow section suitable for carrying six transmission wires on two wooden cross-arms. The pole is 35 ft. long over all, about 6 ft. being buried in the ground. With a top measurement of 7 in. square and a taper to give an increase of 1 in. width for every 5 ft. of length, the size at the bottom will be 14 in. square. The

[1] These poles were probably of large cross-section. Some tests made on poles measuring 10 in. square at the base and 32 ft. high gave a deflection of just over 2 ft. with a horizontal load of 2000 lb. applied near the top.

drawing shows a section through the hollow pole taken at a point about 4 ft. above the ground level. Iron spacing pieces, as here shown, or their equivalent, must be placed at intervals to hold the longitudinal steel reinforcing bars in the proper position. The number of rods will vary with the distance below the point of application of the load. The bending moment to be resisted at every point being known and the taper of the pole decided upon, the amount of reinforcing required at any given section is easily calculated. The weight of a pole as illustrated would be about 2700 lb. without fixtures. The reinforcing rods and spacing rings would account for approximately one-seventh of the total weight. A factor of safety of four is generally employed in strength calculations of reinforced concrete poles. In some cases the calculations have been based on a safety factor of 5; but there appears to be no justification for using so large a factor.

159. Steel Poles and Towers—Introductory Remarks.—It cannot be said that there is at the present time one standard type of steel structure for supporting the conductors of overhead transmission lines; neither is it likely that one particular design will ever be found suitable for all countries, climates and voltages. Any kind of supporting structure which will economically fulfil the necessary requirements will answer the purpose of the transmission line engineer, who merely requires a durable mechanical structure to carry a variable number of insulators at a height above ground, and with a spacing between them, depending upon the voltage of transmission and the length of span.

As a substitute for wood poles, steel tubes have been used, either in one piece, or built up of several sections of different sizes in order to economize material and give a large diameter at the bottom where the bending moment is greatest, and a small diameter at the top where the bending moment is negligible. Steel poles of considerable height, suitable for longer spans, may be built up of three or four vertical tubes of comparatively small diameter joined and braced together at suitable intervals to give stiffness to the structure. It is doubtful whether, in the long run, such composite tubular structures will hold their own against the small-base latticed steel masts built up of standard sections of rolled steel, as used extensively on the continent of Europe, and, to a relatively smaller extent, in America. The term "tower" is applied mainly to the light steel structures in which the spac-

ing between the main upright members, at ground level, is large compared with the height of the structure; the usual proportion —which will generally be found to be the most economical in material—being 1 to 4; that is to say, if the base is square, the side of this square will be about one-quarter of the distance from the point of measurement to the top of the tower. If the towers are large, the footings are usually separate pieces which are correctly set in the ground by means of a templet, and to which the legs of the tower proper are afterward bolted. A good example of large steel towers is to be found in the 100,000-volt transmission line of the Great Western Power Co. of California. Two three-phase circuits are carried on these towers, the vertical spacing between the cross-arms being 10 ft. There are three cross-arms, each carrying two conductors—one at each end. The horizontal spacing between wires is 17 ft. on the two upper cross-arms and 18 ft. on the lower cross-arm, which is 51 ft. above ground level. No conductor is closer than 6 ft. 5 in. to the steel structures, this being the minimum clearance in the horizontal direction. The average distance between towers is 750 ft., and they are joined at the top by a grounded guard wire 5 ft. above the bottom of the highest cross-arm. The base of the tower measures 17 ft. square, the parts under ground being separate pieces of steel, buried to a depth of 6 ft. to which the tower proper is bolted after being assembled and erected on site.

Although the larger towers are nearly all built of the square type as used for windmills, there is a notable exception in the case of the 140,000-volt line in Michigan, where the towers are of a special three-legged type, built up entirely of angle sections.

Fig. 122 shows a typical form of small-base latticed steel mast on the transmission lines of the Iowa Railway and Light Company, Cedar Rapids, Iowa; while Fig. 123 is a good example of square base tower carrying two three-phase lines. These illustrations are reproduced from photographs kindly supplied by the Ohio Brass Co. of Mansfield, Ohio. The large towers of Fig. 123 were designed and constructed by the American Bridge Co. of Pittsburg for the American Gas and Electric Company's 130,000-volt transmission between Wheeling, W. Va. and Canton, Ohio. The six conductors are each of 200,000 circular mil cross-section, and the two grounded guard wires are of the same

size. The line is 55 miles long, and the average length of span is 580 feet.

The economical span for the square latticed poles, of the type shown in Fig. 122, is probably something less than 450 feet; but for comparatively light lines, this form of structure with spans of 400 to 430 feet is very satisfactory.

160. Flexible Towers.—Although calculations of stresses in transmission lines are usually based on the assumption that the ends of each span are firmly secured to rigid supports; this condition is rarely fulfilled in practice; there is some "give" about the poles or towers, especially when the line is not absolutely straight, and the insulator pins will bend slightly and relieve the stress when this tends to reach the point at which the elastic elongation of the wires will be exceeded. Then, again, the wires will usually slip in the ties at the insulators, even if these ties are not specially designed to yield or break before damage is done to the insulators or supporting structures. The use of the suspension type of insulator, which is now becoming customary for the higher voltages, adds considerably to the flexibility of the line.

In regard to the towers themselves, all steel structures for dead-ending lines or sections of lines are necessarily rigid, and the usual light windmill type of tower with wide base is also without any appreciable flexibility. The latticed steel masts, as used more generally in Europe than in America, are slightly more flexible, and the elastic properties of the ordinary wood pole are well known. The deflection of a wood pole may be considerable, and yet the pole will resume its normal shape when the extra stress is removed. There is much to be said in favor of so-called flexible steel structures; that is to say, of steel supports designed to have flexibility in the direction of the line, without great strength to resist stresses in this direction; but with the requisite strength in a direction normal to the line, to resist the side stresses due to wind pressures on the wires and the supports themselves.

Such a design of support has the important advantage of being cheaper than the rigid tower construction, in addition to which it gives flexibility where this is advantageous, with the necessary strength and stiffness where required. The economy is not only in the cost of the tower itself but in the greater ease of transport over rough country, the preparation of the ground, and erection.

The advantages of flexibility in the direction of the line are considerable. Probably the most severe stresses which a transmission line should be capable of withstanding are those due to the breakages of wires. Such breakages may be caused by abnormal wind pressures, by trees falling across the line, or by a burn-out due to any cause. Suddenly applied stresses such as are caused by the breaking of some or all of the wires in one span are best met by being absorbed gradually into a flexible system. The supports on each side of the wrecked span will bend toward the adjoining spans because the combined pull of all the wires in the adjoining spans is greater than the pull of the remaining wires, if any, in the wrecked span. This movement of the pole top results in a reduction of tension in the wires of the adjoining span owing to the increased sag of these wires; there will be an appreciable deflection of the second and third poles beyond the break, but the amount of these successive deflections will decrease at a very rapid rate and will rarely be noticeable beyond the fourth or fifth pole. It is obvious that, as the remaining wires in the faulty span tighten up, the stress increases; but the combined pull of these wires on the pole top is smaller than it was before the accident, since it is assisted by the pull of the deflected poles, and these joint forces are balanced by the combined pull of all the wires in the adjoining sound span, which pull, as previously mentioned, is smaller than it was under normal conditions.

The greater the flexibility of the supports in the direction of the line, the smaller will be the extra load which any one support will be called upon to withstand; on the other hand, it is usual to provide anchoring towers of rigid design about every mile on straight runs, and also at angles, in addition to which every fifth or sixth flexible tower may be head-guyed in both directions. In the writer's opinion, too much stress is usually laid on the necessity for providing rigid strain towers at frequent intervals to prevent the effect of a break in the wires, or the failure of a single support travelling along the line and causing injury to an indefinite number of consecutive spans. The semiflexible structures referred to are not designed, or should not be designed, without very careful consideration of the conditions they have to fulfil; and there appear to be no scientific reasons, and no records of injury to actual lines, which would justify the assumption that transmission lines of this type are liable to

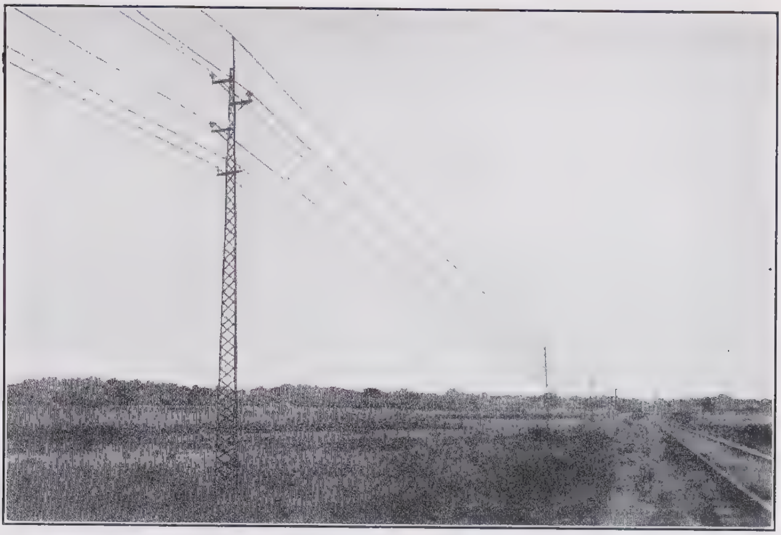

Fig. 122.

Fig. 123.—Typical steel tower transmission line.

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FIG. 124.—Flexible steel tower line.

be wrecked in the same wholesale or cumulative manner as a row of card houses. The strain towers are undoubtedly helpful at the time when the wires are strung; but it is possible that they are used at more frequent intervals than the economies of sound engineering require.

In level country, a modified clamp in the form of a sleeve with flared ends may be used in conjunction with the lighter (and cheaper) flexible type of supporting structures; and a compromise between the loose sleeve and the rigid clamp or tie can be used on all lines with flexible supports. Clamps of this type are designed to allow the wire to slip before the combined pull of all the wires exceeds the load that will permanently deform the supporting structure; and although it is almost impossible to ensure that such devices will remain for any length of time in the same condition as when they are installed, yet they will generally afford a reasonable degree of protection in the event of the simultaneous breaking of all the wires in one span. It is not unusual to carry a galvanized Siemens-Martin steel strand cable above the high-tension conductors on the tops of the steel structures. This has the double advantage of securely, but not rigidly, tying together the supports, and of providing considerable protection against the effects of lightning. The disadvantages are increased cost and possible—but not probable—danger of the gounded wire falling on to the conductors and causing interruption of supply.

The dead-end towers should be capable of withstanding the combined pull of all the wires on one side only, when these are loaded to the expected maximum limit, without the foundations yielding or the structure being stressed beyond the elastic limit. The flexible supports must withstand, with a reasonable factor of safety, the dead weight of conductors, etc., and the expected maximum side pressures; but in the direction of the line their strength must necessarily be small, otherwise the condition of flexibility cannot be satisfied.

It is easy to design braced A-frame or H-frame steel structures of sufficient strength to withstand the dead load and lateral pressure and yet have great flexibility, with correspondingly reduced strength, in the direction of the line. Great care must be used in designing a line of this type so that strength and durability shall not be sacrificed to lightness and flexibility without very carefully considering the problem in all its aspects.

The assumptions made for the purpose of simplifying strength calculations are not always permissible. For instance the effect of a *twist* in these flexible structures is sometimes overlooked; but when there is inequality of tension in the wires on the two sides of the structure, the fact that the section passing through the two main upright members is no longer a plane at right angles to the direction of the line accounts for the lessened strength of such a flexible design to resist loads due to high winds blowing across the line. It is not safe to adopt the flexible type of transmission line support without expert advice and adequate engineering supervision. As an approximate indication of present-day practice in arriving at the load in the direction of the line for which flexible structures should be designed, it may be stated that a load of from one-twentieth to one-tenth of the total load for which the rigid-strain towers are designed should not stress the intermediate flexible structures beyond the elastic limit. It is well to bear in mind that at the moment of rupture of one or more wires on a "flexible" transmission line the resulting stresses in the structures and remaining wires will be in the nature of waves or surges until the new condition of equilibrium is attained, and the maximum stresses immediately following a rupture will generally exceed the final value.

The mathematics required for the exact determination of stresses and deflections in a transmission line consisting of a series of flexible poles is of a very high order, even when many assumptions are made which practical conditions may not justify; but the limiting steady values of these stresses and deflections can be calculated in the manner described in Article 170 at the end of this chapter, and as the range between these limits will usually be very small, the probable maximum stresses under given conditions can be estimated with a reasonable degree of accuracy. The illustration Fig. 124 kindly supplied by Messrs. Archbold Brady and Co., shows a common form of "flexible" high voltage transmission line following a railway.

161. Steel Poles for Small Short-distance Transmission Schemes.—As a substitute for wood poles, light steel structures that can be shipped and erected in one piece appear to be gaining favor. Small amounts of energy at comparatively low voltages can be transmitted over distances of 20 to 30 miles by overhead wires supported on steel poles at a cost which need be no higher, and is sometimes even lower, than if the less durable

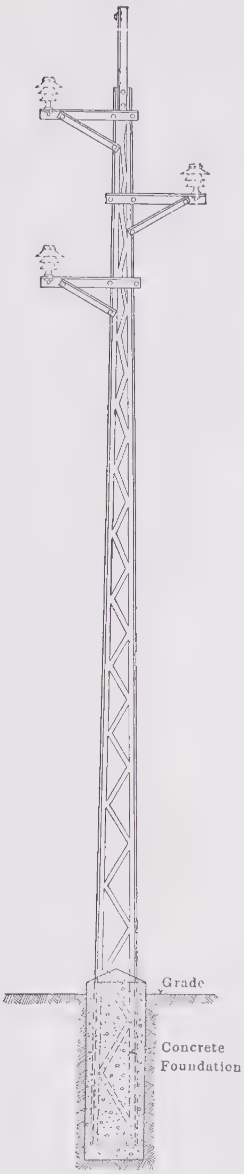

FIG. 125.—Bates one-piece expanded steel transmission pole.

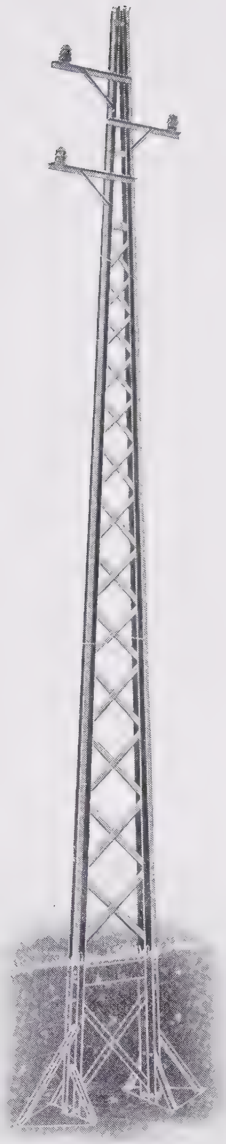

FIG. 126.—Type of steel pole manufactured by the Carbo Corporation.

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and less sturdy wood-pole construction is adopted. One type of steel pole for small lines is the Bates One Piece Expanded Steel Truss of which Fig. 125 is an example. These poles are manufactured by the Bates Expanded Steel Truss Co. of Chicago, with the idea that wooden poles are gradually giving out and that this is a practical substitute, the pole being made in one piece without bolted or riveted lattice work. A pole of this type is easily and economically painted. Another make of light-weight steel pole is shown in Fig. 126. This is manufactured by the Carbo Corporation of Chicago, which makes a specialty of steel poles for the economical construction of moderate voltage small-power transmission lines. Steel poles of the type under discussion would range from 25 ft. to 35 ft. in height and would be spaced from 250 to 300 feet apart.

162. Loads to be Resisted by Towers.—The maximum load which a tower must be designed to withstand will depend upon the number and size of wires to be carried and the estimated ice coating and wind velocity. Apart from the wind pressure on the structure itself, the loading in a direction transverse to the line will be equal to the resultant wind pressure on all the wires (which may or may not be ice coated, depending on the climate); the effective length of each wire being the distance between supports.

In the direction of the line, the forces are normally very nearly balanced, but in the event of one or more wires breaking, the unbalanced load may be considerable, and it is well to design the towers, if possible, to withstand the stresses imposed upon them if two-thirds of all the conductors in one span are severed. It must not be overlooked that if the wires break in one span only, the cross-arm, if pin type insulators are used, will be subjected to a twisting moment; and if the break in the wires is at one end only of the cross-arm, the whole tower is subjected to torsional strain.

The vertical or dead loads consist of the weight of the tower itself and the wires of one span, with possible increase in weight due to sleet or ice. The cross-arms must be of ample strength to take all vertical loads including weight of insulators, with a margin to cover the extra weight of men working on the tower. The approximate weight of insulators is given in the following table:

Working line voltage		Weight of insulator, lb.
22,000	pin type	4
44,000		12
66,000	suspension type	40
88,000		55
110,000		70
140,000		90

If a weight is attached to the lowest unit in a string of suspension insulators with the object of limiting the angle of deflection from the vertical when a high wind is blowing across the line, this must be taken into account when determining the loads to be resisted by the supporting structure.

Particulars regarding wind pressures were discussed in Article 131 of the preceding Chapter. The wind velocity rarely exceeds 80 miles per hour either on the American continent or in England. Tornadoes and cyclones are not considered, because attempts to design overhead lines strong enough to withstand them, would hardly be justified. In regions where sleet deposits are to be expected it appears to the writer unreasonable to base calculations on a heavier loading than that described as *Class B* on p. 271 of Article 131. That is to say, an ice coating 0.5 in. thick is very rarely exceeded, and when the conditions are such as to permit this formation, a wind velocity exceeding 60 miles per hour (corresponding to 8 lb. per foot pressure) is not likely to occur.

In regions where strong winds may be expected, but where sleet deposits do not occur, a maximum wind velocity of 76 miles per hour seems a reasonable assumption. This corresponds to a pressure of 21 lb. per sq. foot of flat surfaces on towers, and 14 lb. per sq. foot of projected surface of wires and cylindrical poles. The total transverse load is dependent upon the length of span, which must be determined with due regard to economic considerations.

163. Design of Steel Towers.—Although details of design and the proportioning of parts are matters best left to the manufacturer, the general type of supporting structure to be used under given conditions should receive careful attention. The most economical design of tower to withstand the probable loads

that it will be subject to, and to satisfy local conditions, including such considerations as transport and erection facilities, is a problem deserving close attention on the part of the engineer responsible for the design of the transmission line. A study of the probable loads to be resisted under the worst weather conditions will enable the designing engineer to specify certain test loads which will ensure that the finished structure will be strong enough to fulfil the practical requirements. The proper value of these test loads and their distribution or point of application should be determined only after mature consideration. The cost of a tower—apart from the height, which is a function of the length of span—is determined largely by the specifications of test loads. A specification calling for tests that are unnecessarily severe, is just as true an indication of incompetence on the part of the designing engineer as a specification giving test conditions that will result in a tower too weak for the actual requirements.

The calculation of stresses in the various members of so simple a structure as a transmission line tower is not a difficult matter, especially if graphical or semi-graphical methods are adopted. If the designing engineer will make sketches of two or three alternative designs likely to fulfil the required conditions, he should be able quickly to calculate the approximate value of the stresses in the principal members, and so obtain a rough idea of the relative weights and costs of alternative designs. The danger of leaving the problem entirely in the hands of the manufacturer is that the latter is always tempted to put forward a design of which he has perhaps made a specialty, and which may have given entire satisfaction in practice without necessarily being the best type of structure for the purpose, or being entirely suitable for use under different conditions.

164. Stresses in Compression Members of Tower Structures.—The failure of steel towers under excessive loads is almost invariably due to the buckling of the main leg angles in compression. The designer should therefore pay special attention to the proportioning of compression members in the structure. Without going into a discussion of the many empirical formulas used for determining the loads that struts or columns can withstand, it may be said that, for tower designs, the "straight line" formula, as suggested by Burr, is quite satisfactory provided the ratio $l \div r$ lies between 40 and 200; this last figure corresponds

to a length of compression member not exceeding about twenty times the width of flange. This formula is

$$S_{\text{comp.}} = K - k(l/r) \tag{156}$$

where K and k are constants,

l is the length, in inches, of unsupported portion of compression member,

r is the least radius of gyration, in inches,

$$= \sqrt{\frac{\text{moment of inertia}}{\text{area of section}}}$$

$S_{\text{comp.}}$ is the unit stress (lb. per sq. in.) in the column.

The ultimate stress which will cause compression members of steel towers to collapse is approximately expressed by the formula,

$$S_{\text{comp.}} = 35{,}000 - 120\frac{l}{r} \tag{157}$$

Assuming a factor of safety of 2½, we may write:

$$\textit{Safe working } S_{\text{comp.}} = 14{,}000 - 48\frac{l}{r} \tag{158}$$

which may be used in the design of steel towers. A similar formula, which is in common use for calculating safe loads in compression members of steel structures is:

$$S_{\text{comp.}} = 16{,}000 - 70\frac{l}{r} \tag{159}$$

This formula (159) is a safer one than (158) to use when the ratio l/r is large; but in any case it is recommended that l/r shall not exceed 120 for main members and 150 for lateral or secondary members. The fact that, for a given cross-sectional area, the *shape* of the section is an important factor in determining the stiffness and ultimate strength of the members in compression, suggests that, where lightness and economy of material are of great importance, a section of structural steel having a large moment of inertia per square inch of cross-section should be chosen. The standard sections of rolled angles or tees are sometimes replaced by steel tubes.

As an example of the relative economy of the tubular form and other forms of section, when used as comparatively long struts, a

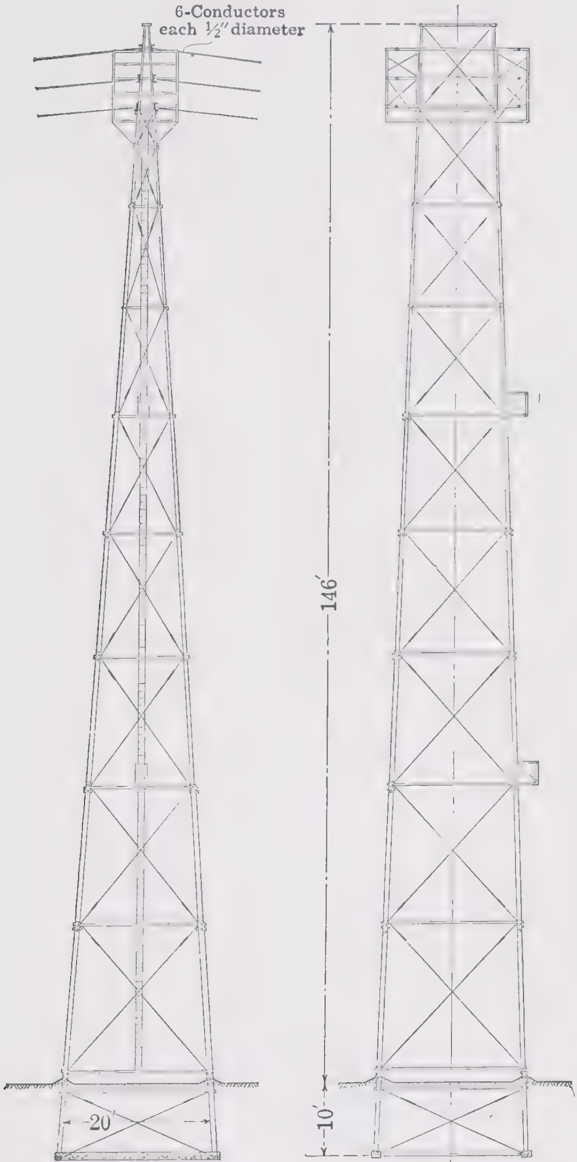

FIG. 127.—Steel tower with members of tubular section.

steel tube 7 in. internal diameter, 1/8 in. thick, weighing 10 lb. per foot, will be as efficient in resisting compression as a steel angle 7½ in. by 7½ in. by ½ in. thick, weighing 25 lb. per foot, or as an I beam 8 in. by 6 in. by ½ in. thick, weighing 35 lb. per foot. So large a tube would not be required except in very high towers: a tube from 4 to 5 in. diameter would generally be large enough for the main members of a transmission line tower up to 100 ft. high. The illustration, Fig. 127, is from a drawing kindly supplied by Messrs. Stewarts and Lloyds, Limited, of Glasgow, Scotland; it represents a tower 146 feet high as supplied for a power transmission line in the south of England.

165. Outline of Usual Procedure for Calculating Stresses in Tower Members.—The illustration, Fig. 128, which is reproduced by kind permission of the Shawinigan Water and Power Co., and the Canadian Bridge Co., Limited, shows a typical square-base galvanized steel tower as used on the Three Rivers line of the Shawinigan Water and Power Co. of Montreal. These towers are designed to carry six aluminum conductors of nineteen strand 200,000 circular mil cable, each being supported by seven suspension disks of the Ohio Brass Co.'s standard type. In addition to the conductors, there are two ground wires of 3/8 in. stranded Siemens-Martin steel cable attached to the points (1) at each end of the upper cross-arm. The line is built for 100,000 volts.

The method of procedure in calculating stresses is to make a sketch showing the points of application, and the vertical and horizontal components, of the outer forces. Then indicate by arrows the assumed horizontal and vertical components of the reactions, using the suffixes R and L to indicate the direction or assumed direction of the horizontal components. Since the whole structure is in equilibrium under the influence of the various loads and reactions, it is merely necessary to see that the three following conditions are satisfied at any point considered:

(a) The sum of all vertical force components = zero.

(b) The sum of all horizontal force components = zero.

(c) The sum of all moments about any point = zero.

When taking moments in any particular plane, all those in a clockwise direction would be considered positive and those in a counter-clockwise direction negative. All joints are considered as frictionless pivots, which assumption is, of course, not strictly

correct, especially in the case of riveted joints. It is usually an easy matter to choose a section through the structure in such a position that the stresses in a given bar can readily be calculated by applying one or more of the three equations of equilibrium.

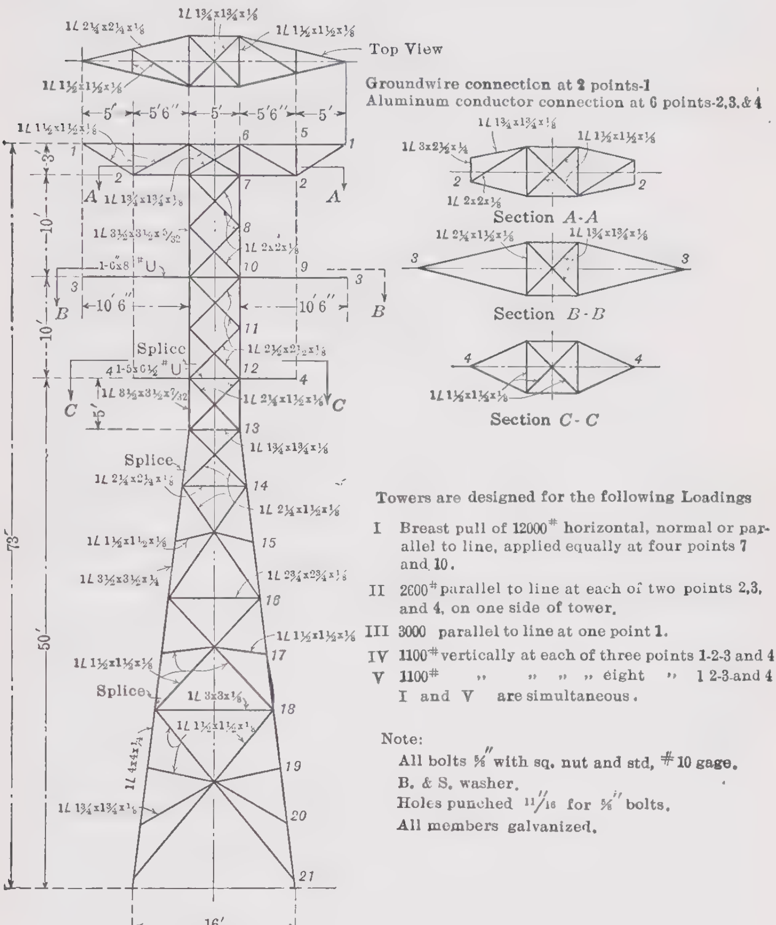

Fig. 128.—Steel tower with members of angle section.

The sketch, Fig. 129, will serve to illustrate the method usually followed in calculating the stresses in the main members of a tower structure such as the one shown in Fig. 128. The loading considered is that corresponding to the condition of test loads *I* and *V* applied simultaneously.

The point at which the horizontal breast pull of 12,000 lb. is applied corresponds approximately to the point 65 ft. above ground level where the corner legs would meet if produced beyond the points (13). The weight of the tower (which it is supposed has not yet been designed in detail) is taken at 4000 lb., and this, together with the test load V, gives a resultant vertical loading of 12,800 lb. applied somewhere on the center line of the tower.

Consider a section such as XY which cuts only three members, namely, the leg A at ground level, the leg B just above joint O', and the diagonal brace C.

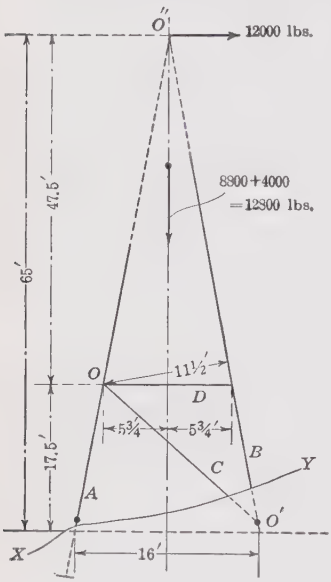

FIG. 129.—Sketch for calculation of stresses in tower members.

Select a point O where the members A and C meet, and consider the moments, in the plane of the paper, which are produced about this point by the external forces and the reactions in the members severed by the imaginary section XY. It is obvious that the stresses in A and in C have no effect on the tendency of the part of the structure above the section line to rotate on the point O, and the whole of the externally applied turning moment must be resisted by the stress in the member B. Therefore

$$(12,800 \times 5.75) + (12,000 \times 47.5) - (x \times 11.5) = 0$$

from which it is found that $x = 56,000$ lb.

Since there are two members B taking the whole crushing stress, the total load tending to crush the one member B is 28,000 lb. The length of the unsupported portion of this member is 5.5 ft. or 66 in. The cross-section of 4 in. $\times$ 4 in. $\times$ ¼ in. angle is 1.93 sq. in.; and the least radius of gyration, $r = 0.79$. The test load should not strain the tower beyond the elastic limit. Using the formula (157), the ultimate stress is,

$$S =_{\text{comp.}} 35,000 - 120 \left(\frac{66}{0.79}\right)$$
$$= 25,000$$

This corner member is therefore capable of supporting, just before collapse, a compressive load of $1.93 \times 25{,}000 = 48{,}300$ lb. It should be of ample strength to resist the test load of 28,000 lb. without permanent deformation.

Turning now to the uplifting force acting in the member *A* and tending to pull up the foundation, the center from which the moments are calculated is shifted to the point O' where the members *C* and *B* meet. The equation of moments is now,

$$(12{,}000 \times 65) - (12{,}800 \times 8) - (x \times 16) = 0$$

whence $\quad x = 42{,}300$

and the tension in one corner angle *A* is 21,150 lb.

The above example briefly describes what is known as the method of moments. It has been assumed that the tower side under consideration lies in the same plane as the external forces: but the error introduced is practically negligible. It is an easy matter, if desired, to make the necessary correction.

When calculating the stresses in a diagonal member such as *C* of Fig. 129, the moments would be taken about the point O'', which is the junction of the members *A* and *B*; but in that case the actual loads on cross-arms and the wind pressure on the side of the tower would have to be taken into account and substituted for the concentrated test load of 12,000 lb. at the point O'' which does not produce any stress in the brace *C* so long as the corner angle *A* remains truly straight and exerts no lateral pressure at the point *O*. The method of moments can usually be applied for all sections of a tower structure if the imaginary dividing planes are properly placed. The counter members or ties that are not in tension under the conditions of loading considered are usually assumed to be non-existent, *i.e.*, to serve no useful purpose as compression members.

When computing the stresses in the flexible "A"-frame steel structures it is assumed that the structure remains always normal to the line in a vertical plane; but unbalanced forces in the conductors will actually deflect the frame from this position and so reduce its possible resistance to transverse loads. It is practically impossible to calculate the strength of the distorted frame, and although flexibility in the direction of the line is usually a desirable feature of this type of structure, it is very important to design the so-called flexible steel towers so that they will not be deflected unduly by such torsional loads as they

may be subjected to at times when strong winds are blowing across the line.

166. Stiffness of Steel Towers. Deflection Under Load.—The deflection of the top of a transmission-line tower of the ordinary light "windmill" type with wide square base, when bolted to rigid foundations and subjected to a horizontal load such as to stress the material to nearly the elastic limit, might be from 2 to 5 in. With regard to the two-legged or "flexible" type of tower, if this is of uniform cross-section, it may be treated as a beam fixed at one end and free at the other end. If the resultant pull can be considered as a single concentrated load of P lb. applied in a horizontal direction, at a point H inches above ground level, the deflection, in inches, will be,

$$\delta = \frac{PH^3}{3MI} \tag{160}$$

where M is the elastic modulus for steel (about 29,000,000; being the ratio of the stress in pounds per square inch to the extension per unit length), and I is the moment of inertia of the horizontal section of the structure.

167. Tower Foundations.—The upward pull of the tower legs, which was found in the above example to amount to 21,150 lb., has to be resisted by the foundation. A factor of safety of $2\frac{1}{2}$ to 3 should be allowed. The weight of concrete may be taken at 140 lb. per cubic foot, and of good earth at 100 lb., the volume of the earth to be lifted being calculated at the angle of repose, which may be about 30 or 33 degrees with the vertical, as indicated in Fig. 130. If the footing of a tower is in gravel, or a mixture of sand and loam tightly packed, there is actually a far greater resistance to the pulling up of the footings than that which is offered by the mere weight of the footings with prism of earth as calculated in the usual way.

When concrete has to be used, it is generally cheaper to reinforce it with steel of an inverted T form, as this makes a lighter construction than a solid block of concrete, and an equally good hold is obtained owing to the increased weight of the packed earth which has to be lifted. At the same time it must not be forgotten that the digging of a large hole 5 to 8 ft. deep is considerably more costly than the digging of a hole about 2 ft. square, and this extra cost in erection must be taken account of in designing the footings. In marshy or loose soil, or where the

right of way is liable to be flooded, special attention should be paid to the design of durable foundations. Concrete footings with or without piles, or rock-filled crib work may be necessary; it is a matter requiring sound judgment and, preferably, previous experience on the part of the engineer in charge of construction. Crumbling hillsides are best avoided; it is extremely difficult to guard against damage by land slides or even snow slides when towers are erected on the steep slopes of hills.

The use of concrete adds considerably to the cost of foundations and it should be avoided if possible; on the other hand, it is

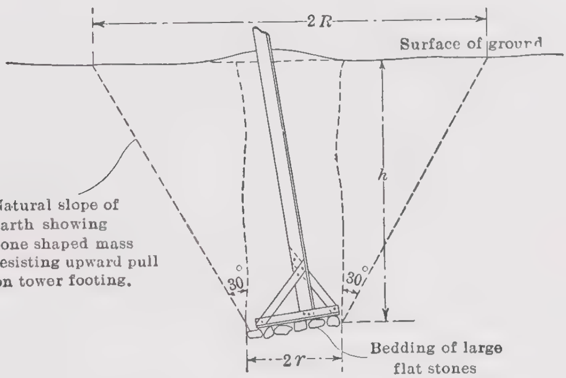

FIG. 130.—Foundation for steel tower anchor stub.

not easy to design foundations to resist a given uplift without an exact knowledge of the soil conditions at the site of the tower. For the greatest economy of foundation, it is necessary that the designer obtain reliable information on this point.

Assuming an average angle of slope of 30 degrees, as indicated in Fig. 130, and a weight of soil of 100 lb. per cubic foot, the depth of foundation may be calculated as follows.

Let h = depth of footing below ground level, in feet.
r = equivalent radius of footing area, in feet.
R = radius at ground level of conical section of earth to be lifted. (Feet.)
θ = angle of natural slope of earth.

The volume of frustrum of cone to be lifted is,

$$V = \frac{\pi}{3} h(r^2 + R^2 + rR) \qquad (161)$$

or, if $r + h \tan \theta$ be put in the place of R,

$$V = \frac{\pi}{3} h(3r^2 + h^2 \tan^2 \theta + 3rh \tan \theta) \tag{162}$$

If $\theta = 30$ degrees, $\tan \theta = 0.5774$ and (approximately),

$$V = \pi h(r^2 + 0.11h^2 + 0.58rh) \tag{163}$$

If $r = 1$ ft., and $h = 7$ ft.; the volume of earth to be lifted, by formula (163) is then,

$$V = 230, \text{ which gives,}$$
$$W = 23{,}000 \text{ lb.}$$

As previously mentioned, if the soil is firm, this method of calculation usually gives results well below actual values of pull required to uplift the footing. Under the conditions upon which this example has been based, it is probable that the footing would not move with a pull appreciably smaller than 30,000 lb.; there would then be a packing of the soil immediately above the footing, and a final pull of about 40,000 lb. might be necessary to uproot the stub and footing.

If the footings are imbedded in concrete, and separate grounding rods are not provided, it is well to let the iron-work project through the bottom of the concrete block, to ensure that the tower is properly grounded.

When concrete is not used, the design of the anchors is a matter that should receive very careful consideration. If towers are to be subjected to load tests, these tests should, if possible, be conducted on a tower set on its own anchors, as used in the field, because the strength is to an appreciable extent dependent upon the method of attachment of the tower's legs to the anchor stubs. One manufacturer of transmission-line towers[1] claims that the old style anchor, with the bottom diagonal of the tower attached to the anchor stub above ground, does not afford proper resistance to the horizontal force at the ground level and sometimes leads to failure of the tower legs in the bottom panel. This manufacturing company uses a design of anchor in which the joint between the tower leg and the bottom diagonal is below the ground surface.

Another point of importance is the surface of the footing in contact with the earth immediately above it. The weight of the

[1] The American Bridge Company.

cone of earth to be lifted may be ample to provide the desired factor of safety; but movement of the tower foundations may occur through the packing of the earth due to excessive unit pressure over the upper surface of the footing, and this movement may be appreciable notwithstanding that there may be no disturbance of the ground surface. A surface of not less than 1 sq. ft. for every 10,000 lb. of the vertical force which will pull out the anchors should be provided, unless the nature of the soil is such as to justify a reduction of this allowance.

168. Concluding Remarks Regarding Steel Tower Design.—Generally speaking, there is a tendency to economize in the cost of steel towers by using sections of structural steel in which stiffness is obtained by making the thickness of metal small in proportion to the other dimensions of the cross-section. It is true that light weight of parts and of the complete tower are important if the advantage of lightness can be obtained without sacrifice of other advantages, the chief of which is durability. When a transmission line is not intended to last longer than 15 or 20 years, these light sections are permissible; but for the more important and costly lines, it is well to avoid the use of metal thinner than $\frac{1}{4}$ in. for the main members, or than $\frac{3}{16}$ in. for the secondary or bracing members. In the writer's opinion it is not wise to use 4″ by 4″ angles for the corner legs less than $\frac{5}{16}$ in. thick, although a thickness of $\frac{1}{4}$ in. is not uncommon in towers actually in use at the present day. The ultimate life of such towers is, however, as yet unknown. Towers made of few pieces of comparatively heavy section steel will probably prove more durable than those built of a larger number of lighter parts.

If the temptation to use very light sections of structural steel is avoided, and if towers are regularly inspected and painted when and where necessary, their life should be 50 years or more. Galvanized towers are usually not painted; but it is not safe to rely upon the thin coating of zinc to prevent corrosion for more than a few years at or near the ground level. A casing of concrete extending about 12 in. above ground level will afford protection; or the parts that are buried may be painted instead of being galvanized, and if the anchor stubs are made in two lengths, the upper length can at any time be replaced without interrupting the service.

When considering designs of towers for a long transmission line,

it is well to avoid if possible a number of different types, and where the height does not require to be increased, it may sometimes be found more economical to use two standard towers close together for supporting special long spans, or for turning sharp corners, than to design special towers for the purpose. An angle not exceeding 7 degrees can usually be turned on a standard tower. This angle may even be as great as 10 degrees, especially if the length of the approach spans is decreased. In fact by reducing the length of approach spans, very much sharper angles can be turned; but it then becomes a question whether or not a special structure might not be the cheaper alternative.

There is an unexplained prejudice against the guying of steel towers where extra strength to resist lateral loads is required. By giving proper attention to the method of guying, and inspecting the line at regular intervals, there is no apparent reason why this fairly obvious device to save the extra cost of special structures should not prove entirely satisfactory. It is true that, with the so-called rigid design of tower, a very small deflection at the point of attachment of the guy wire may be sufficient to produce permanent deformation of the structure, and there is a possibility that the tower may collapse under excessive load before the guy wires have taken up their proper share of the abnormal stresses. This is especially likely to occur if the towers are set on concrete foundations. On the other hand it is not impossible to design towers of the square base type with foundations purposely arranged to yield slightly; and if these structures are provided with guys (say of plow steel cables) fixed very securely to practically unyielding concrete anchorages, it is probable that economies might, in many instances, be effected. Guying of corner poles or of occasional poles on a straight run, when the more flexible type of "A"-frame structure is used, is generally to be recommended.

A brief specification for a complete transmission line using steel towers is given in Appendix III. This line is generally similar to the one for which an estimate of cost was given in Chapter III.

169. Determining Position of Supports on Uneven Ground.—The lowest point of the span is not necessarily the point at which the wires come closest to the ground. When there is doubt as to the proper location of the supports in rough country, the method illustrated in Fig. 131, and described by Mr. J. S. Viehe

in the *Electrical World* of June 15, 1911, will be found very convenient. The curve *a* is the parabola corresponding to the required tension in the particular wire to be used. The ratio of the scale of feet for vertical measurements to the scale for horizontal measurements should be about 10 to 1. The dotted curves *b* and *c* are exactly similar to *a*, but the vertical distance *ab* represents the minimum allowable clearance between conductor and ground, while the vertical distance *ac* is the height above ground level of the point of attachment of the lowest wires to the standard transmission pole or tower. These curves should be drawn on transparent paper: they can then be moved about over a

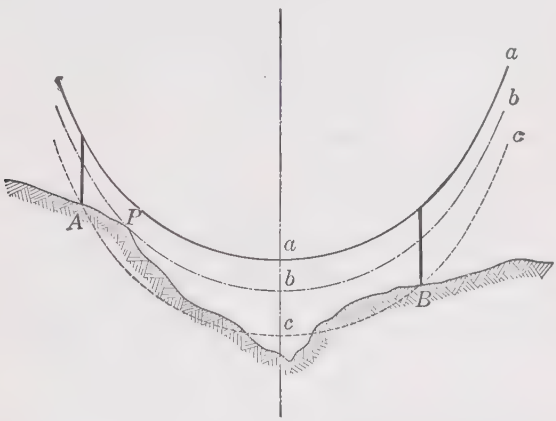

FIG. 131.—Method of locating position of towers in rough country.

profile of the ground to be spanned, drawn to the same scale as the curves, until the best location for the supports is found. The point *P* where the curve *b* touches the ground line is seen to be far removed from the lowest point of the parabola, in the example illustrated in Fig. 131. A little practice will make the finding of the points *A* and *B* an easy matter, even if the length of span, or distance between *A* and *B*, must be kept between close limits.

This method is particularly applicable to long-span lines carried over rough country.

170. Study of Deflections and Stresses in Flexible Tower Lines.—Consider a series of poles as in Fig. 132, the end one being rigid while all the others are flexible and of equal height and stiffness. It is assumed that all spans were originally of equal length *l*, and that there were *b* wires in each span, strung to a

tension of S pounds per square inch and having a corresponding sag s. In span No. 1, terminating at the rigid support, some of the wires have been severed, leaving only a wires in this span. It is assumed also that there is no slipping of the wires in the ties on the pin type insulators, and no yielding of pole foundations.

The elastic deflection of a pole or tower considered as a beam fixed at one end and loaded at the other is

$$\delta = \frac{PH^3}{3MI}$$

where P is the load, H the height, M the elastic modulus, and I the moment of inertia of the cross-section.

In the special case considered, the value of P, which produces the deflection δ_1 of the first flexible pole, is

$$P = A(bS_2 - aS_1)$$

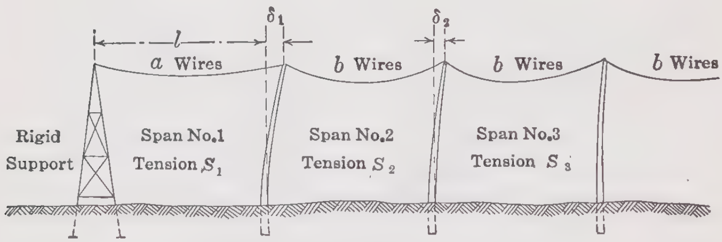

FIG. 132.—Flexible pole line.

where A is the cross-section of one conductor and S_1 and S_2 are the stresses in the conductors of spans No. 1 and No. 2 respectively. It is assumed that all the wires are attached to the pole tops at a point H in. above ground level.

By putting $K = \frac{H^3}{3MI}$, the successive deflections may be written:

$$\delta_1 = KA(bS_2 - aS_1) \tag{164}$$

$$\delta_2 = KAb(S_3 - S_2) \tag{165}$$

and the sum of the deflection of a series of flexible poles of the same height and stiffness is

$$\Delta = KA(bS_n - aS_1) \tag{166}$$

where n is the number of the last span. It is usually safe to assume that S_n is equal to the initial tension S in the fourth or fifth span from the break.

Fig. 133 shows the conductors in the first span with a sag s under normal conditions with b wires in the span, and a smaller sag s_1 after some of the wires have been cut, leaving only a wires in the span. For simplicity in calculating the movement of the point of attachment of the wires on the flexible pole, instead of considering the span as increasing in length from l to $(l + \delta)$, the span l may be supposed to remain unaltered while the length of the conductor is reduced by pulling it through the tie of the insulator (G) on the flexible pole until the sag is reduced from s to s_1. The length of wire pulled through in this manner may, for all practical purposes, be considered equal to the actual pole-top deflection, δ. This assumption is justifiable since the deflection δ is always small relatively to the span l.

The length of the (parabolic) arc with sag s is

$$\lambda = l + \frac{8s^2}{3l}$$

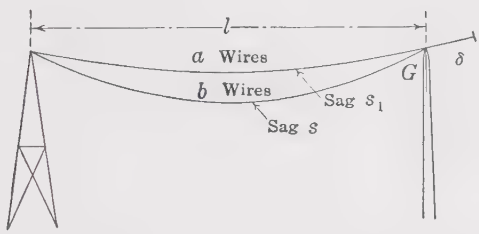

FIG. 133.—Elongation of wire in span due to deflection of pole top.

and with sag s_1

$$\lambda_1 = l + \frac{8s_1^2}{3l}$$

The difference is

$$\lambda - \lambda_1 = \frac{8(s^2 - s_1^2)}{3l}$$

to which must be added the elongation due to the stretch of the wire under increased tension; this is

$$\frac{\lambda_1 (S_1 - S)}{M}$$

or, with quite sufficient closeness,

$$\frac{l(S_1 - S)}{M}$$

Hence the deflection of the first flexible pole expressed in terms of the sag and tension of the conductors in the first span is:

$$\delta_1 = \frac{8\,(s^2 - s_1{}^2)}{3l} + \frac{l(S_1 - S)}{M} \tag{167}$$

Returning again to the arrangement of line depicted in Fig. 132, we shall consider (1) the total pull of all the wires in span No. 2 and the effect of this pull on the first flexible pole if all the wires are broken in span No. 1, and (2) the effect on the first flexible pole and the stresses in the remaining wires in No. 1 span on the assumption that all the wires in this span are not broken.

When the particulars of the poles are known, so that the factor K in the formulas for deflection can be determined, it is desired to calculate the stresses in poles and wires corresponding to the new conditions of equilibrium; or, if the poles have yet to be designed, the factor K must be determined, in order that the stiffness of the poles shall satisfy certain necessary or assumed conditions, such as the maximum deflection of pole top which will not stress the remaining wires in span No. 1 beyond the elastic limit of the conductor material. (A factor of safety must be used to allow of momentary increased stresses due to probable mechanical surges.)

171. Numerical Example: Transmission Line with Flexible Supports.—No attempt will be made to obtain an exact mathematical solution of these problems, but close approximations can be obtained with sufficient accuracy for practical purposes, especially when it is considered that many possible influencing factors, such as the yielding of foundations and the slipping of wires in the ties, cannot be taken into account even in the most complete mathematical treatment of the subject.

It is assumed that the poles are equidistant and in a straight line, and that the first support is rigid, all as indicated in Fig. 132. Four separate limiting conditions will be considered:

(A) All wires are severed in the first span, and the pole between spans 2 and 3 is considered to be rigid.

(B) All wires are severed in the first span, but the pole between spans 2 and 3, and all subsequent poles, are considered to offer no resistance to deflection in the direction of the line.

(C) There are a wires remaining in span No. 1, and b wires in all other spans. The pole between spans 2 and 3 is considered to be rigid.

(D) There are a wires in the first span, but the pole between spans 2 and 3, and all subsequent poles, are considered to be infinitely flexible.

The transmission line will be supposed to have the following characteristics:

Six No. 2-0 aluminum conductors.

Cross-section of conductor, $A = 0.1046$ sq. in.

Length of span, $l = 400$ ft.

Normal sag = 9.76 ft., which corresponds to

Stress $S = 2400$ lb. per square inch.

It is assumed that there is no grounded guard wire above the conductors, and that the average height of the point of attachment of the wires above ground level is $H = 45$ ft.

The modulus of elasticity of aluminum cables for the purpose of these calculations is assumed to be $M = 7{,}500{,}000$. The flexible towers are in the form of braced "A"-frames, each vertical limb consisting of one 7-in. steel channel of light section ($9\frac{3}{4}$ lb. per foot). The moment of inertia of the section of such a channel is 21.1, and since there are two channels, the value of I is $21.1 \times 2 = 42.2$ and the section modulus $Z = \frac{42.2}{3.5} =$ (say) 12. The elastic modulus for steel is $M = 29 \times 10^6$. The factor K for use in pole deflection formulas as previously given is therefore

$$K = \frac{(45 \times 12)^3}{3 \times 29 \times 10^6 \times 42.2} = 0.0428$$

The maximum deflection of this particular structure before permanent deformation would take place will occur when the difference of pull due to the wires is such as to stress the metal to (say) 30,000 lb. per square inch. The resisting moment is $S \times Z = 30{,}000 \times 12$ and the resultant pull at the pole top will be

$$\frac{30{,}000 \times 12}{45 \times 12} = 667 \text{ lb.}$$

The maximum allowable deflection is therefore,

$$\begin{aligned} \delta &= K \times 667 \\ &= 0.0428 \times 667 \\ &= 28.5 \text{ inches.} \end{aligned}$$

Case (*A*). All wires broken in span No. 1; second pole beyond break considered rigid.

Since all the wires are severed in span No. 1 ($a = 0$) it is not possible to make use of formula (167), but a similar formula can be used which expresses the deflection in terms of the constants for span No. 2. This formula is

$$\delta_1 = \frac{8}{3l}(s_2{}^2 - s^2) + (S - S_2)\frac{l}{M} \tag{168}$$

By calculating δ_1 for various arbitrary values of S_2 smaller than S, curve No. 1 of Fig. 134 can readily be drawn. This gives the relation between the stress S_2 in the wires of the second span and the pole-top deflection δ_1 on the assumption that the second pole beyond the break is rigid. On the same diagram draw the

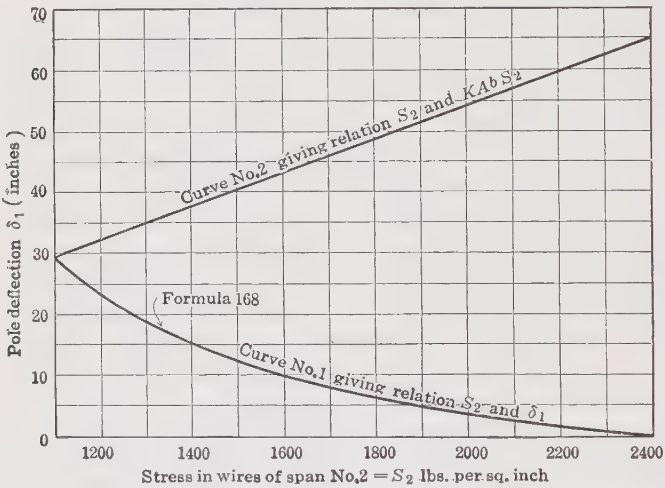

Fig. 134.—Graphic solution of Problems (A) and (B).

straight line marked *curve No.* 2, which gives the relation between pole deflection and the stress S_2, as given by formula (164) when the tension S_1 in wires of the first span is equal to zero. The point of crossing of curves No. 1 and No. 2 evidently indicates the deflection corresponding to the condition of equilibrium. This deflection is $\delta_1 = 29.5$ in. and stress $S_2 = 1100$.

It will be noted that in this particular example the deflection is about the same as the maximum permissible deflection (28.5) previously calculated; but even if allowance be made for shocks and mechanical surges, it is probable that the pole would not suffer serious injury, because some of the wires would be liable to

slip in the ties and so relieve the tension. If wind pressures acting on snow or ice deposits are added to the stresses due to weight of conductor material only, the strain will be greater, but on the other hand, much sleet deposit is liable to be shaken off the wires in the event of a sudden severing of all the wires in the first span.

The above results are, however, based on the assumption that the second pole beyond the break is rigid, which may not be in accordance with practical conditions.

Case (*B*). Conditions as above; but the second and subsequent poles beyond the break are supposed to be infinitely flexible ($K = \infty$).

In this case the tension S_2 will not depend upon the deflection of the first flexible pole; it will be equal to the original tension $S = 2400$ for all values of the deflection δ_1. The deflection obtained when $S_2 = 2400$ is of course readily calculated by means of formula (164), or it can be read off Fig. 134, since it is the deflection indicated at the point where curve No. 2 meets the vertical ordinate for $S_2 = 2400$. This value of δ_1 is 64.5 in., which would lead to permanent deformation of the flexible structure. The actual deflection of the first pole in a series of flexible poles of equal stiffness would lie somewhere between these limiting values of 29.5 in. and 64.5 in. if the law of elasticity may be considered to apply in the case of the higher deflections. As a general rule the breaking of all wires in one span will lead to the wrecking or permanent deflection or uprooting of the first pole, which cannot be at the same time flexible enough greatly to reduce the combined pull of all wires in span No. 2, and yet strong enough to resist the ultimate combined pull of these wires. There would be an exception in the case of short spans with tall flexible poles; and in any case it is probable that only the first pole would be damaged or moved in its foundations.

It is rare that all the wires in one span are broken simultaneously unless the design of the line is such that the severing of one or more wires leads necessarily to the rupture of the remaining wires owing to the excessive stresses imposed on them. The calculation of stresses and deflections when a certain number of wires remain in the faulty span is more difficult than in the cases already considered, but the solution is of greater practical value.

Case (*C*). There are a wires in the faulty span and b wires in the sound spans. The second pole beyond break is considered

rigid. (For the purpose of working out numerical examples it will be assumed that only one wire remains in faulty span; thus $a = 1$ and $b = 6$.)

Instead of only two equations, there are now three equations to be satisfied simultaneously; these are:

(a) Formula (164):

$$\begin{aligned}\delta_1 &= KA(bS_2 - aS_1)\\ &= 0.0269S_2 - 0.00448S_1\end{aligned}$$

(b) Formula (167), giving deflection in terms of elongation of remaining wires in span No. 1:

$$\delta_1 = \frac{1}{150}(95.3 - s_1^2) + \frac{S_1 - 2400}{18{,}750}$$

(*c*) Formula (168), giving deflection in terms of the shortening of the wires in span No. 2. (This relation is given by curve No. 1 already plotted in Fig. 134.)

It should be mentioned in connection with formulas (167) and (168) that, by assuming a constant length of span, the sag s is always inversely proportional to the stress S. The assumption of a constant length of span for the purpose of simplifying the relation between sag and tension introduces no appreciable error in practical calculations. In the particular example from which the curves are plotted, and the numerical results obtained, the relation is $s = \dfrac{23{,}420}{S}$.

Proceed, now, to plot curve No. 3 in Fig. 135 from formula (167) by assuming various arbitrary values of S_1 from the lowest possible limit of $S_1 = S = 2400$ up to the elastic limit of about 13,000. For a reason to be made clear hereafter this curve should be drawn on transparent paper; the horizontal scale used for the values of S_1 may be arbitrarily chosen, but the scale of ordinates giving the deflections δ_1 must be exactly the same as used for Fig. 134. On the same diagram (Fig. 135) draw also the straight line marked *curve No.* 4, giving the relation between S_1 and the quantity $KAaS_1$. This latter quantity when subtracted from the quantity $KAbS_2$ will give the pole deflection to fulfil the condition of formula (164). The reason for drawing the curves of Fig. 4 on transparent paper will now be clear.

The transparent paper with the curves of Fig. 135 is placed over Fig. 134, with the horizontal datum lines of zero deflection

coinciding as shown in Fig. 136. The point of intersection of curves No. 1 and No. 3 will give the corresponding values of the stresses S_1 and S_2[1] but with a pole having definite elastic prop-

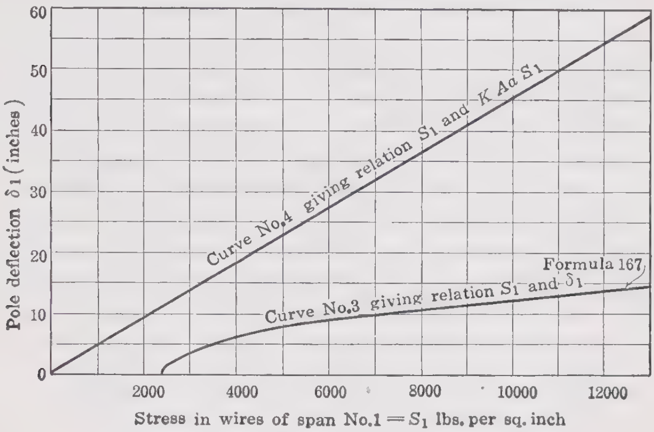

FIG. 135.—Curves to be drawn on tracing paper for solution of Problems (C) and (D).

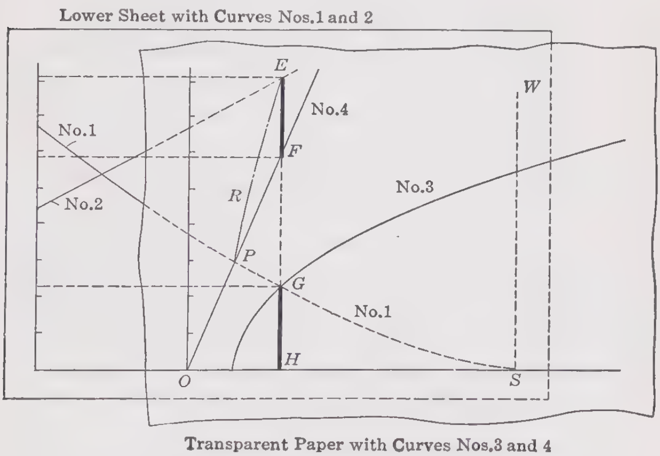

FIG. 136.—Graphic solution of Problems (C) and (D).

[1] There is a definite value of S_1 for any given value of S_2 independent of all considerations of pole stiffness and size of wire and number of wires in adjoining spans. This is the relation which will satisfy formulas (167) and (168) simultaneously; it is expressed by the equation

$$\frac{8}{3l}(2s^2 - s_1^2 - s_2^2) = \frac{l}{M}(2S - S_1 - S_2)$$

erties there is only one value of the deflection which will satisfy the three conditions previously referred to. The deflection as a function of the pole stiffness is the distance EF (Fig. 136), being the difference between the corresponding ordinates of curves No. 2 and No. 4. By moving the tracing paper with the curves No. 3 and No. 4 over the other curves until the distances HG and FE on the same vertical ordinate are equal, the deflection corresponding to the condition of equilibrium is readily obtained. If preferred, the curve $OPRE$, representing the *sum* of the quantities of curves No. 3 and No. 4, may be drawn on the tracing paper instead of the curve 4, and when the point of intersection (E) of this new curve with curve No. 2 on the lower sheet lies on the same vertical ordinate as the junction (G) of the curves No. 1 and No. 3, the distance HG will be the required deflection.

The solution of the numerical example worked out in this manner is

$$\delta_1 = 10.2 \text{ in.}$$
$$S_1 = 7400 \text{ lb. per square inch.}$$
$$S_2 = 1500 \text{ lb. per square inch.}$$

Case (D). Same conditions, with the exception that the second pole beyond the break, instead of being rigid, is assumed to be infinitely flexible. This assumption is made also in the case of all subsequent poles. This means that $S_2 = S = 2400$ whatever may be the amount of deflection of the first flexible pole, and the problem can be solved graphically as indicated above, the only difference being that curve No. 1 giving the relation between δ_1 and S_2 when the second pole beyond the break is rigid must be replaced by the vertical line SW (Fig. 136), being the ordinate corresponding to a tension $S_2 = 2400$.

The numerical solution in this case is:

$$\delta_1 = 13.2 \text{ in.}$$
$$S_1 = 11{,}450 \text{ lb. per sq. in.}$$

It is interesting to note that there is little difference between the deflections for the two extreme cases (C) and (D); the average value for δ_1 is 11.7 in., corresponding to a stress $S_1 = 9400$ in the remaining wire of the faulty span. This is well below the elastic limit, and it is probable that this wire would not break even if the five other wires were severed. The figures chosen for illustrating

the calculations relate to a practical transmission line, and it will be seen that the stresses and deflections corresponding to the state of equilibrium after the severing of one or more wires in one span can, with the help of simple diagrams, be predetermined within reasonably narrow limits.

172. Erection of Steel Tower Transmission Lines.—This book is not intended to give practical advice to construction engineers or the men actually engaged in the work of erecting poles or towers and stringing wires. A competent construction engineer with experience in handling men and materials in the field, should be given a free hand in planning and executing the work of erecting a power transmission line; and such a man will not derive much assistance from books. On the other hand, there are some excellent books available dealing with the more practical side of transmission line engineering. These include the various electrical engineering handbooks. The reader desiring information on the methods ordinarily adopted in carrying out the details of construction, is referred to these other sources of information; also to the papers and articles which appear from time to time in the Journals of the engineering societies and in the technical press. Appendices II and III which follow this Chapter also contain items of some practical interest connected with the setting out and erection of wood pole and steel tower lines.

The principal reason for referring to these matters in this place is to emphasize the importance of devoting much time and thought to the various details of overhead line construction *before the work is actually started.* The proper setting out of the line is among the most important matters connected with overhead construction. If a line is not carefully surveyed and planned in every detail, it will often be impossible to get good and reliable service from it. This does not mean that the commercial aspect of the undertaking is not of prime importance; on the contrary, it is the only aspect from which an engineering undertaking of the kind under consideration should be viewed. But this is not equivalent to saying that a small first cost is always desirable, or that even a short low-voltage transmission line can be constructed and operated economically by persons without engineering skill and experience. It is an easy matter to find examples of lines that have cost too much; but it is not impossible to find the transmission line that has cost too little—in the first instance.

Generally speaking, the writer believes that not enough attention is paid to preliminary investigations and estimates of power transmission lines. The construction of comparatively short lines for moderate voltages appears to be, and indeed is, a fairly simple piece of work; yet—in respect to economy and service—such lines may be a source of endless trouble if they have been planned and constructed without regard to the fundamental principles of engineering.

APPENDIX I

INDUCTANCE OF TRANSMISSION LINES WITH ANY ARRANGEMENT OF PARALLEL CONDUCTORS[1]

The manner in which the inductance and the induced e.m.f. can be calculated when the conductors of a three-phase system occupy the vertices of an equilateral triangle, was explained in Chapter II; and it was also stated that a departure from the symmetrical arrangement of conductors does not modify the calculated results to a great extent. It will be interesting to study the problem in its broader aspect, with a view to ascertaining what is the nature and magnitude of the modifying factors. It is proposed to indicate a simple method of calculating the total induced e.m.f. in any conductor of an electric-energy transmission system, whatever may be the actual arrangement or relative positions of the conductors. It is assumed in all cases that the conductors are of circular section and that they remain parallel with each other throughout the whole distance of transmission.

Calculation of Total Resultant Flux Surrounding One Conductor When There Are Several Return Conductors.—In Fig. 1 the total outgoing current I is supposed to flow along one conductor, while the total return current is divided between a number of conductors, the condition being that

$$I = -(I_1 + I_2 + I_3 \ldots\ldots + I_n)$$

Let d_1, d_2, d_3, etc., represent the distances between centers of the corresponding conductors carrying the return currents and the conductor carrying the outgoing current, and note that the total flux surrounding the latter conductor may be considered as the algebraic sum of several separate fluxes, namely, the flux due to a current I_1 returning at a distance d_1; the flux due to a current I_2 returning at a distance d_2, and so on, for any number of components of the total current I. All these separate components of the total flux can readily be calculated by means of formula (25) of Article 43, Chapter IV, and the expression for the total flux surrounding a conductor in which the current I returns

[1] This Appendix is a reprint, with slight changes and omissions, of articles which were first published in the *Electrical World* of May 23, 1908 and Sept. 15, 1910.

along a number of separate conductors, as indicated in Fig. 1, becomes:

$$\Phi = \frac{2l}{10}\left[- I_1 \log_\epsilon \frac{d_1}{r} - I_2 \log_\epsilon \frac{d_2}{r} \; \ldots\ldots - I_n \log_\epsilon \frac{d_n}{r}\right] \quad (1)$$

where r stands for the radius of cross-section of the conductor carrying what will be thought of as the outgoing current I.

In the case of energy transmission by polyphase currents, with any number of conductors, the algebraic sum of the currents in the conductors must, at any given instant, be equal to zero. Any one conductor may be looked upon as carrying the outgoing current, while the remaining conductors together carry the return current. Formula (1) can, therefore, be used for calculating the effective flux of induction surrounding any one conductor in a polyphase transmission, whatever may be the arrangement of the conductors. The phase relations of the various component fluxes must, however, be taken into account, and for this reason the graphical addition of vector quantities with the help of a diagram will be found most convenient. Instead of drawing the vectors representing magnetic flux components—in phase with the current vectors—the component vectors of the resulting e.m.f. of self-induction may be drawn—in this case 90 time-degrees behind the corresponding current vectors.

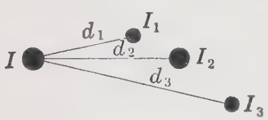

FIG. 1.—Section through four parallel conductors.

Calculation of E.M.F. of Self- and Mutual Induction.—In order to calculate the induced e.m.f. it will be advisable first to put equation (1) in a more practical form. The symbols I_1, I_2, etc., in equation (1), when the latter is to be used for calculating the maximum value of the induction due to an alternating current, must be considered as representing the maximum value of the current wave; but it will be more convenient to assume sinusoidal currents, and then let these symbols stand for the virtual (or r.m.s.) value of the currents.

The procedure is now as indicated in Article 45 of Chapter IV, leading up to formula (28) which may be written:

$$\left.\begin{array}{c}\text{Reactive volts per mile}\\ \text{of single conductor}\end{array}\right\} = aI \log_{10}\frac{d}{r} + bI \quad (2)$$

$$\text{where } a = 0.00466f$$
$$\text{and } b = 0.000506f$$

The item bI is the reactive voltage component due to the flux set up *inside* the material of the conductor by the current I. The flux producing this increased reactive e.m.f. is not included in the flux as calculated by formula (1). It may generally be neglected in calculations of high voltage overhead transmission lines.

The final expression for the reactive voltage drop per mile of conductor when there are several parallel return conductors is:

$$E = a\left[-\,I_1 \log\frac{d_1}{r} - I_2 \log\frac{d_2}{r} \;.\;.\;.\;.\; - I_n \log\frac{d_n}{r}\right] + bI \qquad (3)$$

Numerical Example. Three-phase Transmission.—Consider the special case, which not infrequently arises in practice, of the conductors of a three-phase transmission being arranged as indicated in Fig. 2—that is, with the centers of the three conductors lying in the same plane, the minimum distance, d, between any two of the wires being approximately equal to the side of the equilateral triangle which would have been adopted had the triangular arrangement been decided upon.

FIG. 2.—Three conductors in one plane.

In a three-phase transmission system the current flowing out through any one wire may, as previously mentioned, be considered as returning along the two remaining wires, and when the three conductors occupy the vertices of an equilateral triangle the whole of the return current is at a distance d from the outgoing current. This condition also applies to the middle conductor (No. 2) in the arrangement shown in Fig. 2; but it does not apply to either of the outside conductors, Nos. 1 and 3. In the case of conductor No. 1 a part of the outgoing current returns along conductor No. 2 at a distance d, while the remainder returns along conductor No. 3 at a distance $2d$; so that the total flux of induction surrounding conductor No. 1 must necessarily be greater than that surrounding conductor No. 2. The same argument applies to conductor No. 3.

Applying formula (3) to the arrangement of conductors, as shown in Fig. 2, the quantity between brackets in the case of conductor No. 1 becomes:

$$\begin{aligned} &-\,I_2 \log\frac{d}{r} - I_3 \log\frac{2d}{r} \\ &= -\,(I_2 + I_3)\log\frac{d}{r} - I_3 \log 2 \end{aligned}$$

$$= I_1 \log \frac{d}{r} - I_3 \log 2$$

The total induced e.m.f. per mile of conductor No. 1 will therefore be:

$$E_1 = a \times \left[I_1 \log \frac{d}{r} - I_3 \log 2 \right] + bI_1 \tag{4}$$

Similarly, for conductor No. 3:

$$E_3 = a \times \left[I_3 \log \frac{d}{r} - I_1 \log 2 \right] + bI_3 \tag{5}$$

while the volts induced in the middle conductor (No. 2) will be simply:

$$E_2 = a \times I_2 \log \frac{d}{r} + bI_2 \tag{6}$$

It is interesting to note that what may be referred to as the disturbing element in the case of the two outside wires (the quantities $I_3 \log 2$ and $I_1 \log 2$ respectively) is not dependent upon the actual diameter or distance apart of the conductors. It consists of an e.m.f. component either 30 time-degrees or 150 time-degrees behind the phase of the line current, depending upon the order of the phase rotation; and the magnitude of this e.m.f. component relatively to the total e.m.f. of self-induction will depend upon the value of the ratio $\frac{d}{r}$. If d is large and r relatively small, as in the case of a high-pressure overhead transmission system, then the first quantity between brackets, in equations (4) and (5), is relatively large, and the disturbing element ($I_3 \log 2$ or $I_1 \log 2$) is usually negligible. On the other hand, if the conductors consist of three separate single cables, laid side by side in a trench, with the distance, d, between them small in comparison with the diameter, $2r$, of the cables, then the "disturbing element" becomes of greater importance relatively to the total induced e.m.f.

In order to form some idea of the magnitude of this out-of-balance component of the induction, it will be well to work out two numerical examples, one for a high-tension overhead scheme and the other for a low-tension transmission system with the three conductors in comparatively close proximity.

Example 1.—Assumed data: Three-phase power transmitted = 20,000 kw.; e.m.f. = 110,000 volts; power-factor = 0.8; frequency f = 25 cycles per second; length of line = 200 miles. Conductors of aluminum; diameter, $2r$ = 0.6 in. Minimum dis-

tance between wires, $d = 10$ ft. $= 120$ in. On the above data the current per conductor is about 130 amp. With the aid of formulas (4), (5) and (6) it is an easy matter to determine the induced e.m.fs. in the several conductors, and since the quantity, $\log \frac{d}{r} = \log \frac{120}{0.3} = 2.6021$, while $\log 2 = 0.3010$, it will at once be seen that the "disturbing element" is relatively small.

The e.m.fs. induced in each conductor 200 miles long, in round figures (neglecting the component bI due to the internal flux) are as follows:

In the middle conductor (No. 2), 800 volts, the time-phase of which is exactly one-quarter cycle behind the time-phase of the current I_2.

In conductor No. 1, an e.m.f. component of 800 volts, exactly a quarter cycle behind the current I_1 *less* another component (referred to as the disturbing element) equal to about 90 volts, the phase of which is exactly one-quarter cycle behind the current I_3. The resultant is the difference between two *vector* quantities separated by a time-phase angle of 120 deg., so that this resultant is actually *greater* than either of the two components, as will be shown hereafter.

In conductor No. 3 there will be an e.m.f. component of 800 volts, one-quarter cycle behind the current I_3, and a component of 90 volts, one-quarter cycle behind I_1.

Example 2.—Assumed data: Three-phase power transmitted $= 20$ kw.; e.m.f. $= 110$ volts; power-factor $= 0.8$; frequency $f = 60$ cycles per second; current per wire $= 130$ amp.; distance of transmission $= \frac{1}{2}$ mile; three single cables in trench, lying in the same plane with a distance between centers $d = 3$ in.; diameter over copper $= 2r = 0.5$ in.

In this example the quantity $\log \frac{d}{r}$

$$= \log \frac{3}{0.25}$$
$$= 1.0792$$

The ratio between log 2 and this number is $\frac{0.3010}{1.0792} = 0.28$. That is to say, the component of the total induced e.m.f., which appears only in the two outside conductors, as indicated by formulas (4) and (5) is, in this example, numerically greater than a quarter of the more important component; while in the previous example of a high-tension overhead transmission system the ratio was

$\frac{0.3010}{2.6021} = 0.115$, being considerably smaller because of the greater distance between the wires.

Vector Diagram Illustrating Example 2.—The vectors I_1, I_2 and I_3 in Fig. 3 represent the currents in the three conductors, the time-phase angle between them being 120 deg. The rotation of the phases is assumed to be in the order I_1, I_2, I_3; in other words, I_2 lags behind I_1 by one-third of a cycle, and I_3 lags behind I_2 also by one-third of a cycle. The lengths of these vectors are such as to represent the line current of 130 amp.; but, as the diagram has been drawn to illustrate the phase angles and magnitudes of the various components of the induced e.m.fs., the magnitude of the current vectors need not be considered. If the numerical values of the induced volts are determined with the aid of formulas (4), (5) and (6), it will be found that the component common to all three conductors amounts to 21.6 volts, while the "disturbing element"—that is, the component appearing in the two outer conductors only—amounts to 5.5 volts.

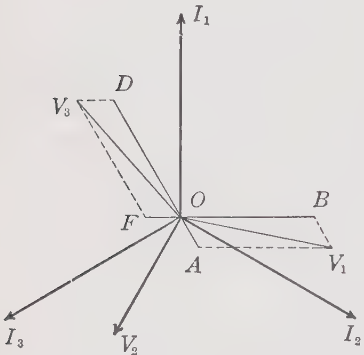

Fig. 3.—Vector diagram. Three conductors in same plane.

The vectors OB, OV_2 and OD must, therefore, be drawn of such a length as to represent 21.6 volts in a direction exactly 90 time-degrees behind the corresponding current vectors; and, so far as the middle conductor is concerned, the vector OV_2 will represent the whole of the induced e.m.f.; but in the case of conductor No. 1 (carrying current I_1), OA must be drawn exactly 90 time-degrees *in advance* of OI_3—that is, exactly *opposite* to OD, because of the negative sign in equation (4)—and of such a length as to represent 5.5 volts. By combining OA with OB in the usual way, OV_1 is obtained as representing the total e.m.f. induced in conductor No. 1. In a similar manner OV_3 is obtained for the total induced e.m.f. in conductor No. 3. It is interesting to note that OV_1 lags behind the current I_1 by a time interval *greater* than a quarter period, while the lag of the induced volts V_3 behind the current I_3 is *less* than a quarter period.

In the particular example under consideration the calculated value of V_1 or V_3 is 24.8 volts; V_2 being 21.6 volts.

It is not difficult to understand why the magnitude and phase relations of the induced e.m.fs. in the various conductors of a polyphase transmission are not the same for an unsymmetrical arrangement of conductors as for an arrangement in which each conductor is similarly placed in relation to all the other conductors. With an unsymmetrical arrangement, the unbalancing effect may be said to be due to the mutual induction between the loops formed by different pairs of wires; there may, in fact, be a transfer of energy between one loop and another just as in the case of the primary and secondary windings of a transformer.

Effect of Transposing the Conductors.—If each conductor of the arrangement referred to in the above example is made to occupy, in turn, the position midway between the remaining two conductors for a distance equal to one-third of the total distance of transmission, it is obvious that the out-of-balance effect will be corrected. It will, however, be of interest to ascertain what will be the numerical value of the (equal) voltages induced in the three conductors if transposed in the manner suggested. It is not necessary to consider more than one of the conductors, and, in Fig. 4, OB represents (as in Fig. 3) that portion of the e.m.f. induced in conductor No. 1 which remains unaltered whether the conductor be midway between the other two, or be itself one of the outer conductors. The length of this vector will, therefore, be such as to represent 21.6 volts. Now, when the arrangement of the conductors is in the order 1, 2, 3 (as in Fig. 2), the "disturbing element" will be BG, drawn 90 degrees in *advance* of OI_3, exactly as OA (or BV_1) in Fig. 3; but the *length* of this vector, instead of being equivalent to 5.5 volts, will be only one-third of this value, or 1.83 volts, because conductor No. 1 occupies this position over one-third of the total distance of transmission. When the arrangement of the conductors is 1, 3, 2, the "disturbing element" will be GV_1

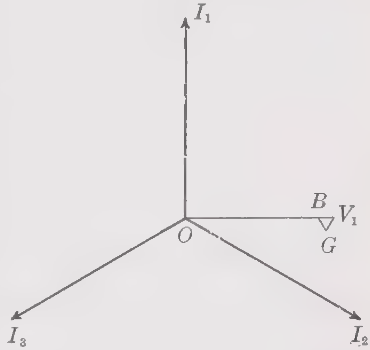

FIG. 4.—Vector diagram illustrating effect of transposing conductors lying in the same plane.

(Fig. 4), drawn 90 degrees in advance of OI_2. Clearly BGV_1 is an equilateral triangle, and the resultant of the induced e.m.f. in conductor No. 1 is OV_1, drawn 90 time-degrees behind the current vector OI_1 and equal in magnitude to the algebraic sum of OB = 21.6 volts and BV_1 = one-third of 5.5 volts.

If, therefore, the wires of a transmission line are disposed in one plane, as indicated in Fig. 2, but transposed at intervals so that each wire shall occupy the middle position over a space equal to one-third of the distance of transmission, then the resultant induced e.m.f. per conductor will, so far as phase is concerned, lag behind the current by a quarter period, exactly as if the wires occupied the vertices of an equilateral triangle; but the amount of the induced volts will be somewhat greater than in the latter case, under otherwise similar conditions.

The numerical value of the induced volts per conductor—that is, the length of the vector OV_1 in Fig. 4—can be calculated by the formula:

$$E = a\left(I \log \frac{d}{r} + \frac{I \log 2}{3}\right) + bI \tag{7}$$

where I is the current in any one conductor, and the two quantities between brackets have merely to be added algebraically. If preferred the quantity between brackets may be written:

$$I \log \left(\frac{d}{r} + \sqrt[3]{2}\right)$$

or $I \log \left(1.26 \frac{d}{r}\right)$, so that formula (7) appears in the form:

$$E = 0.00466 fI \log \frac{1.26d}{r} + 0.000506 fI \tag{8}$$

Inductance of Electric Transmission Lines as Affected by the Subdivision of the Circuits and the Arrangement of the Conductors.—There are reasons in favor of transmitting large amounts of electric power through two or more sets of wires, quite distinct from mechanical considerations or the increased security against a total shut-down in the event of accidents. The inductive drop of pressure may be reduced by substituting, for a single set of transmission lines, two or more sets of suitably arranged lines of a correspondingly reduced cross-sectional area. Whether or not the subdivision of a transmission line into two or more parallel circuits would be justifiable in practice will depend upon economic and other considerations which it is not proposed to touch upon here.

Single-phase Systems.—In Fig. 5 the two conductors of a single-phase transmission are shown, with distance d between centers of wires. The current may be considered as going out through the conductor 1 and returning through conductor 2. The diameter of the wire is assumed to be $2r$ and the current I amp.

The formula which gives the induced volts per mile of single conductor when the whole of the current may be considered as returning at a distance d from the center of the outgoing conductor is

$$E = aI \log \frac{d}{r} \tag{9}$$

where a has the value given above, and the item bI has been omitted as it is not necessary to include it when considering *differences* of reactive e.m.fs., especially in the case of overhead systems where its magnitude is relatively small, and frequently negligible. This formula alone is sufficient to indicate that an improvement in the matter of inductive voltage drop is to be expected if, instead of transmitting the total current I through one pair of conductors, there be provided two or three pairs of conductors spaced sufficiently far apart to prevent mutual inductive effects, each pair being of sufficient cross-section to carry one-half or one-third of the total current, as the case may be; because, although the quantity $\log \frac{d}{r}$ will increase slightly on account of the reduction in the dimension r, this increase will not be of nearly so much importance as the reduction of I.

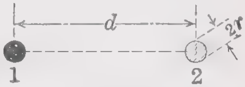

Fig. 5.—Two parallel conductors.

Numerical Example.—In order to illustrate the above point a few examples will be worked out based on the following assumed data:

Total current, I = 100 amp.

Diameter of single conductor to transmit the total current, $2r$ = 0.5 in.

Frequency, f = 60 cycles, from which a = 0.279.

Distance between centers of wires (corresponding to a pressure of about 50,000 volts), d = 70 in.

If the transmission line is divided into two equal sections, the current in each section will be 50 amp., and for equal total weight of copper (leading to the same ohmic drop of pressure),

the radius of each conductor will be $r \div \sqrt{2}$. Similarly, if there are three equal sections, the current will be 33.33 amp., and the radius of the conductors $r \div \sqrt{3}$.

The induced volts as given by formula (9) work out as follows for the three conditions:

Single pair of lines

$$e = 68.34 \text{ volts} \tag{10}$$

Two pair of lines of equal total cross-section,

$$e = 36.25 \tag{11}$$

Three pair of lines of equal total cross-section,

$$e = 25.00 \tag{12}$$

These figures show that the inductive drop of pressure on a single-phase transmission may be reduced by splitting up the current and transmitting along two or more pairs of lines spaced sufficiently far apart to prevent appreciable magnetic interference between the sets of lines; and the reduction of the inductive drop is very nearly in proportion to the number of subdivisions of the single line.

Although electric transmission systems have been arranged with two distinct sets of conductors run upon separate pole lines spaced sufficiently far apart to avoid magnetic interference, such an arrangement is necessarily costly. Consider, therefore, two alternative arrangements, shown in Figs. 6 and 7, by which a single circuit can be split up into two parallel circuits, the four wires being carried on the one set of poles with the spacing between the individual wires as small as possible—that is, such that in no case shall the distance d between outgoing and return conductors be less than the minimum determined by the voltage of the supply.

In Fig. 6 is shown a symmetrical arrangement with the four conductors of equal cross-section occupying the corners of a square; the outgoing conductors are marked 1 and 3, and the return conductors, 2 and 4. Even if the two circuits 1–2 and 3–4 are connected in parallel at both ends of the line, the symmetry of the arrangement will insure that the total current will divide itself equally between the two sets of conductors. The effective or resultant magnetic flux surrounding any one conductor will, for the same reason, be equal to that which

surrounds any one of the remaining three conductors. It will, therefore, suffice to calculate the e.m.f. of self-induction generated in any one conductor.

Consider the conductor 1, in which there is the current $\frac{I}{2}$. If the other outgoing conductor, 3, were situated anywhere on the dotted circle of radius d, passing through 2 and 4, then the magnetic effect of the current in 3—so far as conductor 1 is concerned—would counteract the effect of the return current in either 2 or 4. On the basis of the data previously assumed, the flux around 1 would generate an e.m.f. of 36.25 volts, as in equation (11). If, on the other hand, conductor 3 were coincident with 1, there would be the condition of the full current I in the conductor 1, the whole of which would be returning at a distance d, and the induced volts would be 68.34, as given in equation (10). With the conductor 3 situated at a distance $\sqrt{2}d$ from conductor 1, as shown in Fig. 6, the resultant effective flux surrounding conductor 1 may be considered as the difference between the flux due to a current I up to a distance d less the flux due to a current $I/2$ up to a distance $\sqrt{2}d$; and this resultant flux would produce a back e.m.f.

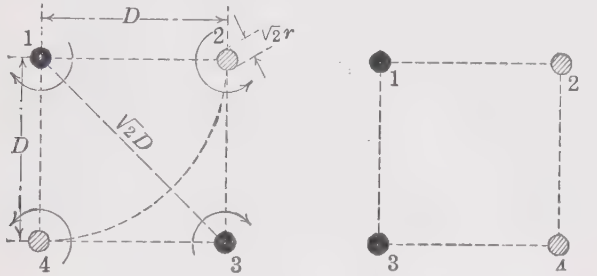

FIG. 6. FIG. 7.

FIGS. 6 and 7.—Alternative arrangements of conductors. Single-phase transmission.

$$E = aI \log \frac{d}{r \div \sqrt{2}} - a\frac{I}{2} \log \frac{\sqrt{2}\, d}{r \div \sqrt{2}} \tag{13}$$

On the data previously assumed, the e.m.f. is

$$E = 72.5 - 38.39 = 34.11 \text{ volts.} \tag{14}$$

Thus, by arranging the conductors of the divided circuit in the manner shown in Fig. 6, which permits of the four wires being

supported on the one set of poles, a better result is obtained in regard to inductive voltage drop than if the two circuits had been run entirely separately; the voltage drop in this latter case being 36.25, as in equation (11).

If, on the other hand, the position of one pair of conductors be assumed to be reversed, as indicated in Fig. 7, then the magnetic flux in the loop formed by the outgoing and return conductors 2 and 3 has no effect on the conductors 1 and 4, and the effective flux surrounding any one conductor is clearly that due to a current $\frac{I}{2}$ returning at a distance $\sqrt{2}d$: the induced volts per conductor will be 38.39, this being the value of the second term in formula (13).

With an arrangement of conductors, as in Fig. 7, it is obvious that the conditions are worse than if the two circuits are quite distinct, because a portion of the flux produced by one pair of conductors, such as 3 and 4, passes also through the loop 1–2, thereby increasing the inductive drop in these wires.

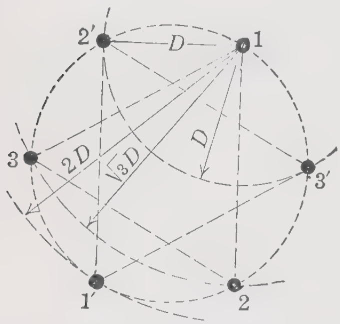

FIG. 8.—Arrangement of conductors—three-phase transmission.

Three-phase Systems.—The satisfactory results obtained in regard to inductive drop when a single-phase circuit is split up into two circuits arranged as indicated in Fig. 6, suggest that a somewhat similar arrangement might be adopted with advantage in the case of polyphase transmissions. An arrangement of wires suitable for three-phase transmission is shown in Fig. 8. Here the three-phase line is supposed to be split up into two parallel three-phase circuits, 1, 2, 3 and 1′, 2′, 3′. The arrangement being symmetrical and all conductors being assumed to be of equal size, the same amount of current will be carried by each of the six conductors, provided the load is a balanced one, such as is usual in the case of a three-wire, three-phase system.

With the arrangement of wires as in Fig. 8 the minimum distance d is maintained between all wires at different potentials, and the current in conductors, such as 1 and 1′, placed at opposite

ends of a diameter, will be of the same time-phase and equal in magnitude.

It will be interesting to work out a numerical example based on data already assumed in connection with the single-phase transmission, namely, a current of 100 amp. per phase and a minimum distance, d, of 70 in. between conductors at different potentials. The points to bear in mind are:

(1) That owing to the symmetrical arrangement of the conductors, with the rotation of the phases always in the same direction, the total effective magnetic flux around any one conductor is the same (except in regard to phase) as that which surrounds any one of the other five conductors. The calculations can therefore be made for any one conductor, such as No. 1.

(2) That the current in any outgoing conductor, such as 1, may be considered as returning through the five remaining conductors, due attention being paid to phase relations.

(3) That the resultant of the currents in conductors 2′ and 3′, or the resultant of the currents in conductors 2 and 3, is equivalent to a current equal to that in conductor 1, but of opposite phase. The total effective flux around conductor 1 may, therefore, be considered as the resultant of three component fluxes:

(a) A flux due to a current $\frac{I}{2}$ returning (through $2'-3'$) at a distance d; *plus* (b) a flux due to a current $\frac{I}{2}$ returning (through 2–3) at a distance $\sqrt{3}d$; *less* (c) a flux due to a current $\frac{I}{2}$ returning (through 1′) at a distance $2d$.

The numerical values for the induced volts are found to be: $(a) = 36.25$ [being the same as in equation (11)]; $(b) = 39.47$; $(c) = 40.49$; and $(a) + (b) - (c) = 35.23$.

If two separate three-phase lines spaced a considerable distance apart were substituted for the arrangement in Fig. 8, the induced volts per mile per conductor would be as given in equation (11), namely, 36.25, assuming the triangular arrangement of wires, with distance d between them. The arrangement shown in Fig. 8 is therefore slightly better from the point of view of inductive drop, notwithstanding that both sets of wires can be run on the same pole line with no greater spacing between wires than the minimum distance d determined by the voltage between phases. The fig-

ure 35.23 volts for the split three-phase system may be compared with 34.11 volts as given in equation (14) relating to the single-phase transmission with two circuits. It is clear that in either example, the drop in volts per conductor in the undivided circuit, with each conductor of sufficient section to carry the total current of 100 amp., would be 68.34, as given by equation (10).

APPENDIX II

SPECIFICATION FOR WOOD POLE TRANSMISSION LINE

1. General Description of Transmission Line.—This transmission line, which is 25 miles long, connects the water power generating station at . . . . . in the mining district of . . . . . . with the substation at the mines. The system will be three phase with a pressure of 22,000 volts between wires supported on wood poles. The conductors will be No. 2/0 stranded aluminum. The average span will be 150 ft., and the separation between wires will be 3 ft., the three conductors being arranged in the form of an equilateral triangle with one conductor at the top and the remaining two conductors below, at the ends of a wooden cross-arm, all as shown on drawing No. . . . . . .

Where long spans are necessary, a double-pole arrangement, as shown on drawing No. . . . . . ., will be adopted. Particulars relating to special precautions and methods of procedure in the case of exceptionally long spans, will be found in clause (8) under the heading "Spans."

There will be no telephone wires supported on the transmission-line poles.

There will be no grounded guard wire above the conductors, but galvanized iron lightning rods, as shown on drawing No. . . . ., will be fitted to every third pole on the average. Further particulars relating to protection against lightning are given in clause (7) under heading "Grounding." For particulars of sags and tensions, refer to clause (12) under heading "Stringing of Wires."

2. Clearing.—The width of the right-of-way shall be 100 ft., and all lumber, brush and other growth of every description must be cut and cleared so that, in no portion of the right-of-way, shall the tops of tree stumps, undergrowth or bush be higher than 18 inches above ground level.

At all points where a space of 50 ft. on each side of the pole line is insufficient to prevent possible damage to wires by falling trees, the normal width of the clearing must be exceeded.

All useful material shall be separated and suitably stacked at a safe distance from waste material piled for burning.

All tops, limbs, brush and other waste shall be burned, great care being taken to prevent spread of fire beyond the limits of clearing. Suitable fire-fighting appliances shall be kept at hand while burning is proceeding.

3. Poles.—Cedar poles shall be used when obtainable; but, owing to difficulties of transport, it is proposed to make use of the poles (mainly pine) obtained in the neighborhood of the transmission line while clearing the right-of-way.

Dimensions.—The greater number of the poles required will be 35 ft. long: they shall be sawn square at both ends. These poles shall measure not less than 24 in. in circumference at the top under bark, and not less than 38 in. under bark 6 ft. from butt. The approximate number required will be 840. In addition to these, about 100 poles 45 ft. long will be required, and these shall measure not less than 24 in. in circumference at top and 42 in. 6 ft. from butt.

Quality.—All poles to be cut of best quality live green timber, well proportioned from butt to top and well seasoned; the bark to be peeled, and all knots and limbs closely trimmed. The poles shall be reasonably straight, and no poles having short crooks or a reverse curve will be accepted. The amount of "sweep" measured between six-foot mark and top of pole shall not exceed 8 in. in the 35-ft. poles, or 11 in. in the 45-ft. poles.

Twisted Poles.—No poles having more than two complete twists in the total length, and no cracked poles will be accepted.

Dead Poles.—No dead poles or poles having dead streaks covering more than one-quarter of their surface will be accepted.

Butt Rot.—This must not exceed 10 per cent. of the cross-section of the pole, and the diameters of poles with butt rot or hollow hearts must be substantially greater than the corresponding diameters of sound poles. Poles with hollow hearts exceeding 8 in. in diameter will not be accepted. If average diameter of rot does not exceed 6 in., the butt measurement must be 2 in. greater than in the case of sound poles. If the average diameter of rot is 7 in., the butt measurement must be 4 in. greater.

Miscellaneous Defects.—Poles with sap rot, woodpeckers' holes, plugged holes, also poles that have been attacked by ants, worms, or grubs, are liable to be rejected as unsuitable.

The treatment of all poles before erection shall be as follows: The gains shall be sawn square with the axis of the pole and in such a position that, when erected, the curvature of the pole

(if any) shall be in the direction of the line. The position of the gains is indicated on the accompanying drawing No. . . . . showing the standard pole construction. The gains shall be not less than $\frac{5}{8}$ in. and not more than $\frac{7}{8}$ in. deep; they shall be accurately cut so that the cross-arms will have a driving fit, and the holes for the $\frac{5}{8}$-in. bolts securing cross-arm to pole shall be bored *after* the cross-arm has been fitted in position. These holes, together with all other necessary holes, as indicated on drawings, shall be bored clean and true without splintering. The holes for lag screws securing braces to poles shall be bored after braces have been fitted to cross-arms; they must be small enough in diameter to ensure that the threads of the lag-screw shall engage properly in the wood.

The butts of all poles, together with the gains and tops, shall be treated with two coats of coal-tar-creosote oil, heated to about 220° F. and applied with a brush. At least 24 hours must be allowed to elapse between applications. The painting of the butts shall be carried at least 18 inches above ground level.

4. Cross-arms.—The cross-arms shall be of yellow birch, Oregon fir, or long-leaf yellow pine, well seasoned, close grained, and free from knots or sap wood. They must be dressed on all sides. They must measure $4\frac{1}{2}$ in. deep by $3\frac{1}{2}$ in. wide, and be bored, as indicated on drawing No. . . . ., with templet, true and symmetrical: the holes to be bored clean and without splintering. After having been bored, the cross-arms shall be painted with two coats of good asphaltum paint. In cases where double cross-arms are required, it will be necessary to bore the standard cross-arms with additional holes for the $\frac{3}{4}$-in. spacing bolts, the position of which is shown on the pole drawings previously referred to.

5. Grading.—An effort should be made to maintain as far as practicable an even grade. By carefully choosing the location of each pole so as to avoid the highest points and greatest depressions when passing over uneven ground, it may be possible to avoid the use of poles differing in length to any great extent. Should it be necessary to shorten a pole, this must be done by sawing a piece off the butt end; but unless this is done before the treatment with preservative liquid, the butt must receive a further treatment with the creosote oil before erection of the pole. In some cases where the ground is favorable, the shortening of poles may be avoided by digging the hole deeper than would

otherwise be necessary. When using shortened poles, and when passing over uneven ground, it is important to bear in mind that under no condition shall the bottom conductors hang lower than 18 ft. above the ground, and when crossing tote roads or public footpaths, the minimum distance between wire and ground shall be 21 ft.

6. Pole Setting.—Where poles are set in good solid ground, the depth of holes shall be as follows:

35-ft. poles on straight runs	5¼ ft.
45-ft. poles on straight runs	6 ft.
35-ft. poles at corners or where stresses are excessive	6 ft.
45-ft. poles at corners or where stresses are excessive	6¾ ft.

If the ground is soft, the depth of setting shall be 6 in. greater than when setting in solid ground. If the soil is very soft, but not such as would be described as swampy, one or more transverse logs may be bolted to the butt of the pole in order to obtain additional bearing area.

When erected in solid rock, the depth of hole shall not be less than 3¾ ft.

In loose or sandy soil, the sand barrel or its equivalent should be used. This must be filled with a firm soil which may contain stone or rock.

In swampy ground the base of the pole must be provided with an arrangement of transverse timbers securely braced to the pole, in addition to which the hole shall, if necessary, be lined with sheet piling and filled with good soil which may contain stones or rock. As an alternative, a stone- or rock-filled crib may be built round the butt of the pole above ground level. In some cases concrete may be used with advantage in the pole foundation, but it will generally be found that the use of concrete can be avoided.

Poles must not be set along the edge of cuts or embankments or where the soil is liable to be washed out, unless special precautions are taken to ensure durable foundations.

When setting the poles in good ground, the holes shall be dug of ample size to allow of easy entrance of the butts, and the size at bottom must be large enough to admit of the proper use of tampers. When back-filling holes, there should be not less than three tampers to one shoveller, in order to ensure that the dirt shall be packed tight. In no case must the earth be thrown in to a greater depth than 6 inches without being tamped hard before

the next layer is thrown in. The proper filling of holes is a matter of great importance. When the filling is properly done, it should not be necessary to remove any excess soil; this should be packed firmly around the pole, the object being to raise the level of the ground near the pole and so cause water to drain from, rather than toward, the butt.

When setting poles on a straight run, the lining up should be done with a transit, and the poles placed with cross-arms truly at right angles to the direction of the line. Where the direction of the line alters, the poles at the angles must be set so that the cross-arm halves the angle. If the deviation exceeds 5 degrees, the corner poles shall be provided with double cross-arms and fixtures. When possible, the cross-arms, braces, and other fixtures (but not the insulators) should be mounted on the poles before erection.

7. Grounding.—The proper grounding of lightning rods on the pole line is a matter of importance. Judgment must be used in determining when and how to ground the poles; but either of the following alternative methods will be considered satisfactory, provided the soil is reasonably moist:

(1) A piece of galvanized iron pipe 1½ in. in diameter and 8 or 9 ft. long shall be buried in the hole alongside the butt or driven into soft soil, the ground wire being attached thereto in such a manner as to ensure a good and enduring electrical contact.

(2) The ground wire, consisting of $\frac{5}{16}$ in. galvanized stranded steel cable, after being carried straight down the side of the pole and secured with cleats, shall be wound spirally around the butt and carried right down to the bottom of the pole. Not more than 15 ft. of wire should be buried in the ground.

It is of little use to ground a pole in solid rock, but where a pole is set in rock, it may be found that the ground wire can be carried down the face of the rock, or in a crevice, to a point where a good ground can be obtained. Where grass is growing, the soil will usually contain sufficient moisture to afford a reasonably good ground. When the ground wire does not enter the ground alongside the pole, sudden bends or turns should be avoided in the wire connecting the lightning rod with ground plate or pipe.

It is not intended to provide all poles with lightning rods; but, except when the soil is clearly unsuitable for a ground connection, the poles in the positions described below shall be grounded:

Both poles supporting extra long spans requiring the double pole arrangement as shown on drawing No. .... previously referred to.
The poles on each side of railway crossings.
All guyed poles.
The six poles nearest to generating station.
The six poles nearest to substation.

In addition to the above-mentioned poles, one pole out of every three poles shall be grounded. It is not necessary that every third pole be grounded: judgment must be used in determining the location of the poles to be grounded. As a general rule, it is more important to ground poles on heights and in exposed positions than those on the lower ground; but, on the other hand, it is of little advantage to ground where the soil is dry or otherwise unsuitable. In exposed positions it may be advisable to ground two or more consecutive poles, while in unexposed positions four or five consecutive poles may be left without lightning rods.

8. Spans.—The standard length of span shall be 150 ft. Shorter spans must be used at angles and on curves, as mentioned in clause 9. If the span exceeds 170 ft. the poles must be specially selected for strength. No span greater than 190 ft. shall be carried on single poles. For longer spans, the double-pole arrangement as shown on drawing No......previously referred to, shall be adopted, with a horizontal spacing of 5 ft. between wires for spans up to 600 ft.; but spans exceeding 500 ft. shall be avoided if possible.

Railroad Crossings.—(*a*) The span where line crosses railroad shall be kept as short as possible; but in no case must a pole be placed a smaller distance than 12 ft. from the rail, except in the case of sidings, where the distance may be reduced to 6 ft. At loading sidings sufficient space must be allowed for a driveway between rail and pole. When possible the distance between rail and pole should not be less than the height of the pole, but if this spacing requires a span greater than 120 ft., it will be preferable to place the pole nearer to the rail provided the ground is suitable. If it is necessary to cross the railroad with a span greater than 150 ft., the double-pole arrangement as used for extra long spans, and as shown on drawing No......shall be adopted.

(*b*) In all cases the cross-arms and insulators shall be doubled on the poles nearest the rail.

(*c*) The poles at railroad crossings must be set not less than 6 ft. in the ground (4 ft. in rock).

(*d*) If the crossing is at a spot where grass or other fires might cause injury to the poles, these shall be provided with a casing of concrete at least 2 in. thick, to a height of 5 ft. above ground level.

(*e*) The clearance between rail and high-tension conductor shall not be less than 30 ft., and the poles should be specially selected for strength and straightness.

(*f*) When crossing over telephone wires, the clearance shall be not less than 10 ft.

(*g*) The poles at railway crossings must be securely guyed, whether or not there is a bend in the line. If a departure from the straight run is necessary, special attention should be paid to the method of guying.

(*h*) The poles on each side of the rail shall be provided with lightning rods, and well grounded. Bent iron lightning guards, as shown on drawing No...... shall be fixed at each end of cross-arm and connected to the ground wire; these will also serve the purpose of hook guards, to engage the conductor if it should become detached from the insulator. If the nature of the soil is quite unsuitable for the purpose of grounding, the lightning rod may be omitted; but if the pole is not grounded, two strain insulators must be placed in each guy wire securing poles nearest to rail; the upper of these insulators being not less than 6 ft. distant from the lowest high-tension conductor, and the second insulator being not less than 8 ft. above ground level.

(*i*) Special attention shall be paid to the tying of the conductors to the double insulators on the poles at each side of the rail. As a protection against damage by arcs over insulators, the serving of No. 2 aluminum tie wire shall be carried far enough to ensure that the conductor is protected by the serving or tie to a distance of not less than 12 in. from the center of insulator.

(*j*) In addition to the pole number, the poles on each side of the crossing shall bear a label with the Company's name and the voltage (22,000 volts) painted thereon in easily distinguishable characters.

9. Angles and Curves.—Whenever there is a change in the direction of the line, a sufficient number of poles must be provided to prevent the angle of deviation on any one pole exceeding 15 degrees. If the deflection from the straight run does not exceed

5 degrees it is not necessary to use a pole with double fixtures. When the deflection exceeds 5 degrees, poles with double fixtures shall be used, and these must be side guyed. When the "pull" at corner pole exceeds 2 ft. the span on each side of pole shall be less than 150 ft.; the reduction in the length of span being at the rate of about 2½ ft. per foot of "pull," all as indicated in the table accompanying Fig. 1. Should it be necessary to turn the line at a point where space is limited, through an angle greater

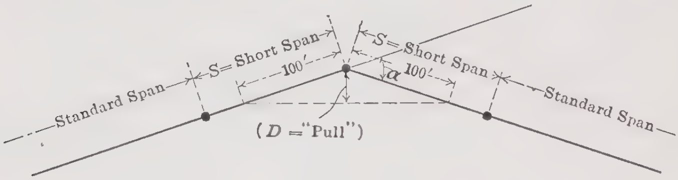

FIG. 1.

Limit of length of spans on each side of angle pole (standard span = 150 ft.).

"Pull" D feet	Deflection α degrees	Short span, S, not to exceed: feet	Remarks
2	2°–18′	145	Double fixtures not necessary.
3	3°–26′	143	
4	4°–36′	140	
5	5°–44′	138	
6	6°–52′	135	Use double fixtures on poles. Side-guy.
7	8°–00′	133	
8	9°–10′	130	
9	10°–20′	128	
10	11°–28′	125	
11	12°–38′	123	
12	13°–47′	120	
13	14°–56′	118	

than 15 degrees, two or more poles with double fixtures may be set close together, each pole being side guyed, or securely braced. In all cases where there is a departure from the straight line, the poles must be set so that the cross-arms will bisect the angle.

10. Guying.—The material to be used throughout for guys is 5/16-in. galvanized seven-strand steel cable. Where the wire is wrapped around the pole, a protecting strip of No. 24 galvanized sheet iron shall be put under the wire. The wire shall make two complete turns about the pole.

The anchoring shall generally be done by burying an anchor log from 4 ft. to 6 ft. long, and bolting thereto a $\frac{5}{8}$-in. guy rod. Other methods may have to be adopted to suit the varying nature of the ground, but in all cases it is important to ensure a good hold and to see that the guy rod is in line with the guy wire. The angle of the guy wire when anchored in the ground shall be approximately 45 degrees where circumstances permit.

No strain insulators shall be used on guy wires, except as called for at railway crossings; but all guyed poles shall be provided with lightning rod and be well grounded. It is not intended that work be done on live wires on guyed or other grounded poles. As a general rule all poles shall be guyed before the conductors are strung. Poles must be guyed at all points as mentioned below:

(*a*) At angles exceeding 5 degrees.

(*b*) Where the line goes up a 15 per cent. or steeper grade (head guys every fifth or sixth pole, or only at top of hill on short lengths).

(*c*) On hillsides where the footing may be good, but where there is danger of slipping stones or soil producing side pressures on the pole (side guy).

(*d*) At each end of exceptionally long spans, where double poles are used.

(*e*) All poles with double fixtures.

11. Insulators.—The line insulators will be supplied by Messrs. .... They will be of the pin type, the pins having porcelain bases with wood thimbles and $\frac{5}{8}$-in. galvanized iron bolts for fixing to cross-arms. The pole-top insulators will be supported on malleable-iron pole-top pins, and the separable thimbles of these pole-top pins will be cemented into the insulators at the makers' works. The insulators shall be mounted on the cross-arms after the poles have been erected. The pole-top insulator pins may be bolted in position before erection of pole.

12. Stringing of Wires.—No. 2/0 seven-strand bare aluminum cable will be used throughout. Care must be used in handling the conductors, to guard against cuts or scratches or kinks. The conductor must not be drawn over rough or rocky ground where it is liable to be injured by stones, etc.

It is important that the cables be pulled up to the proper tension so that the sag will be in accordance with the particulars

given on the curves Figs. 2 and 3. These curves give not only the correct sag at center of span, but also the required tension in the cable at the time of stringing. The curves are calculated for wires subject only to their own weight and hanging in still air.

In the case of extra long spans, and where the grade is not constant, it will generally be found more convenient and quicker to adjust the tension by means of a spring dynamometer than by measuring the sag.

The cables must not be pulled around insulator pins on angle poles.

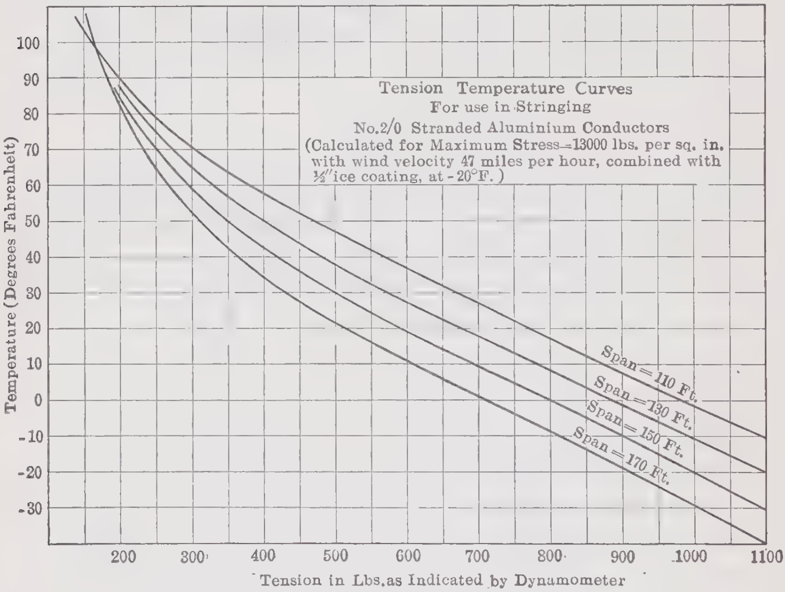

Fig. 2.—Chart giving tension at which wires should be strung.

The tie wire shall be No. 2 B. & S. solid, soft aluminum wire. The tie on straight runs shall be of the type known as the armor top, with the conductor in the groove on top of insulator. At corners, the tie shall be of the type known as the armored Western Union, with the conductor carried around the insulator in the side groove. The tie shall be a modification of the type used by the Niagara, Lockport and Ontario Power Co., between Niagara Falls and Buffalo. The serving of No. 2 tie wire on the conductor is for the purpose of preserving the latter from abrasion and from damage due to possible electric discharges over the insulator. The use of pliers should be avoided in making the ties, except for

the final clinching, when they must be used with care to avoid cutting or otherwise injuring the conductor.

When joints are required in the conductors, they shall be made with MacIntyre tubes which shall be given two twists with the splicing clamps provided for the purpose.

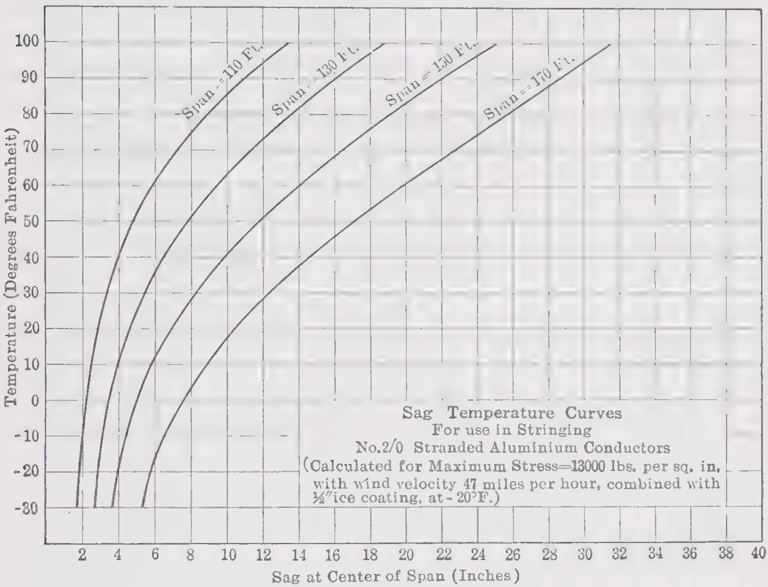

Fig. 3.—Chart giving sag in wires when correctly strung.

13. Locating and Numbering Poles.—All poles shall bear a distinguishing number in clear bold figures about 10 ft. above ground level. These numbers will correspond with the numbers on the plans which will be prepared as soon as possible after the poles have been erected in position. The plans will be drawn to a scale sufficiently large to show the location of each pole.

APPENDIX III

SPECIFICATIONS FOR STEEL TOWER TRANSMISSION LINE

These specifications, which are preliminary specifications subject to revision in minor details after bids for the various materials have been received and considered, cover the construction of an overhead transmission line connecting the generating station at . . . . with distributing station at . . . . . . , a distance of approximately 60 miles as measured along the right-of-way of the transmission line.

Methods of construction are not dealt with in detail, because the work in the field will be in the hands of a competent and experienced construction engineer who will be allowed considerable latitude in regard to the actual handling of materials and in deciding upon the best methods to be adopted in the erection of towers, stringing of conductors, and other details of practical line construction.

General Description of Line.—Two three-phase circuits of No. 2/0 equivalent copper cable will be run in parallel on one set of steel towers spaced approximately eleven to the mile. The towers will be of the semi-flexible type, with rigid strain towers at intervals of about a mile, or more frequently where corners or extra long spans render their use necessary. A 7/16-in. galvanized Siemens-Martin steel-strand cable will be carried the full length of the line and be firmly secured to the top of each tower.

The pressure between conductors will be 80,000 volts, and the suspension type of insulator will be used throughout.

Details of entering bushings and methods of connecting lightning arresters at generating and receiving stations are not dealt with in these specifications as they come under another contract.

The proposed line has been staked out by the stadia survey party, and the right-of-way secured where necessary. Stakes have been driven to indicate proposed location of towers, but these positions are subject to modification.

The line passes through country that is for the greater part uncultivated; the ground is undulating and in some parts wooded. A considerable amount of clearing has yet to be done. Roads are bad; but the transmission line is within 2 to 3 miles of the railway at all points.

Duties of Engineer in Charge of Construction.—Before the work of construction is begun, the construction engineer will go over the line as staked out by the preliminary survey party and as shown on plan No. . . . . . herewith. He will take with him an engineer equipped with a light transit, and an assistant capable of acting as axeman or rodman as circumstances may require. The construction engineer will decide in the field the position of each tower, making changes in the preliminary plan in the matter of tower locations and even to a small extent in the route to be followed, if in his opinion such changes will result in a better and more economical line. Hub-stakes shall be driven to mark the center-point of each tower, and a second stake shall be driven about 12 ft. ahead or in the rear of the hub-stake in the direction of the line. This is for reference when setting the anchor stubs.

The construction engineer must check clearances between conductor and ground, and on long spans, especially if there is doubt as to position and amount of minimum clearance, he should take the necessary particulars to allow of the matter being settled in the office. After agreeing and checking the alterations to plan in the office, the construction engineer will assist in making out the shipping schedules for delivery of materials at the most suitable points.

In regard to the work of erection proper, the construction engineer will attend to all details of organization of the parties in the field, and will study the best means of distribution of materials along the line; all with the view of avoiding unnecessary expenditure, and of carrying out the work expeditiously and in a workman-like manner. Such details as the actual methods to be adopted in the erection of towers and stringing of wires will be decided upon after discussion with the chief engineer, and after due weight has been given to manufacturers' suggestions.

Clearing.—On those parts of the line on which clearing is required, it is proposed that this work be done immediately after the line has been finally staked out. This clearing will extend 60 ft. on each side of the center line of the right-of-way, and it will be carried out under a separate contract.

Towers.—Two standard types of steel tower will be used: these will be referred to as the strain type and flexible type respectively. Copy of the specification on which bids will be obtained from manufacturers is attached hereto.

Foundations for Towers.—The use of concrete is to be avoided, but in marsh land or loose soil concrete footings may be necessary. The decision as to where concrete is to be used will rest largely with the engineer in charge of construction in the field.

When the tower stands on solid rock—which may occur in a few instances—the standard footings will not be used; but a special wedge bolt, shaped at the top to take the standard tower, will be grouted in with sulphur or other approved cement. In levelling up on rock foundations, it may sometimes be cheaper to build up one or two piers of concrete, securely tied down to the rock, rather than level off the rock on the high side.

In selecting sites for towers, the construction engineer shall pay attention to the matter of foundations, and endeavor to secure sites where the foundations are good. Hillsides are to be avoided, especially where the soil is liable to crumble or slide. The matter of grading should also be considered when finally selecting sites: much may be accomplished in the judicious selection of tower sites by slightly adjusting the length of span to obtain sites which will tend to equalize the grade.

To facilitate the work of erection, a wooden digging templet will be provided, together with a rigid but light-weight angle steel templet to ensure the correct placing of the anchor stubs; the latter being bolted to the templet before the work of backfilling the holes is commenced.

The second stake which, as previously mentioned, will be driven truly in line with the hub stake, will be used for the correct setting of these templets. The steel templet must be carefully leveled up in order that the center line of the tower shall be vertical.

Grounding.—In cases where the iron work of the foundations is completely encased in concrete, the tower shall be well grounded by means of a 10-ft. length of 1-in. galvanized-iron pipe driven or buried in the ground, and electrically connected to one of the tower legs. When the tower stands on rock, an effort should be made to obtain a good ground by carrying a length of the galvanized guy wire from the tower leg to a rod driven in damp soil at a short distance from the tower if a suitable spot can be found.

Guying.—Where guy wire is required, the 7/16-in. Siemens-Martin steel ground wire shall be used. When the distance between strain towers exceeds 3/4 mile, one flexible tower situated about midway between the strain towers shall be head-guyed in both directions. Flexible towers used at corners where the deviation lies between 5 and 8 degrees, and the approach spans are of normal length, shall be guyed with two guy wires so placed as to take the corner strain and resist overturning of the tower owing to the resultant pull of the wires.

Angles.—The semi-flexible support is designed for use on straight runs only; but if the deviation from the straight line does not exceed 5 degrees, these intermediate supports may be equipped with strain insulators and used at corners. For angles greater than 5 degrees, but not exceeding a limit of 8 degrees, these supports may be used with two guy wires to take the transverse stress due to the resultant pull of the wires. If the approach spans are reduced to 240 ft., an 8-degree curve may be turned on a semi-flexible structure without guy wires.

Strain towers shall be used for turning corners up to 30 degrees; but when the total deviation exceeds this amount, two towers must be used.

Erection of Towers.—The actual organization of the various crews for distributing material, setting anchor legs, assembling and erecting the towers, will be left to the engineer in charge of construction, who will so conduct operations as to carry out the work efficiently at the lowest possible cost.[1]

Insulators.—These will be of the suspension type to comply with the requirements detailed in the insulator specification of which copy is attached hereto.

Conductors.—The conductors shall be 19 strand hard-drawn copper cables equivalent in section to 00 B. & S. gauge. The tensile strength of the finished cable shall not be less than 90 per cent. of the strength of the individual wires forming the cable; and these shall satisfy the strength requirements of the standard specification drawn up by the American Society for Testing Materials.

[1] Mr. R. A. Lundquist's book on "Transmission Line Construction" is of special interest to the engineer in charge of construction. Excellent articles describing practical methods of construction also appear from time to time in the technical press. The article by Mr. A. B. Cudebec on "Steel Tower Transmission Line Construction" in the *Electrical World* of July 17, 1915, contains much valuable information.

The electrical conductivity shall not be less than 97 per cent. by Matthiessen's standard.

The total weight of copper conductor required is estimated at 800,000 lb. It shall be delivered on drums or reels each containing 1 mile of cable.

Joints in Conductors.—The splices shall be made with copper sleeves of the "MacIntyre" or similar approved type. The finished joint shall consist of 3 turns. The tools provided for the purpose shall be used in making the joints.

Spans and Wire Stringing.—The actual method of stringing the wires will be left to the judgment and experience of the construction engineer. It is, however, suggested that three conductors be drawn up at a time, using the arrangement of sheave blocks known as an "equalizer."

The average span shall be approximately 480 ft. This may be increased to a limit of 500 ft. between flexible supports, and to a limit of 1200 ft. between two strain towers without intermediate supports. It is thought that three or four points on the line may advantageously be spanned between two strain towers placed from 1000 to 1200 ft. apart. In the case of abnormally long spans, it is important to see that the contour of the ground is such as to allow of maximum sag while maintaining the specified minimum clearance between H. T. conductors and ground.

The clearance between lowest wire and ground shall in no case be less than 28 ft.

The charts Nos. . . . . . . and . . . . give all necessary particulars for the stringing of conductors and guard wire at various temperatures. The guard wire connecting the tops of all towers shall be strung and securely clamped to the steel structure before the conductors are drawn up. Dynamometers will be provided, and their use is recommended, especially when spans are unequal in length, and on extra long spans between two strain towers. If an equalizer is used, it is not necessary to insert a dynamometer in more than one leg. Special attention shall be paid to the drawing up of cables to the proper tension or sag. Too great a sag is almost as objectionable as too great a tension; but it must be remembered that where a dip occurs in the line of supports there is sometimes a possibility, in very cold weather, of the conductor being drawn up (by contraction) above the proper level of the lowest insulator. The construction engineer should watch for this possibility with a view to guarding against it.

When the conductors have been drawn up and transferred from snatch block to insulator clamp, it is important to see that the suspension insulator hangs truly vertical before finally tightening up the clamp.

There will be no transpositions on the H. T. conductors. The telephone wires are run on a separate set of wood poles and they will be transposed at every support.

SPECIFICATION FOR STEEL TOWERS

These towers are for use on an 80,000-volt, three-phase transmission line using insulators of the suspension type. It is proposed to use two standard types of towers only; these will be referred to as the rigid or strain towers, and the flexible towers.

The strain towers shall be designed with four corner legs and square bases, generally as indicated on plan No. ..... herewith. An effort will be made to avoid the use of special structures, and where extra long spans have to be carried, two standard strain towers may be placed close together. In one or two places it may be necessary to use extra high towers, and it is proposed to use the standard tower mounted on a special base, generally as shown on plan No. ....., designed to raise the tower 18 ft., or such other amount approximating to this dimension as may best suit manufacturers' designs.

The intermediate or flexible type of support will be of the "A"-frame design, generally as shown on plan No. ...... Preference will be given to a design consisting of few parts, provided this will not add appreciably to the cost of transporting the towers over rough roads to the point of erection. The parts of all towers shall be galvanized when ready for assembling; but, in the case of the flexible type of structure, an alternative offer for painted steel work will be considered, provided the number of parts is small and the section of metal reasonably large.

The plans referred to, which accompany this specification, give all necessary leading dimensions; but the cross-section of the various members and the details of design are left to the manufacturer, who is also at liberty to submit alternative proposals. In no case must the distance between conductors be less than 8½ ft. or the height above ground of the point of attachment of insulators on lowest cross-arm less than 40 ft. The sections of structural steel used for the main corner members of the strain

towers or for the main members of the flexible towers shall not be less than $\frac{1}{4}$ in. thick, and no metal less than $\frac{3}{16}$ in. thick shall be used in the construction of these towers.

Number of Towers Required.—Offers shall be based on the following quantities, which are subject to slight modification.

Flexible towers (plan No. . . . . . .)	592
Rigid towers (plan No. . . . . . .)	65
Extension bases (plan No. . . . . . .)	4

Working and Test Loads for Towers.—The normal length of span is 480 ft. and the total vertical load per tower, consisting of six conductors and one guard wire together with estimated possible ice loading and the weight of the six insulators, is 3100 lb.; but the spans will in many cases exceed the average length. The maximum total overturning pressure in a direction at right angles to the line, due to wind blowing across the wires, is estimated at 3300 lb.; this may be considered as distributed equally between the points of attachment of the seven wires. The manufacturer should estimate the pressure of wind on the tower structure itself by allowing a maximum pressure of 13 lb. per square foot of tower surface. A factor of safety of $2\frac{1}{2}$ shall be used in making stress calculations.

One tower of each type shall be tested in the presence of purchaser's representative, and must withstand without exceeding the elastic limit of the steel, or suffering appreciable permanent deformation, the following test loads applied at the points indicated on the plans previously referred to. These tests are to be made with the tower erected on its own foundations in such a manner as to reproduce as nearly as possible the conditions under which it will ultimately be erected.

Strain Tower Test Loads.—(1) A breast pull of 15,000 lb. applied in the direction of the line at the point of attachment of the middle cross-arm.

(2) A vertical load of 1000 lb. applied at the end of any cross-arm.

(3) A torsional load of 3500 lb. applied in a direction parallel to the line at the end of any cross-arm.

Flexible Tower Test Loads.—(1) A transverse pull of 4500 lb. applied in a direction at right angles to the line at the point of attachment of the middle cross-arm.

(2) A vertical load of 800 lb. applied at the end of any cross-arm.

Metal steps shall be provided on all towers within 8 ft. of ground level for the use of linemen.

It is requested that manufacturers tendering for steel towers call attention to any features of the particular design proposed which may tend to reduce cost of transport and erection on site, as these are matters which will receive consideration when placing the contract.

Galvanizing Test.—The purchaser reserves the right to reject all towers of which the galvanizing is not of the best quality. Tests will be made before erection as follows:

Samples of steel work will be immersed in a solution of sulphate of copper (specific gravity about 1.185) maintained at a temperature of 60 to 70° F. After remaining in the solution 1 minute, the sample will be removed, thoroughly washed in water, and wiped dry. This process will be repeated four times, after which there must be no appearance of red spots indicating copper deposit.

SPECIFICATION FOR PORCELAIN LINE INSULATORS

Number of Insulators Required.—The approximate quantities required, as based on preliminary estimates are:

Suspension type	4000
Strain type	880

Climatic Conditions.—The transmission line on which the insulators will be used is located in the . . . . . . district, where severe thunder storms and heavy rain may be expected during the summer months, and where sleet storms and low temperatures are prevalent in the winter.

Working Voltage.—The transmission is three-phase off delta connected transformers, at a frequency of 60 and a maximum working pressure of 84,000 volts between wires.

Design of Insulators.—The design of the suspension type and strain insulators is left to the manufacturer, who must submit dimensioned drawings or samples with his offer. The units making up the strain insulators need not necessarily differ in design from the units of the suspension insulators, provided the latter are capable of withstanding the mechanical tests required for the strain insulators. The towers have been designed on the assumption that the weight of one complete string of unit insula-

tors will not exceed 60 lb. and that the distance between point of suspension and conductor will not exceed 36 in. These limits should not be exceeded. It is preferred that the number of units in the complete string be not less than three nor more than five.

Metal Parts.—All metal parts subject to rust and corrosion, such as malleable iron castings and steel forgings, shall be heavily galvanized and capable of withstanding the usual tests.

Glaze.—The surfaces of the porcelain not in contact with the cement shall be uniformly coated with a brown glaze, free from grit.

Cement.—Pure Portland cement only shall be used in assembling the parts of the unit insulator.

Mechanical Tests.—An inspection will be made of all insulators with the object of rejecting those containing open cracks in glaze or porcelain.

One complete suspension insulator, selected at random, and consisting of the requisite number of units, shall withstand a load of 5000 lb. without rupture or sign of yielding in any part.

At least three units of which the strain insulators are built up shall be tested to the breaking limit, and must withstand an ultimate load of not less than 12,000 lb.

Electrical Tests.—Three or four complete insulator strings, both suspension type and strain type, shall withstand without flash over a "wet" test of 200,000 volts. In all cases the electrical stress shall be applied for 1 minute, and the spray shall be directed upon the insulator at an angle of 45 degrees under a pressure of 40 lb. per square inch at the nozzles, the precipitation being at the rate of 1 in. in 5 minutes. The suspension insulators shall be hung vertically, and the strain insulators horizontally.

The connection of the test wires shall be so made as to reproduce as nearly as possible the working conditions.

The manufacturer shall satisfy himself by his standard factory tests that each unit is sound mechanically and electrically. The "dry" flashover of the complete string of insulator units shall not be less than 240,000 volts; but this test need not be made in the presence of the purchaser's representative.

The transformer used for the electrical tests must be capable of a reasonably large k.v.a. output; the e.m.f. wave shall be as nearly as possible sinusoidal, and the frequency shall be within the limits of 25 and 60 cycles.

Packing of Insulators.—It is desirable that the parts for one complete insulator, or at most for two insulators, be packed complete in a separate barrel or crate, and that the contents be clearly described on attached label.

Wire Clamps.—A suggested clamp for use with suspension insulators is shown on drawing No. . . . . . herewith; and drawing No. . . . . . shows a proposed strain insulator clamp. Makers are asked to submit samples or drawings of their standard types, preferably of the general design indicated by the above-mentioned drawings. The conductor to be carried is an equivalent No. 2-0 gauge (B. & S.) stranded copper cable; the groove for the wire should be slightly curved and flared at the ends.

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SUPPLEMENTARY INDEX

TO FORMULAS, CURVES, AND TABLES

For use in connection with practical transmission line calculations